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J. C. HULL

A Study of the Subjective Probability Assessments necessary for the analysis of the Risk in Major Capital Investment Opportunities.

Supervisors: Mr. T. Cass, Mr. R. M. Adelson

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ABSTRACT

Five case studies are analysed in depth and other investigations are carried out in order to answer questions concerned with:

(i) the nature of the distributions which are output from risk evaluation models.

(ii) the important features of the distributions which are input to risk evaluation models.

(iii) the accuracy with which different methods for assessing subjective probability distributions are capable of providing the inputs to risk evaluation models.

(iv) the way in which dependencies should be dealt with in risk evaluation models.

and (v) the extent to which it is possible to distinguish important probability assessments from unimportant probability assessments in risk evaluation models.

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FIGURES

NOTATION

n	life of investment in years
C_i	cash flow in year i
d	discount rate
NPV	net present value
IRR	internal rate of return
P	value of performance measure
X_i	value of i-th variable
E_i	most likely value of i-th variable
L_i	lower bound for i-th variable
U_i	upper bound for i-th variable
μ_i	mean of i-th variable
σ_i	standard deviation of i-th variable
ρ_{ij}	coefficient of correlation between i-th and j-th variable
μ_P	mean of performance measure
σ_P	standard deviation of performance measure
s	skewness of distribution (see page 81 for definition)
V_1	value of indepenuent variable
V_2	value of dependent variable
f	distribution of independent variable (chapter 6 only)
g	unconditional distribution of dependent variable (chapter 6 only)
h	conditional distribution of dependent variable (chapter 6 only)
$N(\mu,\sigma^2)$	normal distribution with mean μ and standard deviation σ
z	transformation necessary to convert V_1 to normal distribution (chapter 6 only)
w	transformation necessary to convert V_2 to normal distribution (chapter 6 only).

ρ^*	coefficient of correlation between $z(V_1)$ and $w(V_2)$
μ_1	mean of $z(V_1)$ (chapter 6 only)
μ_2	mean of $w(V_2)$ (chapter 6 only)
σ_1	standard deviation of $z(V_1)$ (chapter 6 only)
σ_2	standard deviation of $w(V_2)$ (chapter 6 only)
S_i	sensitivity coefficient of variable i (see page 111 for definition)
R_i	range coefficient of variable i (see page 120 for definition)
$e_{i,1}$	effect of error $\Delta\mu_i$ on μ_p
$e_{i,2}$	effect of error $\Delta\sigma_i$ on μ_p
$e_{i,3}$	effect of error $\Delta\mu_i$ on σ_p
$e_{i,4}$	effect of error $\Delta\sigma_i$ on σ_p
g_{ij}	increase in μ_p when ρ_{ij} changed from 0 to 1
h_{ij}	increase in σ_p when ρ_{ij} changed from 0 to 1
p_i	error in μ_i expressed as fraction of range
q_i	error in σ_i expressed as fraction of σ_i
k_i	ratio of σ_i to range of i-th variable
K_i	ratio of μ_i to E_i

(Other notation is defined as it appears in the text)

CHAPTER 1

REVIEW OF THE LITERATURE ON THE ANALYSIS OF RISK IN MAJOR CAPITAL INVESTMENTS

1.1 INTRODUCTION

The simplest approach to the evaluation of major capital investment opportunities involves three stages:

1. Forecasting the values of variables
2. Calculating cash flows

and 3. Calculating a performance measure.

In the first of these stages, a single 'point estimate' forecast is made for each of the variables affecting the performance of the investment. During the second stage, the forecasts are combined together to produce estimates of cash inflows and cash outflows in each year of the life of the investment and, from information on taxation rates, government investment grants etc, a series of net cash flows is produced. Finally, in the third stage, the net cash flows themselves are combined to produce a performance measure which represents in some way the benefit to be derived from the investment.

This approach provides an estimate of the investment's future performance, but it suffers from the severe disadvantage that it fails to allow for, or to provide any information about, the investment's inherent risk. The purpose of this chapter is to discuss and critically evaluate all the suggestions which have been made in the literature for overcoming this disadvantage. A recent article which covers roughly the same ground, although not in as much depth, is provided by Bonini (1975).

1.2 THE PERFORMANCE MEASURE

A satisfactory measure of the performance of an investment is necessary whatever the method of evaluation being used. Four of the most commonly used performance measures are:

Payback Period
Accounting Rate of Return
Net Present Value
and Internal Rate of Return

This section will be devoted to a brief discussion of their relative advantages and disadvantages.

Define:

n the life of the investment in years

C_i the cash flow in year i ($0 \leq i \leq n$)

(C_o is the cash flow arising from any initial capital outlay necessary for the investment).

The Payback Period is calculated as the number of years necessary for the investment to generate sufficient cash in order to cover the initial capital outlay. It has the advantage that it is easy to calculate and easy to understand, but suffers from the severe disadvantage that it only takes account of the magnitude of cash flows which occur during the payback period itself. If:

$$\sum_{0}^{k-1} C_i \geq 0$$

the cash flows from year k onwards will be completely ignored by the measure.

The Accounting Rate of Return can be defined in a number of different ways but probably the most sensible (see Franks and Scholefield (1974) is: average net annual profit during the life of the investment as a percentage of total net capital outlay. Although the measure does consider all the cash flows, it takes no account of their timing, ignoring the fact that a cash flow is worth more now than at some future date because it permits profitable consumption or investment during the intervening period.

The Net Present Value (NPV) is defined as:

$$\sum_{i=0}^{n} \frac{C_i}{(1+d)^i}$$

where d, the discount rate is chosen so as to represent the cost of capital to the company. Assuming that capital can be raised freely at 100d% interest p.a. it represents the amount of money the firm could raise (in addition to that required for initial outlays on the project) if by the end of the project's life it is to have paid off all the capital with interest. Generally a positive NPV is an indication that the project should be accepted.

The Internal Rate of Return (IRR) is defined as that value of d which is such that:

$$\sum_{i=0}^{n} \frac{C_i}{(1+d)^i} = 0 \qquad (1.1)$$

It represents the highest (net of tax) rate of interest at which the firm could raise money and still not lose thereby (providing of course that the firm has the opportunity to repay the capital whenever it chooses). The two criteria:

Accept project if NPV > 0

and Accept project if IRR > Discount rate

are in most investment situations equivalent.

There has been a great deal of discussion in the literature of the advantages and disadvantages of NPV and IRR. (See for example Merrett and Sykes (1963), Mao (1966), Teichrow et al (1965) and Adelson (1970)). At present, most writers seem to prefer NPV to IRR.

The main points which have been made are as follows:

(a) The IRR is independent of the magnitudes of the cash flows involved whereas the NPV is not. If all the cash flows (including the initial one) connected with a certain project were doubled the NPV would also be doubled but the IRR would be unaffected. For this reason the way in which NPV and IRR rank alternative projects may not always be the same: the IRR will rank projects according to the rate return and independently of the total sum of money invested whereas the NPV will rank projects according to the increase in the wealth which they bring to the company.

(b) If the investment is such that

$$C_i \leqslant 0 \quad \text{for some} \quad i \geqslant 1$$

(i.e. if it requires outlays of money at times other than the beginning of its life) then equation 1.1 may have multiple solutions and the IRR may become difficult to interpret. However, it should be emphasised that it is relatively rare for difficulties to be created in this way. Situations where the initial investment is spread out over several years do not generally lead to multiple solutions in equation 1.1. Also in other situations where an investment requires an additional injection of cash periodically it is often easy to see that the total cash flow stream can be split into several simpler ones each with the same internal rate of return (see Merrett and Sykes (1963)).

(c) The IRR cannot be interpreted as the rate of return on the capital outstanding per period unless the assumption is made that cash inflows can be reinvested at the internal rate of return.

1.3 SENSITIVITY ANALYSIS

A sensitivity analysis is a useful first step towards analysing the risk in an investment project. It involves first calculating the value of the performance measure on the basis of management's best estimates for each of the variables and then calculating the effect on the performance measure of errors in each of the best estimates. Suppose:

$$P = f\,(X_1, X_2 \cdots\cdots X_m)$$

where P is the value of the performance measure and X_i is the value of the i-th variable (i=1, 2...... m). Suppose further that the best estimate of X_i is E_i (i=1, 2....m). The effect on P of an error ΔE_i in E_i is:

$$f(E_1, E_2 \ldots E_{i-1}, E_i + \Delta E_i, E_{i+1} \ldots E_m) - f(E_1, E_2 \ldots E_m)$$

Some authors (e.g. House (1966, 1968) have considered ΔE_i to be a certain fixed percentage of E_i for all i. However this is not the best way of proceeding as it does not relate the errors which are considered to the uncertainties which management attach to their estimates of the individual variables. Ideally management should make, in addition to the best estimate E_i, a 'pessimistic estimate' L_i and an 'optimistic estimate' U_i for each variable. Measures such as:

$$S_i = f(E_1, E_2 \ldots E_{i-1}, U_i, E_{i+1} \ldots E_m) - f(E_1, E_2 \ldots E_{i-1}, L_i, E_{i+1} \ldots E_m)$$

can then be used to provide an indication of the relative importance of the uncertainties in different variables. (The precise meaning of the terms 'optimistic estimate' and 'pessimistic estimate' need not concern us at this stage. In order that the S_i (i=1, 2....m) are directly comparable it is only necessary for management to be consistent in their use of the terms. If the optimistic estimate is considered to have only a five per cent chance of being bettered for one variable, this should also be true for other variables etc).

The optimistic-pessimistic procedure does of course require management to make more assessments than House's procedure. However, it is difficult to see how Houses's results could be meaningfully interpreted without the extra assessments being made either explicitly or implicitly. It should also be noted that:

$$\frac{\partial P}{\partial X_i}$$

may depend on the value of X_i (Consider, for example the situation where P is NPV and X_i is sales growth rate). The results produced using the optimistic-pessimistic estimate approach cannot therefore in general be deduced exactly from those produced using House's method.

Although a sensitivity analysis (if carried out correctly) can provide an indication as to the relative importance of the uncertainties in different variables, it provides no real indication of the total risk in the investment resulting from management's uncertainties about all the variables. An additional analysis can of course be carried out to investigate the effect of several different errors occurring at the same time, but there are usually so many different possible combinations of errors - some of them more probable than others - that management are likely to experience great difficulty in interpreting the results of such an analysis. However it is usually worth calculating the following two values of the performance measures:

$$f(U_1, U_2 \ldots\ldots U_m)$$

and $$f(L_1, L_2 \ldots\ldots L_m)$$

If the investment is unacceptable on the basis of the first value it is almost certainly not worth considering further; if it is acceptable on the basis of the second value it can usually be accepted without further analysis.

A general difficulty with sensitivity analyses, arises when two of the variables are considered to be correlated by management (For example, two of the variables in a certain situation might be the home market and the export market for a certain product and it might be considered that if the former is above average, then the latter will also probably be above average). In practice the analyst usually has to assume either perfect positive correlation or perfect negative correlation or no correlation at all. In the first two cases errors in the two variables are considered as occurring at the same time; in the third case the errors are considered as occurring one at a time in the analysis.

For a further discussion of the interpretation which can be put on the results of a sensitivity analysis the reader is referred to chapter 7.

1.4 ADJUSTING FOR RISK

Several methods, none of them wholly satisfactory have been suggested for modifying the three stage procedure in section 1.1 so that risk is 'adjusted for' or 'allowed for' in some way. Three of the most popular of these methods will now be briefly considered.

(i) The Use of Hurdle Rates

This is discussed in Van Horne (1974 p. 137). It involves assigning a hurdle rate to each project according to the project's risk (as assessed subjectively by management). The higher the risk the higher the hurdle rate. If NPV is the performance measure, the hurdle rate is used as the discount rate; if IRR is the performance measure then a project is only accepted if the IRR is above the hurdle rate. The method is easy to use, but suffers from the disadvantage that there is no scientific way of setting the hurdle rate for each class. In addition it may be difficult for management to assess the total riskiness of a project without an analysis of the individual uncertainties and the way in which they are combined together.

(ii) The Use of Certainty Equivalents

This is suggested by Robichek and Myers (1965) and Cohen and Elton (1967). Management are asked to assess for each time period a cash flow which, if obtained for certain, is just as desirable as the projected uncertain cash flow. NPV or IRR is then calculated in the usual way. The chief disadvantage of the method is that the cash flow in any one time period is likely to be made up from several individual cash flows, each calculated from a number of different variables, and management are likely to experience considerable difficulty in making the necessary assessments.

(iii) The Use of Payback

One argument often put forward in favour of the use of the payback period method in business is: since cash flows are riskier the further ahead they occur the payback period method discriminates against riskier projects by ignoring all cash flows after a certain point. However this can be quickly dismissed as a very weak line of argument completely ignoring the fact that some projects are more profitable than others simply because of their longer lifes.

1.5 THE NEED FOR A COMPLETE RISK PROFILE

The only wholly satisfactory approaches to dealing with risk in project appraisal are those where a complete risk profile (i.e. a complete probability distribution for the performance measure) is produced from subjectively assessed probability distributions for the variables. Several arguments against less sophisticated approaches have already been put forward. A further argument is that often the three stage procedure outlined in section 1.1 is misleading because there is a discrepancy between:

(a) The value of the performance measure which is calculated from management's best estimates.

and (b) The expected value of the performance measure.

The discrepancy can arise for several reasons:

(i) The best estimates made by management for the variables may correspond to modes or medians of their subjective probability distributions rather than means.

(ii) The value of the performance measure which is calculated from the means of the variables may not be the mean of the performance measure. (This is particularly likely to happen when the performance measure is IRR - see the work of Robichek (1975) which is discussed in section 4·2 of this thesis. It may also happen when the performance measure is NPV and there are variables which describe growth rates, the life of the project or the timing of cash flows).

(iii) There may be dependencies between the subjective probability distributions of the variables (Dependencies between variables which are added together in the cash flow model do not cause problems, but a dependency between variables such as market size and market share which are multiplied together does cause a discrepancy).

In some cases the discrepancy is quite large. In a case study discussed by Hertz (1964) the expected value of the rate of return is 14.6% whereas the rate of return calculated on the basis of best estimates is 25.2%.

Theoretical arguments against the production of a complete risk profile have been provided by Lewellen and Long (1972). They start by pointing out that the difficulties in (i), (ii) and (iii) above can be to a large extent be overcome, and a true estimate of the expected value of the performance measure can be produced, if:

(a) NPV rather than IRR is used as the performance measure

(b) Management are educated to provide single point estimates which are means of their subjective probability distributions rather than modes or medians.

and (c) The number of variables in the cash flow model is limited (with, for example, market size and market share being combined into a single variable if they are dependent).

Lewellen and Long then continue by suggesting that the capital asset pricing model (proposed originally by Sharpe (1964)) makes any consideration of an individual investment's risk irrelevant. To quote from their arguments:

> 'From the time of the initial capital asset pricing model it has become clear that in a well functioning securities market where risk averse investors exchange claims for assets, it is the degree of interrelationship among the returns of those assets which is paramount in determining the prices they will command. It has been established that the one irrelevant feature of an assets prospective returns is its own risk - the outcome uncertainties unique to the asset itself. These can and will be diversified away by the individual investors and by institutions in their securities portfolio leaving only the degree of correlation between the asset's returns and those of the so-called market portfolio as relevant to value since this correlation and its implied risks cannot be extinguished via diversification'.

This is however a fairly extreme viewpoint. The capital asset pricing model assumes a perfect capital market and, as has been pointed out by Van Horne (1974 p. 209), this assumption is questionable. Assets cannot always be sold at their economic value - particularly in bankruptcy conditions.

It is also worth pointing out that, once the results from a risk analysis study have been obtained, ways can often be found for carrying out the investment slightly differently so that the risk is reduced. (Pouliquen (1970 p. 20) provides an example of how risk was reduced in a study concerned with improvements to the Port of Mogadiscio).

1.6 ANALYTIC APPROACHES

As far as analytic approaches to the problem of determining a probability distribution for the performance measure are concerned, Hillier (1963, 1965 and 1969) and Wagle (1967) have made the most significant contributions. A brief review of the work of these authors is presented in this section. For further discussion the reader is referred to chapter 4.

Hillier (1963) considers three cases:

(i) The net cash flows in successive years are independent of each other.

(ii) The net cash flows in successive years are perfectly correlated.

(iii) The net cash flow in each year consists of an independent cash flow plus M distinct cash flows which are each perfectly correlated with the corresponding cash flows in other periods. For any given year however the M + 1 distinct cash flows are independent of one another.

Under the assumption that the distributions of the cash flows are normal (or if not normal are such that the Central Limit theorem can be applied when the distribution of a linear combination of several cash flows is being calculated) he derives a probability distribution for NPV and IRR. The former is a straightforward application of standard statistical results; the latter is rather more subtle involving the relationship:

$$\text{Prob. } \{IRR < d\} = \text{Prob. } \{NPV < 0 \text{ when discount rate is } d\}$$

and is expanded upon in Hillier (1965). Hillier (1969) extends his ideas to the case of several interrelated investments. By showing how a probability distribution for each feasible combination of investments can be derived, he provides an approach for selecting the best combinations of the investments under consideration.

The work of Hillier is reviewed by Bussey and Stevens (1973) who suggest the use of the beta distribution to describe the cash flows with pessimistic, most likely and optimistic estimates being made by management in order to determine the parameters of the distribution. Bussey and Stevens also suggest the use of maximum likelihood estimators as derived by Box and Jenkins (1970) for the estimation of the autocorrelation coefficients. However their premise that the pessimistic, most likely and optimistic estimates which management make for the cash flows in each time period are samples from a priori multivariate normal distributions is of doubtful validity.

Hillier's work is also commented upon by Horowitz (1966) who in an extension of an earlier article (Horowitz (1963)) concerned with the plant investment decision shows that if the individual variables from which the cash flows are derived have normal distributions it does not follow that the cash flows themselves will be normally distributed. However this may not be a very important objection in practice (see reference to Central Limit Theorem above).

The main drawback to Hillier's approach is that his formulation is entirely in terms of net cash flows and does notinclude the variables (such as sales levels, costs, etc.) from which the cash flows are derived. Management may have difficulty in assessing distributions for the cash flowsdirectly and, in addition to this, variables such as 'the life of the project' cannot easily be considered to be stochastic within the formulation.

Wagle (1967) overcomes these difficulties by splitting the total net cash flow stream into a number of separate streams (e.g. one corresponding to sales, one to fixed costs, one to variable costs etc). He shows how the means and variances of the cash flows in these streams can be obtained from high, medium and low estimates of the basic variables. Correlations between variables can be incorporated into the formulation and, in an example Wagle shows how the life of a project can be allowed to become a stochastic variable.

King et al (1975) provide an interesting recent contribution to the literature by discussing how high, medium and low estimates for two variables x and y can be converted to high, medium and low estimates for such combinations as:

$$x + y, \quad xy, \quad \frac{x}{y}$$

Obviously this has limitations but, possibly, it can be used, as an initial step, to reduce the total number of variables considered in certain situations.

Most of the other approaches which have been suggested for determining analytically a probability distribution of NPV or IRR either make highly restrictive assumptions or are very similar to those already mentioned. Two examples are Canada and Wadsworth (1968) and Tersine and Rudko (1972). Canada and Wadsworth include only the initial investment, annual return (assumed constant), salvage value and life of the project in their formulation, while Tersine and Rudko show how a stochastic life for the investment can be incorporated into Hillier's analysis.

1.7 SIMULATION APPROACHES

The determination of the probability distribution of the performance measure by means of Monte Carlo simulation was first suggested by Hess and Quigley (1963) and Hertz (1964). The method requires subjective probability distributions to be assessed for each of the variables affecting the performance of the investment. Sampling once from each of these distributions enables a single value of the performance measure to be calculated. Repeated independent sampling enables a complete probability distribution for the measure to be obtained. The method has the advantage that there are virtually no restrictions on either the shapes of the probability distributions assessed by management or on the complexity of the model relating the performance measure to the variables.

Correlations between variables can be handled within the simulation approach in a number of different ways. Hertz (1964) suggests that a single subjective probability distribution be assessed for the independent variable and that several subjective probability distributions be assessed for the dependent variable, each one being conditional on the independent variable lying in a certain interval. On each run of the simulation the value sampled for the independent variable would then determine the particular conditional subjective probability distribution chosen. This illustrates the principle but has the disadvantage that management are asked to make a very large number of individual probability judgments. Simplifying assumptions often have to be made in practice and this aspect of risk simulation is discussed in chapter 6.

Computer software for carrying out risk simulations is fairly widely available (see Berger (1972)) ICL offer PROSPER and IBM Call 360 offer RISK. Most packages offer procedures for the following:

(i) Sampling from both standard and empirical distributions (although it should be noted that PROSPER works entirely in terms of empirical distributions).

(ii) Calculating NPVs and, sometimes, IRRs.

(iii) Plotting a probability distribution for the performance measure.

(iv) Incorporating correlations (although it should be noted that the way in which correlations can be incorporated into the analysis varies from computer package to computer package).

The model (i.e. the series of equations relating the cash flows to the input variables) must of course usually be provided by the user.

Several applications of risk simulation in business have been published since Hess and Quigley (1963) and Hertz (1964). Pouliquen (1970) in a first rate article describes in detail several applications of the technique to the investment proposals facing the World Bank. Economos (1968) describes how risk simulation was used to determine whether a company should enter the computer leasing business. Kryzanowski et al (1972) and Bussey and Stevens (1973) use risk simulation to deal with a proposed plant expansions. Fowkes (1971) describes its application to branch bank location decisions. Smith (1970) and Newendorp and Root (1968) are concerned with its application to petroleum investment decisions. Cameron (1972) shows how it can be used in the case of a proposal to invest in 34 hotels. Other applications are provided by Glasgall (1968), Brown (1970) and Richards and Contesse (1975). Finally, the problems of introducing risk simulation into an organisation are considered by Carter (1972), Hall (1975) and Longbottom and Wade (1971).

1.8 APPLICATION OF THE SIMULATION APPROACH TO SEQUENTIAL INVESTMENT DECISIONS

Decision Trees are now a well accepted technique for handling the analysis of sequential investment decisions. Good general descriptions of the basic methodology are provided by for example Raiffa (1968), Schlaifer (1969), Magee (1964a), Moore (1972), Thomas (1972) and Brown et al (1974) while examples of its application are provided in Magee (1964b), Grayson (1960), Beattie (1969) and Moore et al (1976).

Hespos and Strassman (1965) have described how the decision tree methodology can be used in conjunction with the risk simulation methodology. First a decision tree is drawn and possible strategies open to the company are identified. Those branches on the tree emanating from outcome nodes are then, where appropriate, replaced by complete probability distributions and a Monte Carlo simulation is carried out to determine a complete distribution of the performance measure for some, or all, of the strategies.

A recent example of the application of the procedure to cash flow management is provided by Franks et al (1974). Robichek and Van Horne (1967) also give an example of its use in dealing with the abandonment decision.

1.9 A COMPARISON OF ANALYTIC AND SIMULATION APPROACHES

The analytic approaches suggested by Hillier and Wagle have the disadvantage that they can only deal with a restricted class of models. For example:

(i) It is not generally possible to deal with sequential decision situations using an analytic model (although Bonini (1974) does give an example of how an analytic model can be used when there are abandonment options).

(ii) Growth rate variables cause problems in analytic models. In the example Wagle (1967) gives on page 25 of his article he applies formulae for the mean and standard deviation of the product of two variables to the equation

$$M_n = M_{n-1}(1+G_n)$$

for n=1, 2, 3....15. where M_n is the market size in year n, G_n is the growth rate in year n and G_n has the same distribution for all n. In doing this he makes the assumption that the growth rate in any one year is independent of that in any other year. This might be unrealistic in any given situation and a better assumption might well be that growth rate has the same value G in each year. However, this would lead to the equation.

$$M_n = M_o (1+G)^n$$

which, since the correlation coefficient between $(1+G)^i$ and $(1+G)$ is not known when $i > 1$, would make the determination of the mean and standard deviation of NPV extremely difficult.

(iii) Analytic models can only deal with a limited class of dependencies. For example Wagle's model could not deal with a dependence between life of project and other variables.

The simplest analytic models do have the advantage that an approximate distribution for NPV can be obtained on a hand calculator in a matter of minutes (see Hillier and Heebink (1965)). However for some of the models suggested in Wagle (1967) the calculations are very complicated and require the analyst to have a considerable knowledge of statistics.

One of the chief disadvantages of the simulation approach used to be the cost of the computer time used. However technological developments have now reduced the costs of running even a large risk simulation model to negligible proportions. The chief computing cost is in fact usually that incurred in connection with the development of subroutines for carrying out the cash flow calculations (and in some cases it can be considerable).

Another disadvantage of the simulation approach which is sometimes mentioned is that it only provides an approximate distribution for the performance measure. However measures of the extent of the approximation can always be worked out from elementary statistics since the value of the performance measure calculated on one run of the simulation is independent of that calculated on other runs. If the extent of the approximation is found to be unacceptable the number of simulations runs can be increased. In addition it should be noted that when two or more different ways of embarking on the same project are being compared standard variance reduction techniques (see, for example Hammersley and Handscomb (1964)) can be used.

Finally, the analytic approach has the disadvantage that it only provides a mean and standard deviation for the performance measure. Hillier (1969) produces arguments to support the view that in practice these are all that is necessary because the distribution of the performance measure (whether NPV or IRR) is usually approximately normal. Chapter 4 tests Hillier's arguments empirically.

1.10 UTILITY THEORY

Even when a full risk evaluation study has been carried out, the choice between two alternative investment decisions may not be straightforward. Consider for example two investments A and B whose performance measures have the probability distributions shown in figure 1.1. Investment B has a higher expected value than investment A, but it may not be preferred because of the greater uncertainty involved.

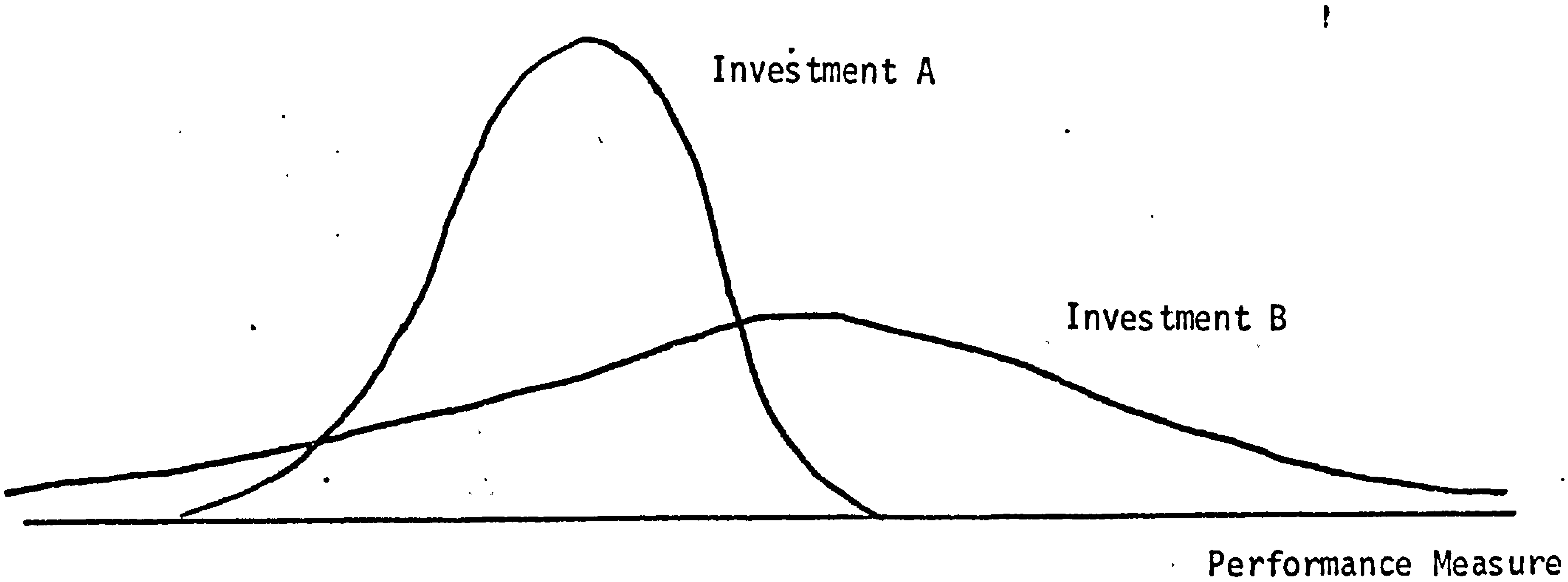

Figure 1.1

Probability Distributions for Two Performance Measures

A theoretical framework for comparing uncertain outcomes is provided by modern utility theory as developed in von Neumann and Morgenstern (1947). The theory first enunciates certain axioms obeyed by a 'rational' man and then shows that these lead to the existence of a preference ordering or utility function u which satisfies the following properties:

(a) u is defined on the set of all possible outcomes

(b) outcome A is preferred to outcome B if and only if $u(A) > u(B)$

(c) a decision giving chances p_i of achieving outcomes A_i $(1 \leqslant i \leqslant n)$ is preferred to one giving chances q_j of achieving outcomes B_j $(1 \leqslant j \leqslant m)$ where

$$\sum_{i=1}^{n} p_i = \sum_{j=1}^{m} q_j = 1$$

if and only if

$$\sum_{i=1}^{n} p_i \; u\,(A_i) > \sum_{j=1}^{m} q_j \; u\,(B_j)$$

Property (b) shows that a utility function ranks the outcomes in preference order while property (c) shows that one set of probabilistic outcomes is preferred to another if and only if it has a higher expected utility. It follows from property (c) that a 'rational' man will always act so as to maximise his expected utility.

Utility functions are only defined up to a positive linear transformation. Indeed it is very easy to verify that if we define a function v:

$$v = \alpha\, u + \beta \qquad \alpha\,,\, \beta \text{ constants } > 0$$

then it has the same properties as u in (a), (b) and (c) above.

Methods for measuring utility are discussed in Hull et al (1973) and studies aimed at measuring the utility of practising businessmen for money have been carried out by Grayson (1960), Green (1963), Cramer and Smith (1964), Swalm (1966) and Spetzler (1968).

A number of authors, have suggested the use of a quadratic utility function:

$$u\,(P) = aP^2 + bP + c$$

where a, b and c are constants and P is the value of the performance measure. This leads to:

$$\begin{aligned} E\left[u\,(P)\right] &= a\,E(P^2) + bE\,(P) + c \\ &= a\mu^2 + b\mu + c + a\sigma^2 \end{aligned}$$

where μ is the mean value of P for a given investment, σ is its standard deviation and E denotes expected value. Thus a quadratic utility function leads to the result that investments can be completely characterised by their mean and standard deviation for the purposes of decision making. For any given investor there will be a series of indifference curves such as those in Figure 1.2.

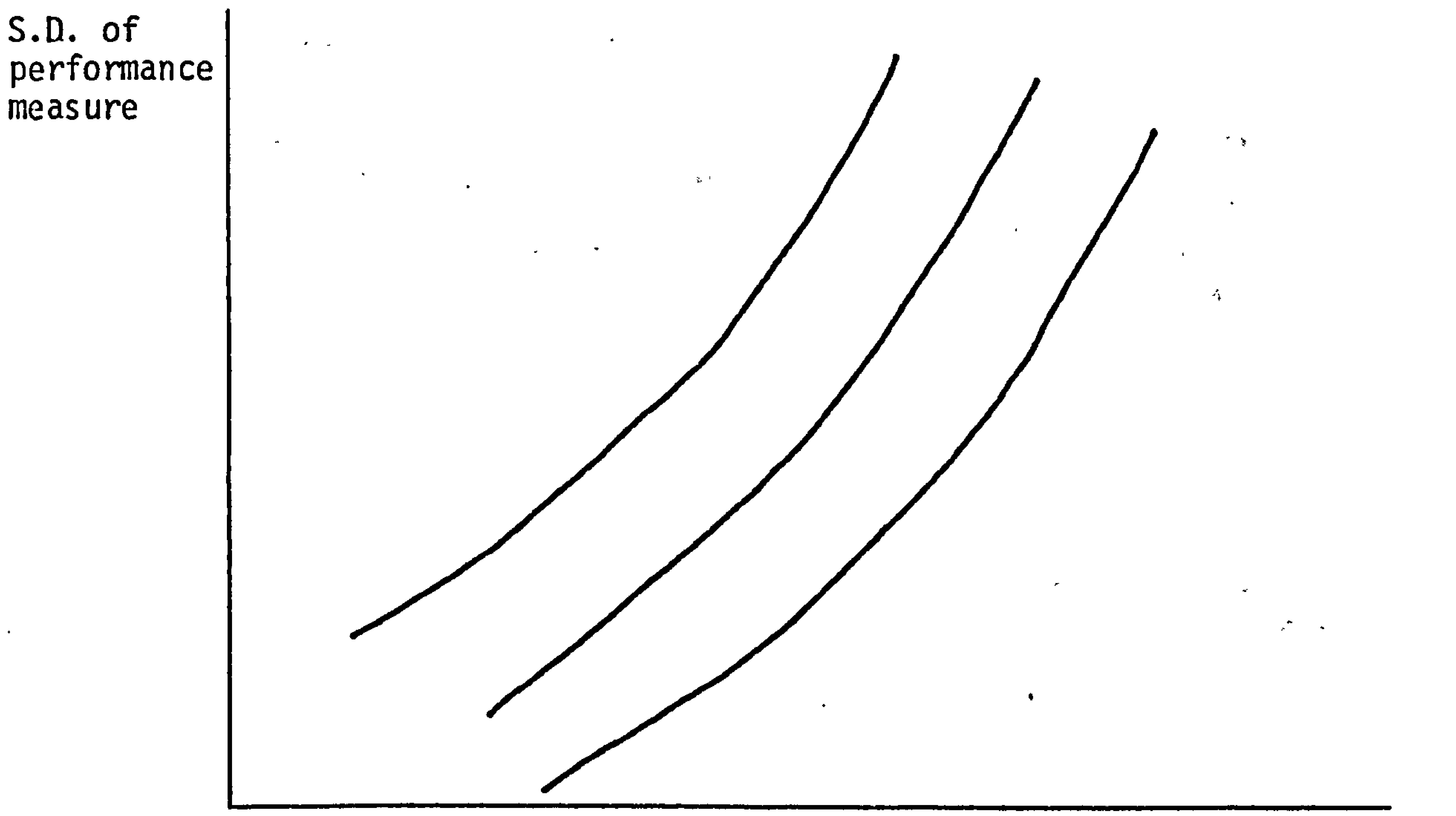

Figure 1.2

Lines joining points of equal utility for the quadratic utility function assumption

Pratt (1964) and Arrow (1965) have pointed out a weakness in the quadratic utility function assumption. If u(x) is the utility for a sum of money x, they define a function r(x).

$$r(x) \quad = \quad - \frac{u''(x)}{u'(x)}$$

and show that the decision maker's risk premium for small actuarially neutral risks (i.e. the amount of money he is willing to pay to avoid the risk) is:

$$\frac{1}{2} \; r(x) \; \sigma^2$$

where x is the wealth of the decision maker and σ^2 is the variance of the risk. r(x) is thus the decision maker's local propensity to insure and is known as his risk aversion function.

Pratt and Arrow then show that a utility function is decreasingly risk averse in the global sense if and only if its local risk aversion function r(x) decreases. This is a powerful result as it can reasonably by hypothesised that most investors become more inclined to accept risky investments as their wealth increases i.e. that they subscribe to decreasing global risk aversion. From this it follows that for most investors:

$$r'(x) \leqslant 0$$

$$\text{or} \quad u' u'' \geqslant u''^2$$

However for the quadratic utility function (except in the case where it degenerates into linearity) this is never true.

A somewhat different approach to the problem of comparing two probability distributions such as those shown in figure 1.1 is provided by the literature on stochastic dominance. This literature is surveyed in recent articles by Hadar and Russell (1974) and Porter and Carey (1974) If for two cumulative probability distributions F and G:

$$G(x) \leqslant F(x) \quad \text{for all } x$$

with strict inequality holding for some value of x then F is said to dominate G with 'first order stochastic dominance'. Alternatively if the somewhat weaker condition:

$$\int_{-\infty}^{x} G(t)\,dt \leqslant \int_{-\infty}^{x} F(t)\,dt \quad \text{for all } x$$

is satisfied, with strict inequality holding for some value of x, F is said to dominate G with 'second order stochastic dominance'. In either case arguments can be put forward to support the contention that an investment whose performance measure has cumulative distribution F should be preferred to one with cumulative distribution G.

Finally, it is worth noting that interesting work has been carried out by Borch (1966) and Hakansson (1971 a-c) directed towards determining how a corporate utility function can be determined from a knowledge of the environment in which the company operates. Borch's paper considers an insurance company where the magnitude of the assets perform a random walk under the influence of the income from premiums and the incidence of claims. Dividends are paid to shareholders according to a certain rule and the results which are obtained show that, even if the shareholder's are not risk averse, the company should be. Hakansson's work is more general. He assumes that a company knows possible investments several periods in advance and shows that the maximisation of expected average compound returns over N periods is equivalent to using the utility functions:

$$u(x) = x^{1/N}$$

where x is defined in each period as the end of period wealth.

1.11 THE PORTFOLIO PROBLEM

This chapter has been concerned with the literature relevant to the analysis of the risk in a single investment project. The problem of selecting a portfolio of risky projects has not been considered as it is not the main subject of the research in this thesis. It is however worth noting that both the analytic and simulation methods which are suggested above can be extended to provide information for solving the portfolio selection problem. Hillier (1969) for example describes how probability distributions for different combinations of risky interrelated investments can be determined analytically. Carter and Cohen (1972) suggest that several interrelated investments can be meaningfully simulated jointly if the values of the variables are related to external factors such as GNP and aggregate industrial production.

1.12 CONCLUDING REMARKS

When providing suggestions for future research directed towards increasing the applicability of risk evaluation models, Bonini (1975) says:

> 'Since the major limitation of the Hillier model is its inflexibility in describing accounting type cash flow relationships some work on simplifications or approximations and the sensitivity of the results to these approximations would be useful. In addition simplifications for estimating covariances in addition to the one suggested by Hillier would enlarge the scope of the model. Finally, the accuracy of the assumption of the normality of NPV should be investigated.
>
> Similarly methods need to be developed to simplify the inclusion of statistical dependencies in Monte Carlo and decision tree models. A good start has been made by Eilon and Fowkes (1973) using a procedure they describe as discriminant sampling to model these dependencies. But this is just the beginning.........'

Some of the research which is described in the chapters which follow is (purely by coincidence) very much along the lines which Bonini suggests.

CHAPTER 2

REVIEW OF THE LITERATURE ON THE ASSESSMENT OF SUBJECTIVE PROBABILITY DISTRIBUTIONS

2.1 INTRODUCTION

It occasionally happens that the inputs to a risk evaluation model can be based entirely on past data (See for example the description in Allais (1957) of mining operations in the Sahara desert). In most situations however this is not possible and the analyst must rely heavily on probability distributions which are based on managerial judgement.

This chapter will examine the different procedures which have been suggested in the literature for obtaining probability judgements from management and the ways in which the judgements, once they have been obtained, can be converted into a probability distribution. Philosophical arguments concerned with the validity of the 'subjectivist' or 'personalistic' view of probability will not be discussed in detail. These are presented by Savage (1954) who shows that if subjective probabilities are assessed in a coherent manner (i.e. so that it is impossible to lay a series of bets against the assessor in such a way that he will lose whatever the outcome) then they will conform to the axioms of probability.

2.2 BASIC APPROACHES

The methods which have been suggested for eliciting subjective probability distributions from management can be divided into a number of categories:

(i) Fixed interval methods

(ii) Variable interval methods

(iii) Relative likelihood methods

(iv) Psychometric ranking methods

(v) Equivalent prior sample methods

(vi) Hypothetical future sample methods

and (vii) Other methods

This section considers each category in turn:

Fixed Interval Methods

In fixed interval methods the range of all possible values of the uncertain quantity is divided into a number of intervals (usually of equal width) and the assessor is asked to state, for each interval, his probability that the value of the variable will be in the interval.

The order in which the intervals are presented to the assessor is thought to be an important consideration in fixed interval methods. Huber (1973) suggests that the assessor should, as a first step, be asked to identify the least likely interval and to provide a probability for that interval. He should then be asked to identify the second least likely interval and to provide a probability for that interval etc. This procedure, Huber says, may overcome the problem (to be discussed in more detail later in this chapter), that probability distributions with too small a variance are liable to the obtained if the assessor's attention is initially focussed on central rather than extreme values of the distribution.

Variable Interval Methods

In variable interval methods the assessor is asked to identify intervals which correspond to given probabilities. One of the possible ways of proceeding is described by Morrison (1967). First the assessor is asked for a value, say X, such that in his opinion the true value of the uncertain quantity is just as likely to be above X as below X (X is the median or 0.50 fractile of the distribution). The assessor is then told to ignore the possibility of the true value lying above X and asked to provide a value, say Y, which divides the range of values of the uncertain quantity below X into two equally likely parts. (Y is the lower quartile or 0.25 fractile of the distribution). Similarly he is asked to divide the range of all possible values above X into two equally likely parts in order to determine the 0.75 fractile. Other questions (if considered necessary) are then used to determine the 0.125, 0.375, 0.625, 0.875 fractiles etc.

Morrison's method (sometimes called the method of successive bisection) has the advantage that the assessor is only ever asked to think in terms of equally likely occurrences. Its disadvantage is that an error in one step of the procedure is carried forward to a later step. This disadvantage has led to alternative methods being developed involving the use of questions such as:

> 'What value of the uncertain quantity would you expect to be exceeded with a probability of 0.1'

However Peterson et al (1972) and Murphy and Winkler (1973) do not prefer such methods, arguing that they require judgements which are cognitively much more difficult to make than the method of successive bisection.

Another disadvantage of Morrison's method has been found to be that it automatically focusses the assessor's attention initially on a central value of the distribution (i.e. the median) and thereby produces a distribution with too small a variance. In order to overcome this disadvantage Barclay and Peterson (1973) have suggested that the assessor should initially be asked to divide the whole range of possible values into three, rather than two, equally likely intervals. (This produces a method known as the 'method of trisection').

Relative Likelihood Methods

In relative likelihood methods questions are asked regarding the relative probabilities of different values, or ranges of values, being obtained. One way of proceeding (see Winkler (1967a) and Barclay and Peterson (1973)) involves first asking the assessor for the most likely value of the variable (i.e. the mode of its distribution), then asking him to name two values which are half as likely as the mode and then asking him to name two values which are a quarter as likely as the mode etc.

Psychometric Ranking Methods

Psychometric ranking was proposed by Smith (1967). The range of possible values of the uncertain quantity is first divided into a number of equal intervals. Then:

(a) the assessor is asked to rank the intervals in order of ascending probability.

(b) the assessor is asked to consider the differences between the probabilities of adjacently ranked intervals and to rank the differences in ascending order.

and (c) the assessor is asked to provide a probability for the lowest and the highest ranked interval.

The quantification of rankings given in Kendall (1962) is used to assign a probability to each of the intervals. The idea is interesting in theory but has received a great deal of criticism as far as its potential for practical application is concerned (See Green (1967) and Morrison (1967)).

Equivalent Prior Sample Methods

Equivalent prior sample methods are only suitable when the variable is a proportion. The assessor is asked to make a statement of the form:

My uncertainty is equivalent to my having taken a sample of size n and having observed a proportion p.

Hypothetical Future Sample Methods

Hypthetical future sample methods are also only suitable when the variable is a proportion. The assessor is asked questions of the form:

How would your best estimate of the proportion be changed if I told you that out of a sample of size n a proportion p (different from the best estimate) had been observed?

Both this method and the previous one would seem to have the disadvantage that they require the assessor to be a fairly sophisticated processor of information. However Winkler (1967a) and Schaefer and Borcherding (1973) have achieved reasonable results with the methods.

Other Methods

Other methods include the portrait method which is described in Pouliquen (1971 p. 13) and methods involving betting or wagering. The latter nearly always require the analyst to make the somewhat doubtful assumption that the assessor wishes to maximise his expected monetary value and are therefore of limited use in practice.

2.3 EXPERIMENTAL EVIDENCE

In recent years psychologists and others have carried out many experiments aimed at answering questions such as:

(a) How good is man as an assessor of subjective probability distributions?

(b) Which of the available assessment procedures is the best?

(c) Can training improve man's performance at assessing subjective probability distributions?

The experiments have involved variables which are uncertain as far as the assessor is concerned but which are such that the experimenter knows - or can find out - the true value.

One interesting feature of some of the experiments is the use of 'scoring rules'. These are functions of the true value of a variable and its assessed distribution which can be used both as a way of comparing the true value with the distribution and as a device for motivating the assessor. A discussion of scoring rules is contained in Savage (1971) and, more recently, Matheson and Winkler (1976). A scoring rule is 'proper' if it is such that the assessor will maximise his subjectively expected score if and only if he reports his true opinions. Ideally a scoring rule should be both proper and sensitive to the precise shape of the probability distribution which is assessed. However, as pointed out in Hogarth (1973), many of the scoring rules which have been suggested do not have the second of these two properties.

A complete survey of all the experiments which have been carried out would not be relevant to this thesis. However the following list is illustrative of the type of work which has been done. (It should of course always be born in mind that results obtained under 'laboratory' conditions cannot necessarily be extrapolated to results obtained in the real world):

(i) Winkler (1967a) asked subjects to assess the proportion of people with different characteristics (e.g. the proportion of University of California students who are male) by 4 different methods: a variable interval method, a relative likelihood method, a hypothetical future sample method and an equivalent prior sample method. Generally the results obtained from the first two (direct) methods were better than those obtained from the last two (indirect) methods. However the subjects rated the indirect methods higher on a 'clarity' scale. In addition they preferred the relative likelihood method to the variable interval method and when inconsistencies were pointed out tended to want to change the fractiles they had assessed in the variable interval method rather than their relative likelihood assessments. However it should be stressed that all the experiments involved subjective probability distributions for a proportion.

(ii) Stael von Holstein (1972) used a fixed interval method in a study of stock market forecasting. His assessors included bankers, stock market experts, statisticians, business school faculty and business school students. They were asked to estimate the probabilities that the prices of several stocks would decrease by more than 3%, decrease by more than 1% but not more than 3%, change at most 1%, increase by more than 1% but not more than 3% and increase by more than 3% at the end of a forthcoming 14 day period. Generally the respondents did not do well at this task. Stael von Holstein estimated, using a scoring rule, that only 21% of the participants produced probability distributions which were 'better' than those obtained using historical data.

(iii) Peterson et al (1972) used variable interval methods in a study involving temperature forecasts made by experienced weathermen. There was a high correlation between the median estimate and the actual temperature and also a correlation between the mean error and the width of the interquartile range (showing that the weathermen were at least able to recognise their 'degree of uncertainty'). Similar results were obtained in a similar set of experiments by Murphy and Winkler (1973).

(iv) Winkler (1971) used fixed interval methods to investigate the ability of graduate students and faculty to assess the distribution of the scores in collegiate and professional football games. Between 28 and 42 (depending on the scoring rule used) out of the 45 participants did better than a hypothetical participant using only past data as far as professional games were concerned, but only between 4 and 14 did so in the case of collegiate games. The reason for this poor result in the case of collegiate games was a tendency for the students to assess distributions with too low a variance.

(v) Alpert and Raiffa (1969) in an unpublished paper which is reviewed in some detail by Pickhardt and Wallace (1974) describe experiments carried out on Harvard graduate students using variable interval methods. Many different variables (e.g. the number of vehicles imported into U.S. during 1967) were used. The results are summarised in table 2.1. (They are similar to later results obtained by Moore and Thomas (1975) and Pickhardt and Wallace (1974)). In Alpert and Raiffa's first group of assessments only 33% (as opposed to the ideal 50%) assessed interquartile ranges which included the actual value and 41% (as opposed to the ideal 2%) assessed distributions where it fell outside the 0.01 and 0.99 fractiles. In the second group of assessments which took place after some training these figures were improved to 43% and 23% respectively. One conclusion from the experiments is that the students were assessing 0.25 and 0.75 fractiles which were too tight (By fitting beta distributions to the fractiles assessed for proportions by a small sample of subjects Alpert and Raiffa concluded that assessors tended to behave as though they knew approximately twice as much as they actually knew). However by far the most surprising conclusion to be drawn from the experiment is that the students were totally unable to assess the tails of the distributions. This inability was even more marked when the students were asked for 0.001 and 0.999 fractiles instead of for 0.01 and 0.99 fractiles. Clearly direct questions about tail probabilities are dangerous.

Table 2.1 Results of Experiments by Alpert and Raiffa

	1st set of assessments	2nd set of assessments	Ideal
% of times value actually fell within interquartile range.	33%	43%	50%
% of times value actually fell outside range defined by .01 and .99 fractiles.	41%	23%	2%

2.4 BIASES IN THE ASSESSMENT OF SUBJECTIVE PROBABILITY DISTRIBUTIONS

A number of biases are liable to be present in subjective probability judgements. For example:

(a) The subject's responses may be influenced by his perceived system of rewards for various responses.

(b) The subject may allow himself to be unduly influenced by information which is easily recalled or visualised (For example too much weight may be given to recent events and recent plans).

(c) The subject may form an initial basis for his assessments and place too much emphasis on it (For example, in variable interval methods of assessment, if a subject is first asked for his 0.50 fractile he may 'anchor' on to it and assess other fractiles by making small adjustments to it).

(d) The subject may be unable to distinguish adequately between distributions relating to a whole population and distributions relating to samples from the population.

(e) The subject may be making unstated assumptions concerning the variable.

(f) The subject may be unduly influenced by the coherence with which difference scenarios of the future have been constructed rather than their 'logical' probability.

Many suggestions have been made in the literature for overcoming these biases. Spetzler and Stael von Holstein (1975) on the basis of their experience at the Stanford Research Institute, recommend that the interview between the analyst and the assessor should consist of five phases:

motivating
structuring
conditioning
encoding
verifying

During the motivating phase the subject is introduced to the idea of probability assessment and its importance is explained. Motivational biases (see (a) above) are discussed openly and it is pointed out to the subject that no firm projection or commitment is inherent in a probability distribution.

During the structuring phase the variable under consideration is clearly defined. Spetzler and Stael von Holstein suggest that a good test to apply to any given definition is: could a clairvoyant reveal the value of the quantity by specifying a single number and without requesting further clarification.

During the conditioning phase the analyst finds out what the subject is going to base his assessment on and cognitive biases (see b - f above) are, where possible, headed off. In the case of the biases in (b) and (c) the analyst may try to make more information available to the subject by asking him to produce scenarios describing extreme outcomes.

As far as the encoding phase is concerned Spetzler and Stael von Holstein recommend that both fixed interval and variable interval methods be used. To avoid anchoring (see bias (c) above) they suggest that the extremes of the distribution should be established first:

> 'Begin by asking the subject for what he considers to be extreme values for the uncertain quantity. Then ask for scenarios that might lead to outcomes outside of those extremes. Also ask for the probabilities of outcomes outside the extremes. This deliberate use of availability is designed to counteract the central bias that is otherwise likely to occur. Eliciting the scenarios for extreme outcomes makes them available to the subject, and he is thus more likely to assign higher probabilities to extreme outcomes. This has the overall effect of increasing the variability of his assigned distribution for the variable'.

Finally in the verifying phase, tests are carried out to see whether the subject really believes the distribution. Some of these tests are likely to involve equally attractive bets. If the subject proves to be uncomfortable with the final distribution it may be necessary to repeat some of the earlier phases.

2.5 GROUP SUBJECTIVE PROBABILITY DISTRIBUTIONS

Sometimes it is considered desirable to produce a subjective probability distribution from the responses of several different managers. In such circumstances there are two possible approaches. The first involves aggregating distributions which are assessed by the individual managers in the usual way; the second involves some mechanism whereby each manager can reconsider his assessments in the light of feedback concerning the assessments made by other managers.

Winkler (1968) has suggested two possible aggregation schemes. The first of these involves the use of weighted averages; the second involves the use of natural - conjugate priors.

Suppose there are k managers and $f_i(X)$ is the probability density function assessed by the i-th manager (i=1,2...k). When the weighted average method is used a 'pooled' density function f (X) is produced where:

$$f(X) = \sum_{i=1}^{k} W_i f_i(X)$$

$$W_i \geq 0$$

and $$\sum_{i=1}^{k} W_i = 1$$

The weights W_i (i=1, 2...k) should reflect the fact that some managers are better assessors than others. Winkler suggests that they can be chosen by the following methods:

(a) By asking a higher authority to rate each manager.

(b) By asking the managers to rate themselves

and (c) By rating the managers on the basis of their previous performance as probability assessors.

In the second of Winkler's aggregation schemes, each of the f_is is chosen as a member of a natural - conjugate prior family of distributions and the distributions are combined in a manner similar to successive applications of Bayes' theorem. This does not, it should be noted, eliminate the need for determining the weights to be given to the distributions of different managers.

The Delphi technique provides a way in which feedback can be incorporated into group probability assessment procedures. It involves interrogating managers by means of sequence of questionnaires. The first questionnaire asks each manager to make a number of estimates connected with the variable under consideration. These estimates are then put together and a summary of them is presented to the managers. The second questionnaire then asks each manager if in the light of the judgements made by the other managers he would like to change his original estimates and the whole process is repeated again. The Delphi technique is considered by many authors to be superior to procedures involving group discussion because with the latter the group is liable to be dominated by particularly forceful personalities. Accounts of experimental work involving the use of the technique are provided in Brown and Helmar (1962), Dalkey and Helmar (1963) and Moore and Thomas (1975).

2.6 CONVERTING MANAGERIAL JUDGEMENTS INTO A DISTRIBUTION FOR SAMPLING

When a subjective probability distribution is to be used in a risk simulation model, it is usual, in order to facilitate sampling, to assume that it can be approximated to by a distribution with a known mathematical form. This section considers the different mathematical forms which can be used.

One distribution which is fairly easy to sample from is the 'step' distribution (see figure 2.1). This has a piecewise linear cumulative distribution function. If it is used in conjunction with fixed interval methods of assessment the widths of all the 'steps' are usually the same. If it is used in conjunction with variable interval methods of assessment the widths of the 'steps' are different and the distribution has the general form shown in figure 2.2. To simplify sampling a step distribution is sometimes (e.g. in the ICL computer package PROSPER) approximated to by a discrete distribution (see figure 2.3).

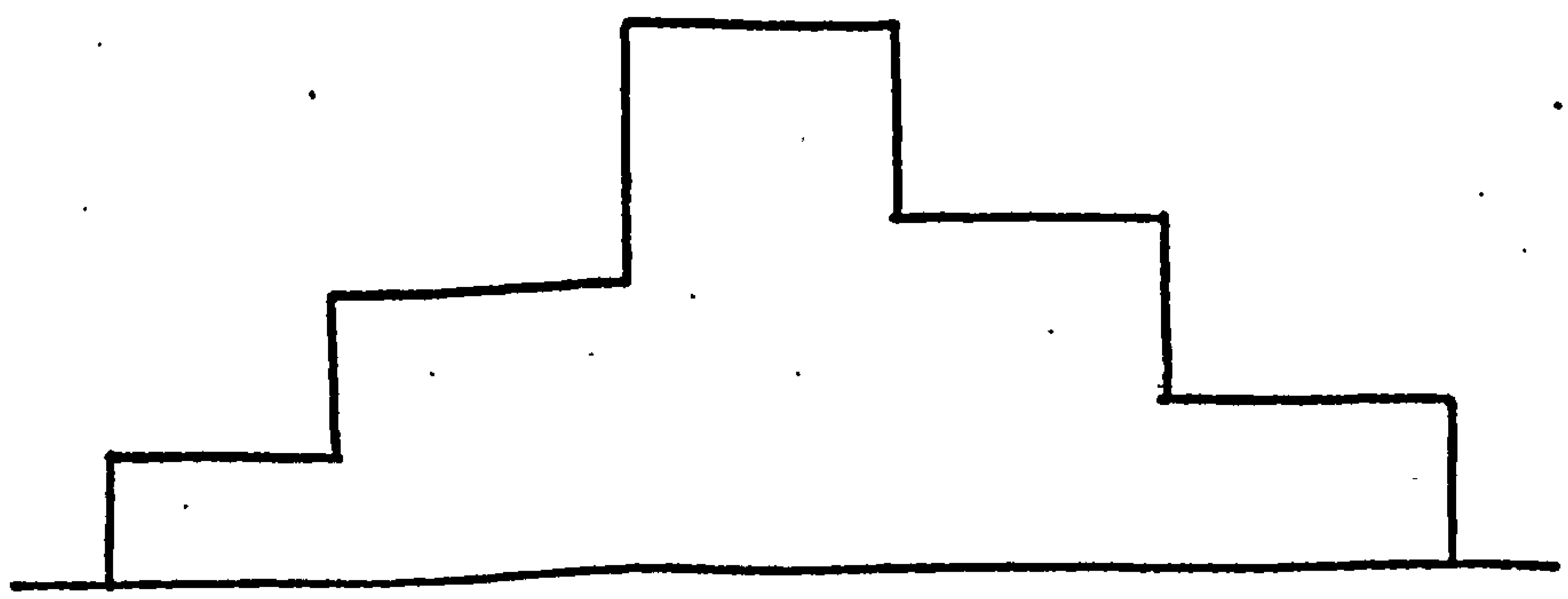

Figure 2.1 Step Distribution

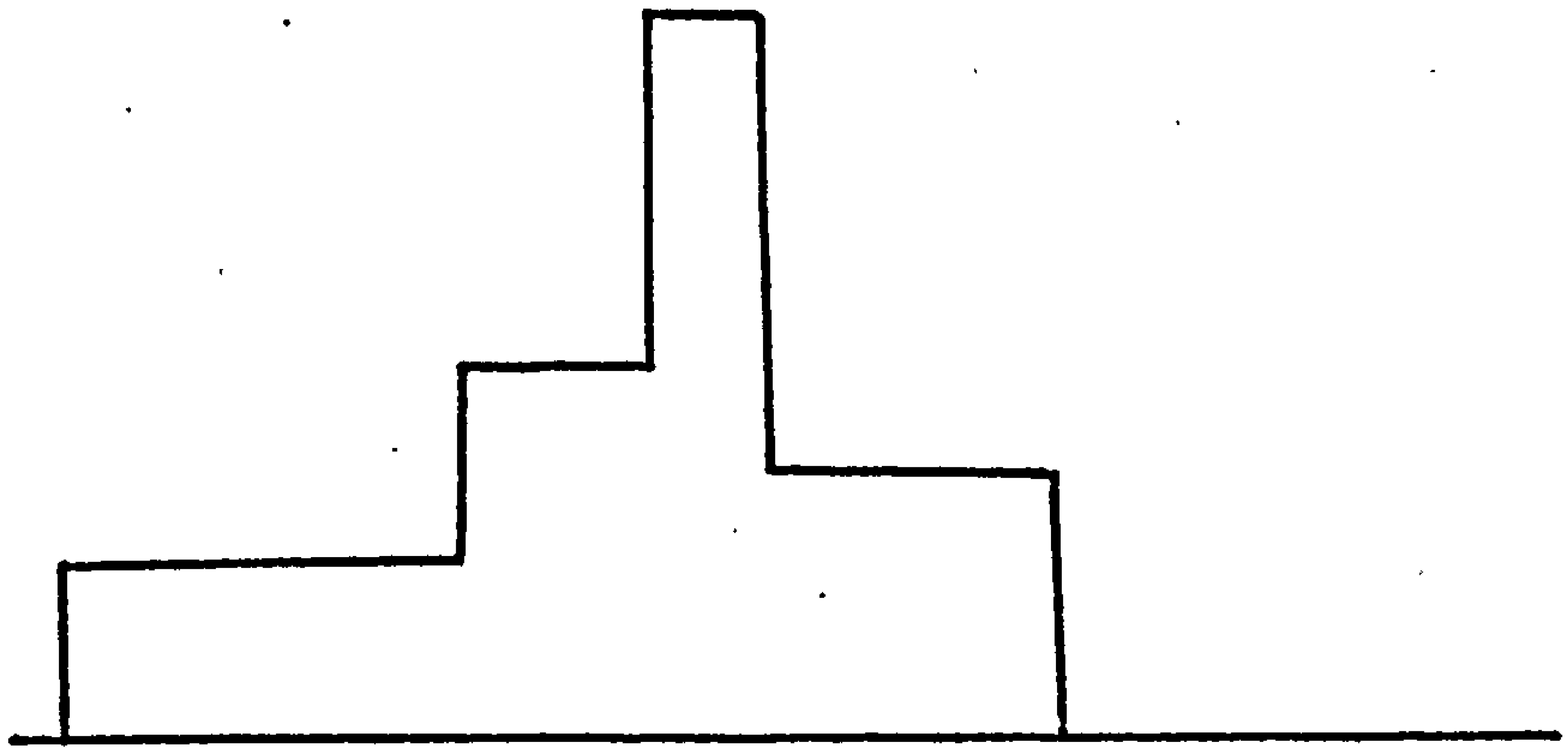

Figure 2.2 Step Distribution when Variable Interval Methods of Assessment are Used.

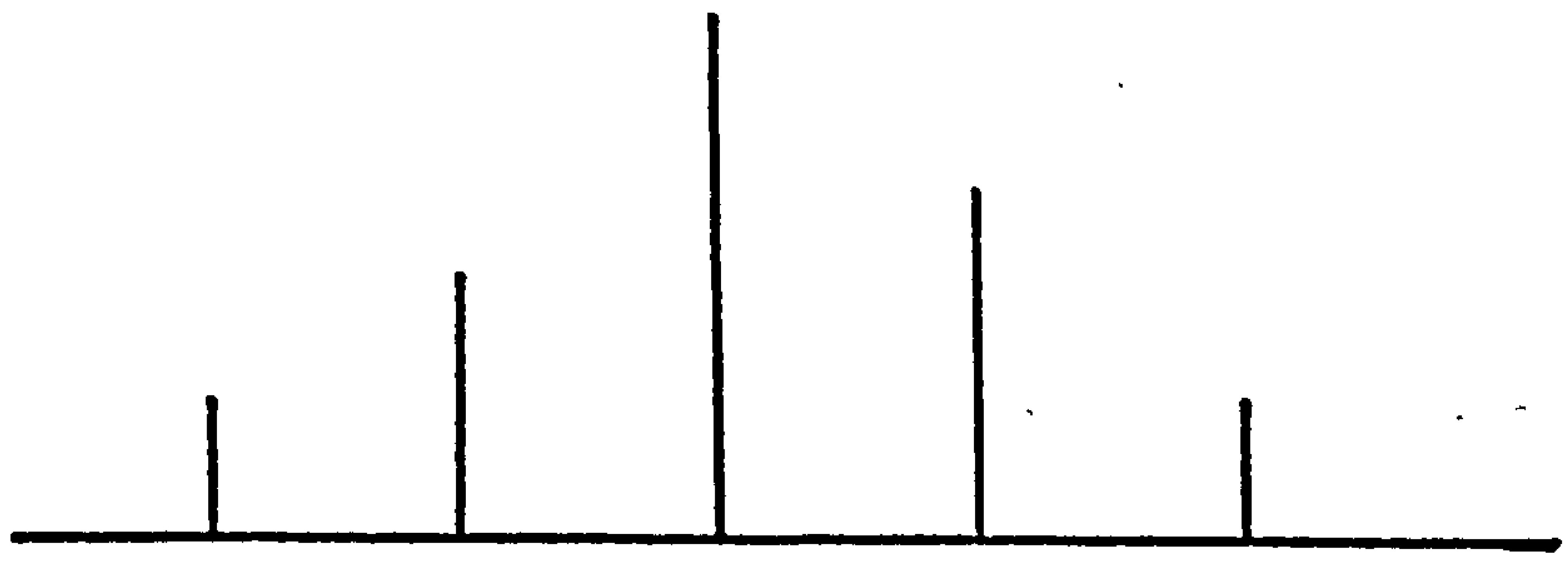

Figure 2.3 Discrete Distribution which is sometimes used as an approximation to step distribution for sampling.

Another distribution - described in Schlaifer (1971 p. 225) - which can be fitted once the assessor has provided a set of points on the cumulative distribution function is known as the piecewise quadratic distribution. (The name derives from the fact that the cumulative distribution function is piecewise quadratic). The distribution is illustrated in figure 2.4. The dots correspond to the points at which the cumulative probability is known. Their positions and those of the crosses are determined as follows:

(a) The density of the two points which mark the ends of the modal interval is set equal to the true average density between them.

(b) Except when the modal interval is the interval between the first two input points, the density at the first input point is set equal to zero; except when the modal interval is the interval between the last two input points, the density at the last input point is set equal to zero.

(c) At each other input point the density is set equal to the slope at the point of a quadratic passing through the point and through the input points immediately to the left and right of the point on the graph of the cumulative function.

(d) The crosses are inserted between each pair of input points so that the ordinate is equal to the true average density between the points and so that the probability distribution is consistent with the known cumulative probabilities.

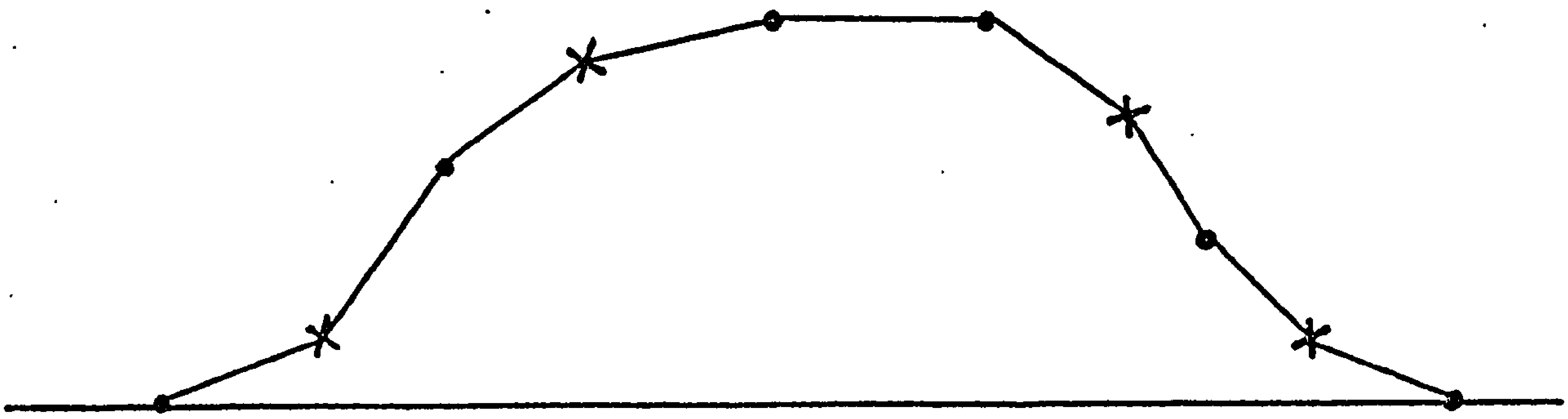

Figure 2.4 Piecewise Quadratic Distribution

A distribution must, it should be noted, be unimodal if it is to be represented by piecewise quadratic distribution.

Schlaifer (1971) also suggests a procedure where the assessor, as an initial step, selects a family of distributions of the right general shape for the variable under consideration. Variable interval methods of assessment are then used to provide enough judgements to enable one particular member of the family to be selected. Table 2.2 lists the families of distributions which Schlaifer recommends and their properties (The bounded lognormal distribution is discussed in Aitchison and Brown (1966). If variable x has a bound lognormal distribution then the variable

$$\log \frac{x}{1-x}$$

is normal. The arc-sinh normal distribution is discussed in Johnson (1949a). If variable X has the arc-sinh normal distribution then the variable

$$\mathrm{Sinh}^{-1} \frac{x-m}{s}$$

has a normal distribution for some constants m and s)

Schlaifer (1971 p. 10) argues in favour of the use of standard distributions as follows:

'A very considerable amount of experimental evidence shows that even people who assess quite reasonable values for the .25 and .75 fractiles of a distribution usually assess values for the .01 fractiles which are far too high and for the .99 fractiles which are far too low. It follows that if a decision maker finds that a distribution of inherently the right shape which agrees with his .25 and .75 fractiles has tails which at first sight seem to be too long, he should think seriously about the tails before he decides to fit a piecewise quadratic that agrees with these judgements'.

Table 2.2 Families of distributions which Schlaifer recommends be used for assessment of probability distributions

Family	Range	No. of parameters	Assessments suggested by Schlaifer	Properties
Beta	$[0, 1]$	2	.25 and .75 fractiles	Symmetrical or positively or negatively skew. Occasionally J-shaped or U-shaped or uniform.
Bounded Lognormal	$[0, 1]$	2	.25 and .75 fractiles	Symmetrical or positively or negatively skew. Occasionally bi-modal, usually however it is similar to Beta.
Lognormal	$[0, \infty)$	2	.25 and .75 fractiles	Positively skew
Logstudent	$[0, \infty)$	3	.25, .75, .875	Positively skew longer tails than Lognormal.
Gamma-q	$[0, \infty)$	3	Parameter q and .25 and .75 fractiles	Positively skew One tail longer, the other tail shorter than lognormal.
Normal	$(-\infty, \infty)$	2	.25 and .75 fractiles	Symmetrical
Arc-sinh Normal	$(-\infty, \infty)$	4	.25, .5, .75 and either .875 or .125 fractiles	Symmetrical or positively or negatively skew.

Finally, it is worth noting that some authors have looked for simpler approaches to assessing subjective probability distributions than those suggested in section 2.2 because of the difficulties which management experience when making detailed probability judgements. The use of a uniform distribution (see figure 2.5) based on an estimate of the range of the variable is suggested by Smith (1970). The use of a triangular distribution (see figure 2.6) based on an estimate of the range of the variable and on an estimate of its most likely value is suggested by Smith (1970) and Eilon and Fowkes (1973). Allen (1968) has suggested the use of a trapezium shaped distribution (see figure 2.7). He based this on the credibility and potential surprise concepts of Schackle (1961) and envisaged management using the following line of reasoning in arriving at figure 2.7.

> 'On the basis of the information available to me at the present time I consider any value of X between B and C to be completely credible (or alternatively I would not be at all surprised if the actual value of X turned out to be anywhere between B and C). I consider it to be utterly incredible for X to have a value less than A or greater than D'.

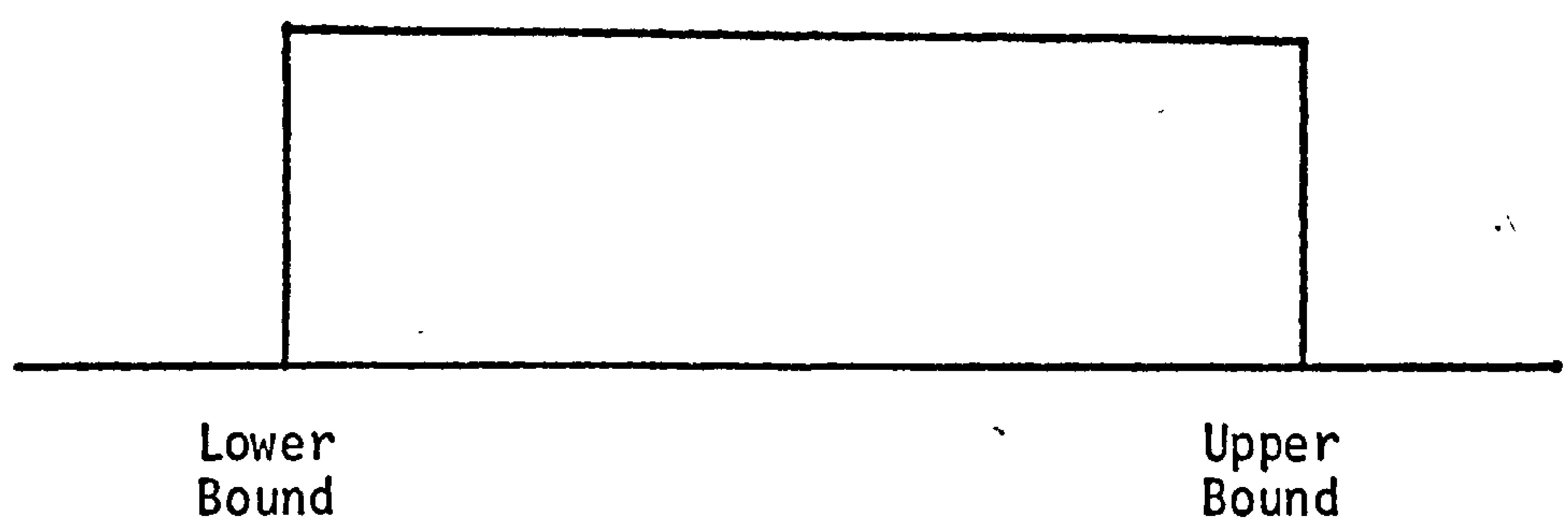

Figure 2.5 Uniform Distribution

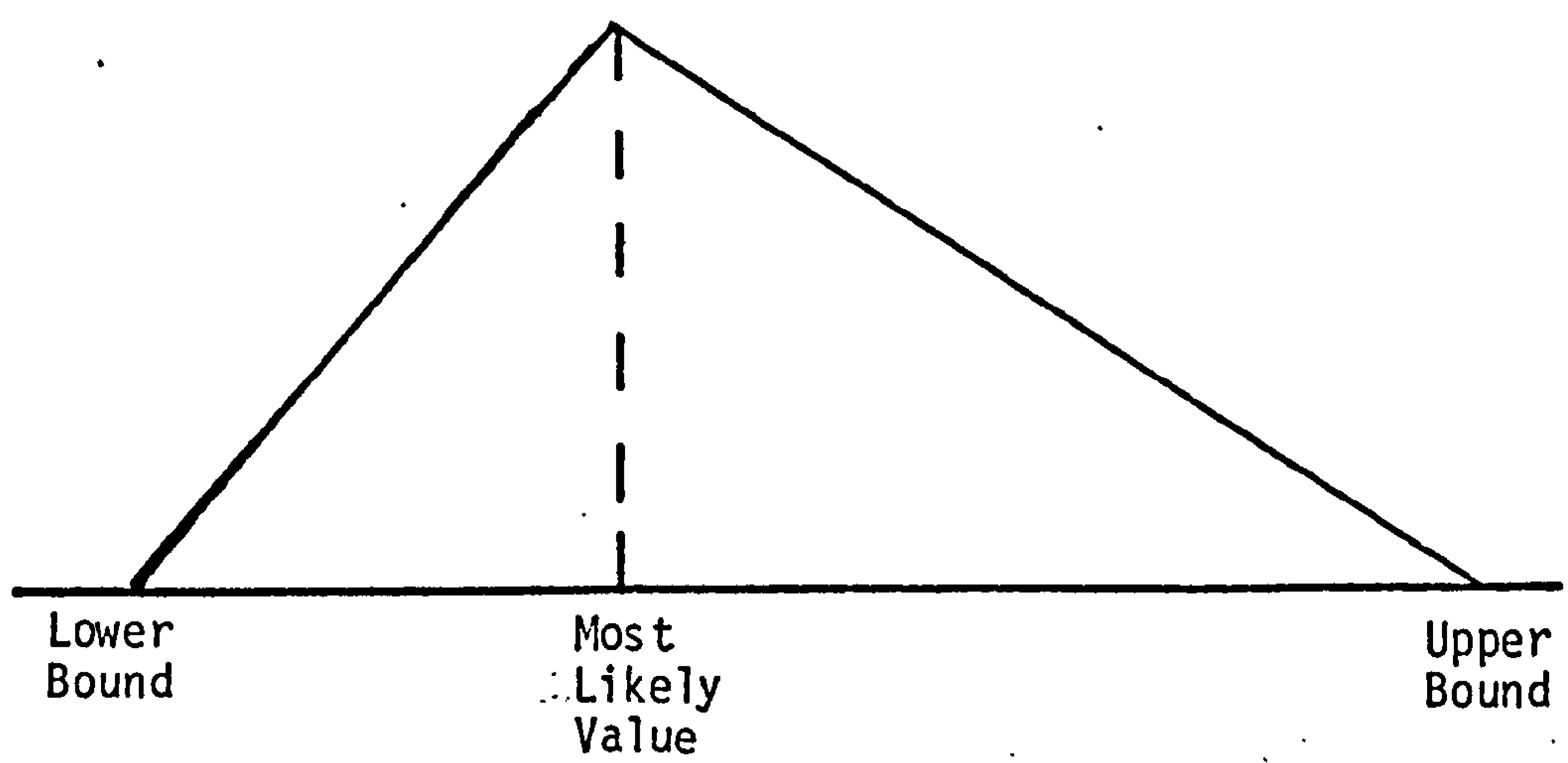

Figure 2.6 Triangular Distribution

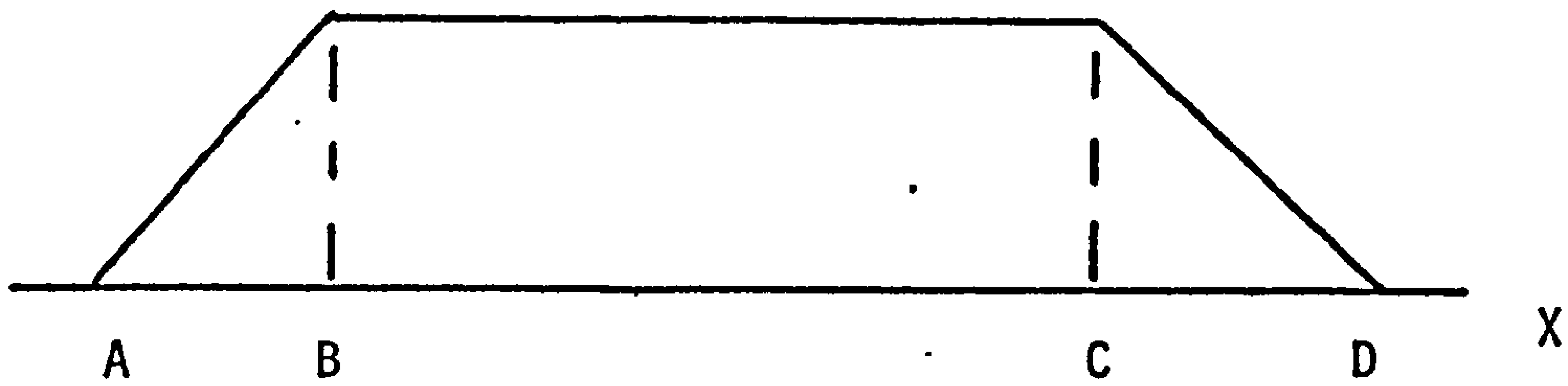

Figure 2.7 Allen's trapezium shaped distribution

2.7 CONCLUDING REMARKS

In risk evaluation, in view of the difficulties inherent in the assessment of subjective probability distributions it is important that the number of assessments which are made for a given variable should be related in some way to the importance of that variable's distribution. This is a subject which is considered in some detail later in this thesis.

CHAPTER 3

OUTLINE OF THE RESEARCH

3.1 INTRODUCTION

The research in this thesis relies heavily on the pioneering work of Hillier (1963, 1969), Wagle (1967), Hess and Quigley (1963) and Hertz (1964). It is therefore appropriate to start this chapter by briefly re-stating the main contributions of these authors.

Hillier (1963, 1969) considers the situation where an investment gives rise to a number of different cash flow streams. He puts forward arguments which strongly suggest that the distribution of the investment's NPV will often in practice be approximately normal and derives a procedure for calculating the mean and standard deviation of NPV from the means and standard deviations of the individual cash flows and the coefficients of correlation between cash flows. He also argues that the distribution of the investment's IRR will often in practice be approximately normal and shows how the distribution of IRR can be calculated if the distribution of NPV is known for different discount rates.

Wagle (1967) considers the situation where the means and standard deviations of individual cash flows are not known directly and have to be derived from those of other variables. He shows that it is sometimes possible to calculate a distribution of NPV or IRR by using results concerned with:

(a) the mean and standard deviation of the sum of two variables

and (b) the mean and standard deviation of the product of two variables.

Hess and Quigley (1963) and Hertz (1964) show that the probability distribution of NPV or IRR (or indeed any other performance measure) can be obtained by Monte Carlo Simulation. Sampling once from the distribution of each uncertain variable enables a single value of the performance measure to be calculated. Repeated sampling enables a complete probability distribution for the measure to be obtained.

3.2 QUESTIONS TO BE CONSIDERED BY THE RESEARCH

The research in this thesis can be divided into four parts. The first part (see chapter 4) is concerned with investigating certain aspects of the probability distributions which are input to and output from risk evaluation models. The second part (see chapter 5) looks at the accuracy with which different methods of assessing subjective probability distributions are capable of providing the inputs to risk evaluation models. The third part (see chapter 6) considers the problems created by dependencies between the variables in a risk evaluation model and how these problems can be dealt with. Finally, in the fourth part (see chapter 7), the results from previous parts are used to suggest ways in which the total number of probability assessments which managements are asked to make in a risk evaluation study can be kept down.

Each part of the research will now be described in more detail.

The First Part of the Research

The first part of this research considers a number of questions which are aimed at determining:

(a) whether the assumptions underlying the analytic procedures suggested by Hillier (1963, 1969) and Wagle (1967) are valid.

and (b) whether some of the results suggested by Hillier and Wagle concerning the input to and the output from risk evaluation models are true in a wider class of situations than those actually studied by the authors.

The first question which is considered is:

For what categories of cash flow models are the distributions of NPV and IRR approximately normal?

This is a question of some importance. If it is known that for certain types of investment the distribution of the performance measure is approximately normal then only two parameters of the distribution are of interest: its mean and its standard deviation and managerial attitudes to risk can, for the types of investment under consideration, be expressed entirely in terms of these two parameters by means of a set of indifference curves. Furthermore it should be noted that the analytic procedures of Hillier and Wagle can only be used with complete impunity when the distribution of NPV is normal.

The second question which is considered is concerned with the variables which are input to a risk evaluation model. Define a 'type I' variable in a cash flow model to be a variable which is such that only its mean and its standard deviation affect the distribution of the performance measure. (In other words, a type I variable is a variable which is such that moments higher than the second are irrelevant as far as a determination of the distribution of the performance measure is concerned). The question which is considered is:

To what extent are the variables which enter a cash flow model approximately type I?

This is a question which is of some importance because the knowledge that a certain variable is type I may in risk simulation have implications as far as the choice of a procedure for assessing the variable's subjective probability distribution is concerned.

The third question to be considered in the first part of the research is similar to the second:

To what extent are the dependencies in a cash flow model approximately type I?

A type I dependence between two variables is here defined as one which is such that only the coefficient of correlation affects the final distributions of the performance measure.

The Second Part of the Research

The subjective probability distributions which are used in risk analysis studies are, at best, only approximate representations of managerial uncertainty because they are based on a small number of individual probability assessments. Chapter 4 explores this aspect of risk analysis in more detail. In particular the chapter is concerned with investigating the differences between:

(i) subjective probability distributions which are produced on the basis of a small number of individual probability assessments.

and (ii) 'true' subjective probability distributions i.e. the distributions which would have been produced if management had been capable of making an infinitely large number of probability assessments.

Three different methods for assessing subjective probability distributions are considered: the fixed interval method, the variable interval method and the relative likelihood method. (See chapter 2 for a description of these methods and a discussion of the literature on them). The research starts by assuming that management are capable of giving answers which are a perfectly accurate reflexion of their judgement to any given set of questions and proceeds to investigate how the accuracy of the distributions which are assessed varies with the assessment procedures used and with the number of assessments made. In particular it considers:

(a) the difference between the 'true' mean and the assessed mean

and (b) the difference between the 'true' standard deviation and the assessed standard deviation.

These differences are particularly relevant as far as 'type I' variables are concerned.

In the second half of chapter 5 it is recognised that the extent to which management are able to discriminate between different values of the variable and between different probabilities when making assessments is limited and the effect of different 'levels of discrimination' on the results obtained earlier in the chapter is investigated.

The Third Part of the Research

Chapter 6 of this thesis starts by considering the question:

How important are dependencies in a risk evaluation model?

It compares for different pairs of variables X and Y the effect on the final distribution of the performance measure of:

(i) failing to assess the distribution of X and Y accurately and (ii) failing to assess the extent of the dependence between X and Y accurately.

The chapter then proceeds to review critically the different methods which have been suggested in the literature for the assessment of the dependence between two variables.

If probability distributions for two variables X and Y have already been assessed, one of the most natural ways in which management can provide judgements about the dependence of Y and X would seem (to the present author) to be by answering questions of the following form:

'if X equals Q, what is your median estimate for Y'

Hillier (1963) and Wagle (1967) suggest a way in which such judgements can be used to provide an estimate of the correlation coefficient between X and Y. Chapter 6 goes one stage further by suggesting a way in which a complete pattern for the dependence between X and Y can be constructed from the answers to such questions. (The point here is that although the analytic approach suggested in Hillier (1963) and Wagle (1967) only requires the assessment of the coefficient of correlation, the simulation approach of Hess and Quigley (1963) and Hertz (1964) requires - for the purposes of sampling - that a complete pattern for the dependence be determined in some way).

The Fourth Part of the Research

Chapter 7 uses the results obtained in previous chapters to investigate different ways of limiting the total number of probability assessments which management are asked to make for a risk evaluation model. The chapter distinguishes between 'essential' probability assessments (i.e. those which have some chance of affecting the decision taken on the investment) and 'non-essential' probability assessments (i.e. those which have no chance of affecting the decision taken on the investment). It discusses the extent to which it is possible for an analyst to identify whether a given assessment is 'essential' before he askes management to make the assessment.

The chapter starts by considering the information which is provided by a straightforward sensitivity analysis. In particular it investigates whether a sensitivity analysis gives a good guide to:

(a) the relative importance of assessing different probability distributions to a given accuracy

and (b) the relative importance of assessing different dependencies to a given accuracy.

The chapter then proceeds to investigate the viability of carrying out a risk simulation analysis in a series of stages in such a way that management are only ever asked to make probability assessments when it is known (from risk simulations already carried out) that those probability assessments are 'essential'.

3.3 RESEARCH METHODOLOGY

In the case of most of the questions mentioned in section 3.2, there are two main methods by which it is possible to obtain answers:

(i) A mathematical analysis of the statistical properties of variables which are obtained by combining together other variables.

and (ii) An analysis, using simulation, of specific case studies.

Both methods are used in this research as appropriate. To illustrate the general approach which is adopted consider the second of the questions discussed in the first part of the research. (This is concerned with the extent to which variables are type I). An examination - and slight extension of - the mathematical analyses given in Wagle (1967) is used to provide suggestions as to those variables which are likely to be type I in a given situation. Risk simulations are then carried out using specific case studies in order to:

(i) confirm or reject the suggestions

(ii) 'gain a feel for' the types of relationships between variables which arise in practice.

and (iii) analyse those of the variables which are not tractable mathematically.

In general the results which this research requires from case studies are obtained by varying the input to the cases and observing the effect which this has on the distributions which are output.

After a search through the literature and visits to different organisations, the five case studies described in the next section were selected for detailed examination. This selection was made on the basis that as many as possible of the different types of cash flow models which are commonly observed in practice should be included. (Of course the case studies do not include all the cash flow models which could arise in practice and it is recognised that this may restrict the generality of the conclusions which are reached in places).

The research methodology used in chapter 5 to compare different probability assessment procedures deserves a special mention. The chapter starts by creating a set of 'true' subjective probability distributions and by defining in detail a number of procedures for assessing subjective probability distributions. It then simulates the way in which each of the distributions would be assessed using each of the procedures. The values which would be calculated by an analyst for the means and standard deviations of the distribution are, for each assessment procedure, recorded and compared with the 'true' means and 'true' standard deviations.

3.4 CASE STUDIES

The net cash flow models used in the five case studies which have been selected for examination are given in appendix A. Only brief descriptions of the case studies will therefore be given here.

Case A: Hertz Model

Case A is the well known case study described in Hertz (1964). A medium sized industrial chemical producer is considering a $10 million extension to its processing plant. The service life of the facility is expected to be 10 years and the engineers estimate that they will be able to utilize 250,000 tons of processed material worth £510 per ton at an average processing cost of $435 per ton. Risk simulation is used to investigate whether the investment should be undertaken. The variables for which probability distributions are assessed are:

The initial market size (thousands of tons)

The market growth rate p.a.

The selling price ($ per ton)

The market share (%)

The initial investment (millions of dollars)

The life of the investment (years)

The residual value (millions of dollars)

The operating costs ($ per ton)

The fixed costs (thousands of dollars per year)

Most likely values and ranges of possible values are provided for these variables by Hertz. Eilon and Fowkes (1973), who also carry out some analyses on the Hertz model, suggest further data which can be used to define the distributions of the variables more precisely.

An analytic approach is not really appropriate as far as the Hertz model is concerned. Wagle (1967) does succeed in providing a method for calculating the probability distribution for NPV analytically, but, as mentioned in section 1.9, he finds it necessary to assume that the market growth rate in any one year is independent of that in any other year.

This research, in common with that of Hertz, Eilon and Fowkes and Wagle, assumes that depreciation, working capital requirements, corporation tax etc. can be ignored in the Hertz model. This does of course make the model unrealistic. However any differences between the Hertz model and a more realistic version of it are irrelevant as far as the questions to be investigated in this thesis are concerned.

Case B: Kryzanowski et al Model

Case B is a well documented case study described in Kryzanowski et al (1972). A major natural resource firm is considering proposals for expanding its plant. Detailed probability assessments are made for 12 uncertain variables and a risk simulation is carried out to determine the distribution of IRR. The uncertain variables are:

The price of the product ($ per unit)

The price growth rate p.a.

The variable operating costs at present ($ per unit)

Three different categories of additional variable costs ($ per unit)

The extra fixed costs as a result of the plant in year 1 ($'000)

The growth rate p.a. for all costs.

The capital costs incurred in year 0 ($'000)

The production in year 1 (units)

The life of the project (years)

Additions to working capital ($'000s) in year 0

The assumptions made in this research are similar to those made by Kryzanowski et al (1972). Production levels in years 2 and 3 are 2.15 and 2.85 times those in year 1. After year 3 production remains constant. Extra fixed costs in year 2 and year 3 are 2.0 and 2.4 times those in year 1. After year 3 the fixed costs remain constant. Capital costs incurred in years 1 and 2 are 2.51 and 1.0 times those in year 0 with no capital costs being incurred in subsequent years. Additions to working capital in years 1, 2 and 3 are 1.15, 0.69 and 0.23 times those in year 0. After year 3 no additions to working capital are made and all working capital is recouped at the end of the project.

The rate of corporation tax (Canadian) is 54.5% and capital equipment is, it is assumed, depreciated at 20% p.a. for tax purposes.

Case C: Interchemical Model

Case C is based on Vandell (1970 a-d). The consumer products division of a large company (Interchemical Ltd) are considering launching an aerosol furniture polish in the Italian market. Management have developed a unique gimmick to distinguish the product from others already on the market: it contains a lemon ingredient which many housewives associate with superior cleaning properties. The company is prepared to promote the product aggressively and it is considered that an initial budget of 100 million lire would provide a basis for a good campaign. The investment which is being considered in this case is therefore an investment in an advertising campaign. The uncertain variables for which probability distributions have been assessed are:

The initial market size (millions of lire)

The market growth (% p.a.)

The market share (%)

The price adjusted factor (% adjustment to the projected market sizes because of price changes).

The cost of goods sold (% of sales)

The variable selling expenses (% of sales)

The fixed costs (millions of lire p.a.)

Corporation tax, working capital etc. are ignored in this case study, but, as with the Hertz case, this is not an important disadvantage as far as the analyses carried out in this thesis are concerned. One feature of the model which is important is the fact that if sales are not up to expectations in the first year the advertising program can be abandoned at very little cost. The effect of this on the model's output is discussed in chapter 4.

Case D: ICI Model

Case D results from a visit to the Agricultural Division of ICI Ltd. at Billingham in Teeside. The case study is considered to provide a typical example of a type of investment problem with which the division is frequently faced. It is also used to illustrate the risk analysis computer programs which have been developed by personnel within the division. In the case, it is proposed that a plant be built to manufacture a chemical: Fertinol. There are two main markets for the chemical: the home market and the export market. If the plant's production is insufficient to satisfy both markets, the home market (which incidentally is slightly more profitable) is given priority. The uncertain variables in the cash flow model are:

The fixed selling costs in Home market (£ p.a.)

The variable selling costs in Home market (£ per ton)

The price per ton in Home market (£)

The potential sales in the Home market (tons p.a.)

The fixed selling costs in the Export market (£ p.a.)

The variable selling costs in the Export market (£ per ton)

The price per ton in Export market (£)

The potential sales in the Export market (tons p.a.)

The capital costs (£)

The data of start up (i.e. years from start of construction)

The life of the plant (years from start up)

The start up costs (£)

The salvage value (£)

The fixed costs of production (£ p.a.)

The variable costs of production (£ per ton)

The working capital required (£)

The production capacity (tons)

The rate of corporation tax used in the case study is 45% p.a. An investment grant of 45% of the capital costs of the plant is assumed and the value of the plant is assumed to depreciate at 20% p.a. for tax purposes. The investment is assumed to be entirely in plant and not at all in buildings. (In general, any part of an investment which is in buildings has to be treated differently from that part of the investment which is in plant as far as grants and tax allowances are concerned).

Case E: Economos Model

Case E is based on Economos (1968). A large corporation has to decide whether to acquire a computer leasing subsidiary. The subsidiary would satisfy the computing needs of the corporation as well as competing with other leasing companies in what is known as the 'third party leasing market'. This market will it is considered become an increasingly important part of the total computer market until the 'year of the hostile act'. In this year action by manufacturers or by the leasing industry itself will, it is considered, impede growth. Only four of the variables entering the analysis of the investment are considered to have a significant uncertainty attached to them. These four variables are:

The rate of growth of the total U.S. computer industry

The rate of growth of the computer requirements of the corporation.

The year of the hostile act

The year of the introduction of fourth generation computers.

At the time of the case 80% of the computers in use in the U.S. are leased and 2.5% of these leases are third party leases. This share of the leasing market is expected to increase by 30% p.a. until the 'year of the hostile act'. From that year and beyond the share of the market is expected to remain constant. From the year of the introduction of fourth generation equipment rental rates on third generation equipment are expected to be only 50% of their original value.

This case study is of interest because the variables involved are markedly different from those in cases A, B, C and D.

3.5 COMPUTER PROGRAMS

Three computer programs have been developed by the author in FORTRAN on an ICL 1903T to carry out the analyses in this thesis. An indication of the way in which they were written is given in appendix B and only brief descriptions are therefore appropriate here.

Program RISKANAL 1

RISKANAL 1 carries out a sensitivity analysis on the basis of best estimates, pessimistic estimates and optimistic estimates as described in section 1.3. The variables are arranged in order of decreasing sensitivity on the output.

Program RISKANAL 2

RISKANAL 2 is the program which was used to carry out most of the risk simulations described in this thesis. The input to the program can be defined in a number of different ways. The output includes a graphical display of the probability distribution of the performance measure. With minor modifications the program was used in chapters 6 and 7 to provide tables showing the sensitivity of the distribution of the performance measure to:

(a) the mean of each variable

(b) the standard deviation of each variable and (c) dependencies between variables.

The program uses the procedure described in Downham and Roberts (1967) for sampling random numbers.

In both RISKANAL 2 and RISKANAL 1 the performance measure can be either NPV or IRR. For both programs it is necessary for the user to supply, in addition to data, a FORTRAN subroutine which calculates the cash flows in each year from the values of input variables.

Program PROBSIM

PROBSIM is the program used in chapter 5 to generate a set of unimodal distributions and to simulate the way in which they would be assessed using different procedures.

CHAPTER 4

AN ANALYSIS OF THE CHARACTERISTICS OF THE INPUT TO AND OUTPUT FROM RISK EVALUATION MODELS

4.1 INTRODUCTION

This chapter considers the following three questions concerned with the input to and the output from risk analysis studies:

(i) To what extent and in what sense can it be said that the distributions of NPV and IRR which are output from a risk evaluation model are approximately normal.

(ii) To what extent and in what sense can it be said that the variables which are input to a risk evaluation model are approximately type I.

and (iii) To what extent and in what sense can it be said that dependencies between the variables which are input to a risk evaluation model are approximately type I.

(In chapter 3, it will be recalled, a type I variable was defined as a variable which is such that only its mean and its standard deviation affect the distribution of the performance measure and a type I dependence was defined as a dependence which is such that only the coefficient of correlation affects the distribution of the performance measure).

4.2 THEORETICAL RESULTS CONCERNING THE NORMALITY OF NPV AND IRR

Suppose that the cash flows resulting from an investmentin years 0, 1, 2....n are C_0, C_1, C_2....C_n respectively (where n years is the life of the investment). Then if d is the discount rate, the NPV is defined as:

$$\sum_{i=o}^{n} \frac{C_i}{(1+d)^i}$$

and the IRR is defined as that value of d which causes the NPV to be zero.

Hillier (1963, 1969) considers the conditions under which NPV and IRR can be shown to be either normal or approximately normal. First he points out that if C_0, C_1....C_n have a multivariate normal distribution then NPV being a linear sum of the C_is will also be normally distributed. He then considers the situation where C_0, C_1....C_n do not have a multivariate normal distribution and carries out investigations to determine whether it is possible to use the Central Limit Theorem to show that NPV will be approximately normal in this case. These investigations will now be examined.

The most well known version of the Central Limit Theorem states the following:

if W_1, W_2....W_n are independent identically distributed non-degenerate random variables with finite means and variances then their sum is asymptotically normal as n tends to infinity. 4.1

This however is not applicable to the present problem as the variables which are summed when NPV is calculated are:

$$\frac{C_i}{(1+d)^i} \qquad i = 0, 1 \ldots n$$

and these are in general neither independent nor identically distributed. Most other versions of the theorem can be considered as falling into one of 3 categories:

(i) those which insist on the 'identically distributed' condition in 4.1 but allow the 'independence' condition to be relaxed.

(ii) those which insist on the 'independence' condition in 4.1 but allow the 'identically distributed' condition to be relaxed. (The most well known theorem here is the Lindeberg theorem which states that if $W_1, W_2 \ldots W_n$ are a uniformly bounded sequence of independent non-degenerate random variables then their sum is asymptotically normal as n tends to infinity).

and (iii) those which allow both the 'independence' and the 'identically distributed' conditions in 4.1 to be relaxed.

Theorems in the first category are not in general applicable. Those in the second category may on occasion be applicable. Usually however a theorem in the third category is likely to be required. Unfortunately it seems to be the case that very few of the theorems which have been produced do actually fall into the third category. Hillier mentions only two such theorems. One is quoted in Doob (1953); the other is quoted in Hoeffding and Robbins (1948). Both impose severe restrictions on the types of dependence allowed.

Even if an appropriate version of the Central Limit Theorem can be found one serious theoretical difficulty is liable to be encountered in connection with the condition (always present in Central Limit Theorems) that the variables should be non-degenerate. To illustrate this difficulty, suppose a simple situation exists where it is known that the cash flows are independent and identically distributed and that they continue forever. (This situation would appear on the face of it to be ideal for the application of the Lindeberg theorem). Suppose that the cash flows have mean μ and standard deviation σ and that the discount rate is d. The variance of the present value of the first n cash flows can easily be seen to be:

$$\sum_{i=o}^{n} \frac{\sigma^2}{(1+d)^{2i}}$$

and this is finite tending to:

$$\frac{\sigma^2}{(1-\alpha^2)} \qquad \text{where } \alpha = \frac{1}{1+d}$$

as n tends to infinity. It follows from this that the distribution of NPV will not tend to normality unless every single cash flow is normal. (In practice it seems likely that the distribution of NPV will be approximately normal in the example which has been considered unless the discount rate is so high that the shapes of the distributions of the early cash flows exert an undue influence. However it is difficult to prove that this is so).

Yet another theoretical difficulty is provided by the fact that, even if the Central Limit Theorem can be applied in a particular situation, there is still no guarantee that the rate of convergence is sufficiently fast to ensure that the NPV (which is usually calculated over a finite number of time periods) is even approximately normal. The conclusion which can be drawn from all of the work mentioned so far is therefore the following:

> Although the Central Limit Theorem can be used to suggest a result which might hold in a large number of investment situations, it cannot be used to prove the result in even a small subset of those situations.

One important argument which can be used to support the hypothesis that NPV will often in practice be approximately normal concerns the fact that the net cash flows in any given situation are usually calculated from a number of other variables. Generally it is true that the final stage in the sequence of calculations necessary to calculate the net cash flow C_i in year i involves using a relationship of the following form:

$$C_i = I_{i1} + I_{i2} \ldots + I_{iN} - O_{i1} - O_{i2} \ldots - O_{iM}$$

where I_{ij} ($j = 1 \ldots N$) are different categories of inflows and O_{ij} ($j = 1 \ldots M$) are different categories of outflows. It may sometimes be possible to use the Central Limit Theorem on this relationship to show that the net cash flows themselves have an approximately normal distribution. It would then follow that the NPV itself was approximately normally distributed.

So far we have only considered the arguments concerning NPV. The nature of the distribution of IRR is considered by Hillier (1963, 1969). He starts by supposing that the investment is such that all the negative cash flows occur at the beginning of its life. It follows from this that:

$$\text{Prob. } (\text{IRR} < R) = \text{Prob. } (\text{NPV} < 0/\text{Discount Rate} = R) \qquad 4.2$$

Hillier then puts forward the following argument to show that IRR is approximately normal if NPV is normal for all values of R. Suppose μ_p and σ_p are the mean and standard deviation of NPV when the discount rate is R and that μ_p' is the derivative of μ_p with respect to R. Assuming that μ_p' and σ_p are constant for different values of R, the distribution of NPV when the discount rate is R is N (μ_p, σ_p) and the distribution of NPV when the discount rate is R+Δ is N ($\mu_p + \mu_p'\Delta$, σ_p) where Δ is any given number. It follows that:

$$\text{Prob. } (\text{NPV} < -\mu_p'\Delta/\text{Disc. Rate} = R) = \text{Prob. } (\text{NPV} < 0/\text{Disc. Rate} = R + \Delta)$$

and therefore (using 4.2) that:

$$\text{Prob. (IRR} < R + \Delta) = \text{Prob. (NPV} < \mu_p'\Delta/\text{Disc. Rate} = R)$$

Hence IRR and NPV are identically distributed except for location and scale parameters. If NPV is approximately normal it follows that (with the assumptions which have been made) IRR is approximately normal. The fact that μ_p' and σ_p are not constant for different values of R means that this result can only be approximately true. However as Hillier (1969) points out $\mu_p''>0$ whereas $\sigma_p'<0$ so that the errors in the analysis caused by μ_p' not being constant will tend to cancel out errors caused by σ_p not being constant.

These arguments of Hillier concerning the distribution of IRR are criticised by Robichek (1975). He considers an investment requiring $1 at time t=0 with a single normally distributed inflow C_n occurring at time n. The NPV of the investment is normal for all discount rates. The IRR however is given by:

$$(C_n)^{\frac{1}{n}} - 1$$

and is (for n > 1) not even approximately normal

Robichek also uses his example to draw a distinction between the 'expected rate of return' (which he defines at the rate that equates the expected periodic cash flows to zero) and the mean of the distribution of simulated IRRs. The former is:

$$\left[E\ (C_n)\right]^{\frac{1}{n}} - 1$$

whereas the latter is:

$$E\left[(C_n)^{\frac{1}{n}} - 1\right]$$

(where E denotes expected value). As it is always true that

$$\left[E\ (C_n)\right]^{\frac{1}{n}} \geqslant E\left[(C_n)^{\frac{1}{n}}\right]$$

it follows that the 'expected rate of return' will in general exceed the mean of simulated IRRs.

4.3 EMPIRICAL RESULTS CONCERNING THE NORMALITY OF NPV AND IRR

Three test risk simulations (to be referred to as simulations nos. 1, 2 and 3) were carried out on each of the case studies described in chapter 3. For simulation no. 1, it was assumed that certain estimates (see Appendix C) had been made of the lowest possible value, the highest possible value and the most likely value of each the variables and triangular distributions were fitted to the estimates as indicated in figure 4.1. In simulation no. 2, uniform distributions (see figure 4.2) were used for the variables and in simulation no. 3, ⌴-shaped distributions (see figure 4.3) were used. In both simulation no. 2 and simulation no. 3 the parameters of the distributionswere chosen so that the variables had the same means and standard deviations as in simulation no. 1. Thus, if we suppose that, for a given variable, μ and σ were the mean and standard deviation of the triangular distribution used in simulation no. 1, then the upper and lower limits of the uniform distribution used for the variable in simulation no. 2 were:

$$\mu + \sqrt{3}\ \sigma$$

$$\text{and}\quad \mu - \sqrt{3}\ \sigma$$

respectively and the upper and lower limits of the $\triangle$ shaped distribution used for the variable in simulation no. 3 were:

$$\mu + 2\sqrt{2}\ \sigma$$

$$\text{and}\quad \mu - \sqrt{2}\ \sigma$$

respectively. μ and σ can themselves be calculated from the lowest possible value a, the highest possible value, b, and the most likely value m, using the relationships

$$\mu = \frac{a + b + m}{3}$$

$$\sigma = \left\{\left[\frac{(b-a)^2 + (m-a)(m-b)}{18}\right]\right\}^{\frac{1}{2}}$$

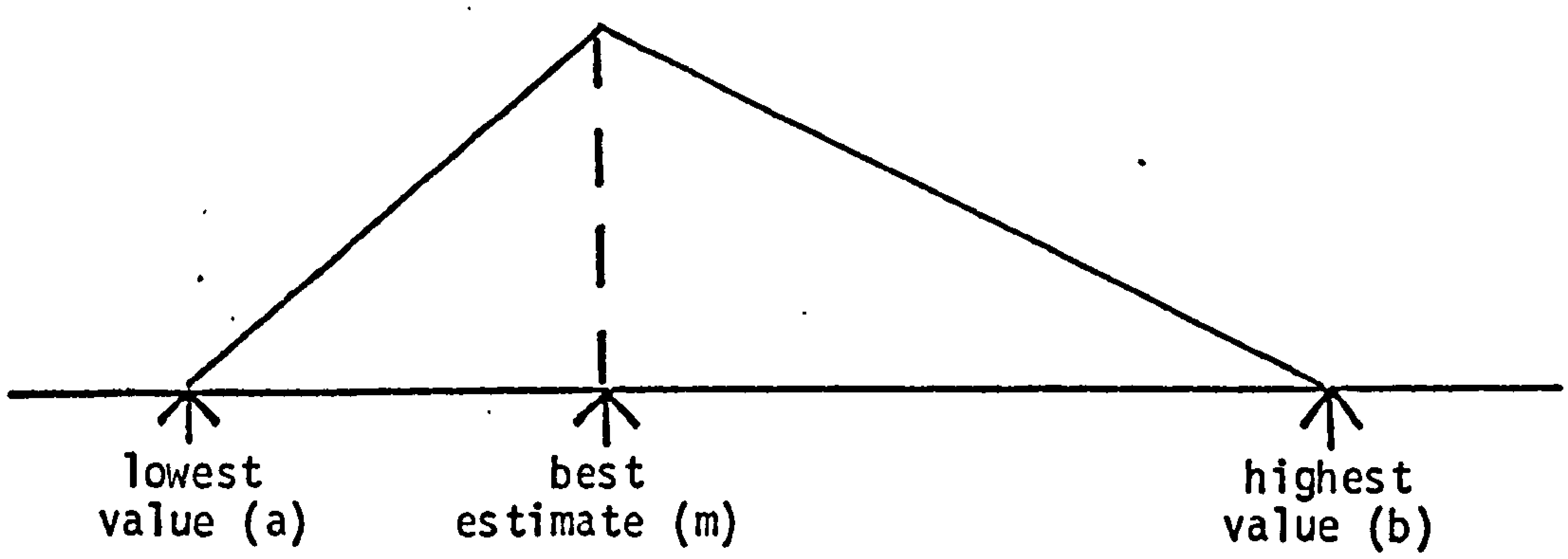

Figure 4.1 Distributions used in Simulation No. 1

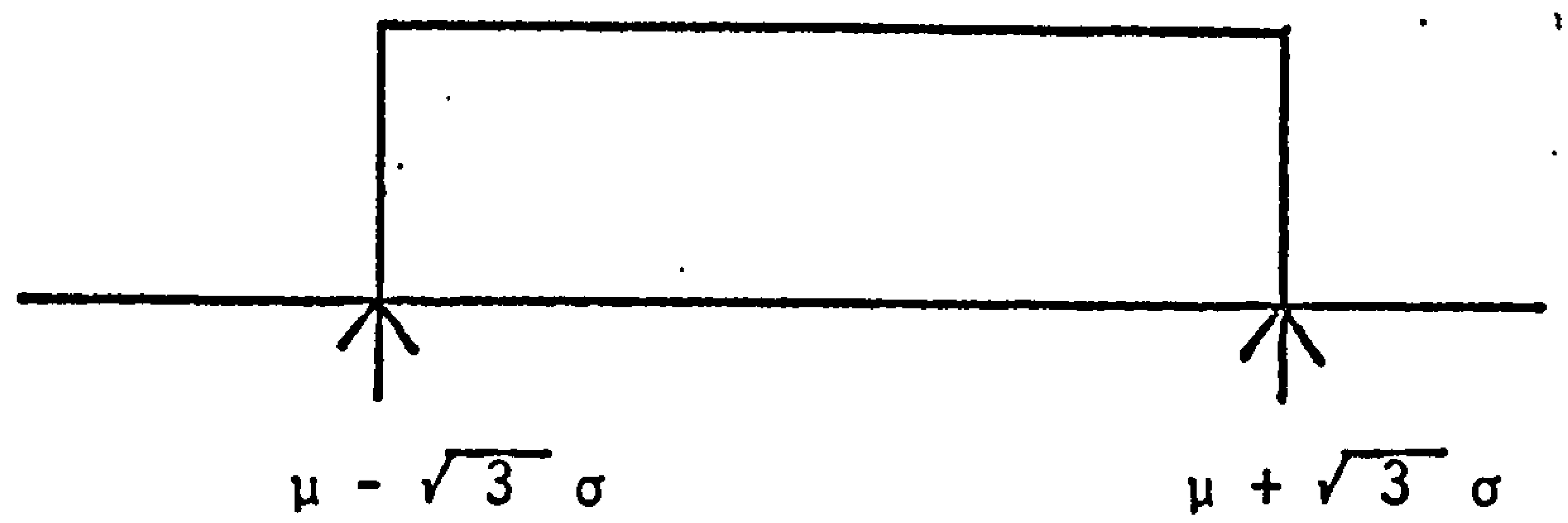

Figure 4.2 Distributions used in Simulation No. 2

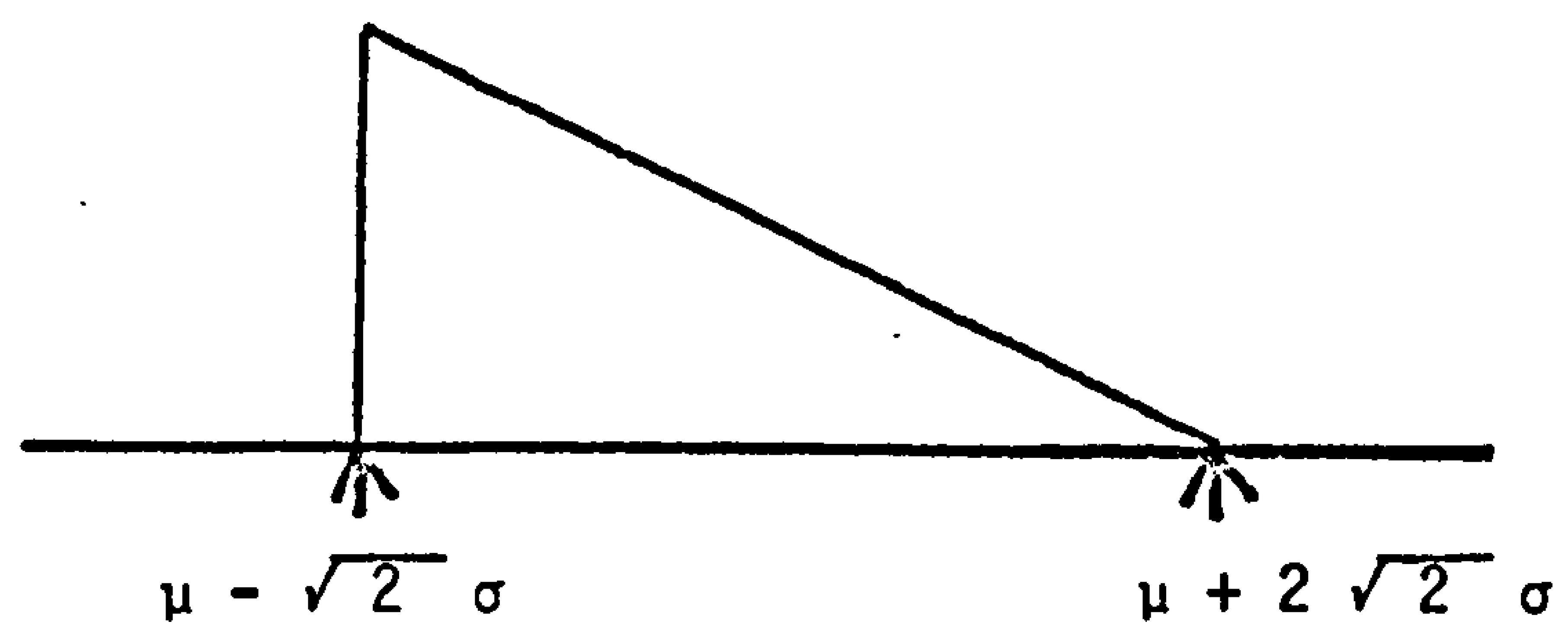

Figure 4.3 Distributions used in Simulation No. 3

Each of the 15 risk simulations carried out involved 2000 runs. The complete distributions of NPV (discount rate = 10%) and IRR were calculated and are shown in appendix D. Two different tests of normality were made on the distributions. These involved:

(a) calculating the chi-square statistic

and (b) calculating the percentage of the distribution lying:

(i) to the right of a point 2 standard deviations above the mean

(ii) to the right of a point 1 standard deviation above the mean

(iii) to the left of a point 1 standard deviation below the mean

(iv) to the left of a point 2 standard deviations below the mean

The values which were calculated for the chi-square statistic are shown in table 4.1. This statistic is not strictly relevant to the present problem as the hypothesis is that the distributions of NPV and IRR are approximately normal, not that they are exactly normal. (It is to be expected that the hypothesis of exact normality will be rejected when 2000 observations are taken). Nevertheless the relative magnitudes of the statistic for different case studies and different performance measures are of interest.

The class intervals which were used in the calculation of the chi-square statistic were not the same as those used in the presentation of the distributions in appendix D. They were instead chosen as:

less than u - 3.0 v
u - 3.0 v to u - 3.0 v
u - 2.8 v to u - 2.6 v
.
.
.
.
u + 2.8 v to u + 3.0 v
greater than u + 3.0 v

where u is the mean and v is the standard deviation calculated for the performance measure. There were therefore always 32 class intervals and it follows that each of the chi-square statistics shown in table 4.1 has 29 degrees of freedom. If the hypothesis had been exact normality this would have been rejected, using a level of significance of 0.05, when the value of the statistic exceeded 42.6 and, using a level of significance of 0.01, when the value of the statistic exceeded 49.6.

Table 4.1 Values of the Chi-square Statistic when taking the Distributions of NPV and IRR for Normality

	Values of the Chi-square Statistic					
	Distribution of NPV			Distribution of IRR		
	Sim. No. 1	Sim. No. 2	Sim. No. 3	Sim. No. 1	Sim. No. 2	Sim. No. 3
Case A	73	102	143	94	53	100
Case B	100	63	171	10620	9280	10428
Case C	1913	2218	2475	20862	18797	18691
Case D	30	36	63	378	252	155
Case E	336	307	623	516	542	607

(The cases A - E are described on pages 34 - 42)

The statistics which were calculated in the second of the tests for normality are shown in tables 4.2 - 4.7. They provide measures of the 'goodness of approximation to normality' as far as the tails of the distributions are concerned. It should be noted that the standard error of each percentage in columns 1 and 4 of tables 4.2 - 4.7 is approximately 0.3 and the standard error of each percentage in columns 2 and 3 is approximately 0.8.

Table 4.2 Comparison of the tails of the Distributions of NPV with the Normal Distribution in Sim. No. 1

	Percentage of the Distribution of NPV in Sim. No. 1 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	2.35	13.45	14.50	3.10
Case B	2.70	12.85	14.70	2.80
Case C	0.00	12.00	18.00	3.70
Case D	2.35	15.30	16.20	2.45
Case E	0.05	19.35	18.45	3.15
Normal Dist.	2.28	15.87	15.87	2.28

Table 4.3 Comparison of the tails of the Distribution of IRR with the Normal Distribution in Sim. No. 1

	Percentage of the Distribution of IRR in Sim. No. 1 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	2.50	14.55	14.10	1.55
Case B	0.00	32.80	2.85	0.00
Case C	0.00	40.15	0.00	0.00
Case D	0.05	13.90	17.15	2.95
Case E	0.25	19.35	19.05	3.10
Normal Dist.	2.28	15.87	15.87	2.28

Table 4.4 Comparison of the tails of the Distributions of NPV with the Normal Distribution in Sim. No. 2

	Percentage of the Distribution of NPV in Sim. No. 2 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	1.45	14.50	15.30	3.40
Case B	1.85	14.75	15.60	3.55
Case C	0.00	12.85	19.00	3.70
Case D	1.90	16.45	16.35	2.35
Case E	0.00	20.20	19.05	1.75
Normal Dist.	2.28	15.87	15.87	2.28

Table 4.5 Comparison of the tails of the Distributions of IRR with the Normal Distribution in Sim. No. 2

	Percentage of the Distribution of NPV in Sim. No. 3 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	2.65	15.35	14.90	2.15
Case B	0.00	34.35	3 20	0.00
Case C	0.00	41.05	0.00	0.00
Case D	0.15	16.50	19.55	2.45
Case E	0.20	19.20	20.95	1.75
Normal Dist.	2.28	15.87	15.87	2.28

Table 4.6 Comparison of the tails of the Distributions of NPV with the Normal Distribution in Sim. No. 3

	Percentage of the Distribution of NPV in Sim No. 3 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	1.65	13.65	13.35	3.45
Case B	2.75	12.50	12.95	2.65
Case C	0.00	0.04	16.80	4.50
Case D	1.20	15.75	16.70	3.20
Case E	0.00	19.80	18.70	3.30
Normal Dist.	2.28	15.87	15.87	2.28

Table 4.7 Comparison of the tails of the Distributions of IRR with the Normal Distribution in Sim. No. 3

	Percentage of the Distribution of IRR in Sim. No. 3 which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	2.50	14.35	13.35	1.60
Case B	0.00	32.05	2.75	0.00
Case C	0.00	43.25	0.00	0.00
Case D	0.45	18.10	18.85	1.80
Case E	0.00	18.60	19.00	2.45
Normal Dist.	2.28	15.87	15.87	2.28

4.4 DISCUSSION OF RESULTS CONCERNING THE NORMALITY OF NPV AND IRR

A close examination of tables 4.1 to 4.7 reveals that the performance measures can be divided into two groups, the first group comprising:

NPV in case A
NPV in case B
NPV in case D
IRR in case A

and the second group comprising:

NPV in case C
NPV in case E
IRR in case B
IRR in case C
IRR in case D
IRR in case E

The distributions in the second group cannot in any meaningful sense be described as 'approximately normal'. (A casual inspection of the graphical displays in appendix D reveals this fact). The distributions in the first group can be described as approximately normal in the sense that:

(a) the value calculated for the chi-square statistic is less than 200 (Degrees of freedom = 29; Sample size = 2000)

(b) the values calculated for Prob. $(x > u + 2\ v)$ and Prob. $(x < u - 2\ v)$ are both within 0.013 of the value applicable for a normal distribution

and (c) the values calculated for Prob. $(x > u + v)$ and Prob. $(x < u - v)$ are both within 0.034 of the value applicable for a normal distribution.

The non-normality of the distributions in the second group can to a large extent be explained by the following observations:

(i) The cash flow model in case study C has a discontinuity in that it incorporates the condition 'if the net cash flow is negative in the first year then the project will be abandoned'. (It is interesting to note that the distribution of NPV in case C - see appendix D - appears to consist of two approximately normal parts, the first part corresponding to the situation where the project is abandoned, the second part corresponding to the situation where it is not abandoned).

(ii) The cash flow model in case study E is 'highly non-linear' each variable being either 'a growth rate' or 'a year in which an event happens'. Also the cash flow model in case E involves relatively few uncertain variables.

(iii) In case study B there is a probability of over 30% that all the net cash flows will be negative. This means that there is a probability of over 30% that the equation for IRR has no finite real root. (Note that for computational convenience the IRR has been assumed to be -100% when all cash flows are negative and +100% when all cash flows are positive).

(iv) In case study C the cash flows are certain to be either all positive or all negative (i.e. the equation for IRR is certain to have no finite real root).

(v) The non-normality of the distribution of IRR in case D is due to the uncertainty in the non-linear variable 'start year'. Extra simulations (see appendix E) were carried out in which this variable was held fixed at its best estimate and approximately normal distributions for IRR satisfying all the criteria mentioned above was obtained.

4.5 THEORETICAL RESULTS CONCERNING TYPE I VARIABLES AND TYPE I DEPENDENCIES

The research of Wagle (1967) and Hillier (1963, 1969) can be used to suggest a number of results concerned with type I variables and type I dependencies.

Suppose that an investment has life n, that C_0, $C_1 \ldots C_n$ are the net cash flows in years 0, 1....n and that X_1, $X_2 \ldots X_n$ are the variables from which the cash flows are calculated. Consider first the situation where each cash flow is a linear function of the variables i.e.

$$C_i = \sum_{j=1}^{m} \alpha_{ij} X_j \qquad i = 0, 1 \ldots n$$

for some constants α_{ij} $(0 \leq i \leq n, 1 \leq j \leq m)$. Since NPV is a linear function of the C_is, it must itself be a linear function of the X_js. Suppose:

$$NPV = \sum_{j=1}^{m} \beta_j X_j$$

where the β_j $(0 \leq j \leq m)$ are constants. Then:

$$E\,(NPV) = \sum_{j=1}^{m} \beta_j \; E\,(X_j)$$

and

$$Var\,(NPV) = \sum_{j=1}^{m} \beta_j^2 \; Var\,(X_j) + 2 \sum_{j \neq k} \beta_j \beta_k \; Cov\,(X_j, X_k)$$

where E denotes expected value, Var denotes variance and Cov denotes covariance.

Since:

$$\mathrm{Cov}\ (X_j, X_k) = \rho_{jk} \sqrt{\mathrm{Var}\ (X_j)\ \ \mathrm{Var}\ (X_k)}$$

where ρ_{jk} is the coefficient of correlation between X_j and X_k it follows that the mean and standard deviation of NPV depend only on (a) the means and standard deviations of the X_js and (b) the coefficients of correlation between the X_js. If NPV is approximately normal (so that it is to a reasonable approximation completely determined by its mean and its standard deviation) it follows that all the X_js and all the dependencies between the X_js are approximately type I with respect to NPV

If each C_i is not a linear function of the X_js the situation is more complicated. Wagle (1967) considers situations where the product of two variables is taken. He points out that if X_1 and X_2 are any two variables with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 then:

(i) if X_1 and X_2 are independent

$$E\ (X_1X_2) = \mu_1\ \mu_2 \qquad 4.3$$

$$\text{and } \mathrm{Var}\ (X_1X_2) = \mu_1^2\ \sigma_2^2 + \mu_2^2\ \sigma_1^2 + \sigma_1^2\ \sigma_2^2 \qquad 4.4$$

(ii) If X_1 and X_2 are dependent with coefficient of correlation ρ:

$$E\ (X_1X_2) = \mu_1\ \mu_2 + \rho\ \sigma_1\ \sigma_2 \qquad 4.5$$

$$\begin{aligned}\text{and } \mathrm{Var}\ (X_1X_2) = {} & \mu_1^2\ \sigma_2^2 + \mu_2^2\ \sigma_1^2 + \sigma_1^2\ \sigma_2^2 + 2\mu_1\ \mu_2\ \rho\ \sigma_1\ \sigma_2 \\ & + 2\mu_1\ E_{12} + 2\mu_2\ E_{21} + E_{22} - E_{11}^2 \qquad 4.6\end{aligned}$$

$$\text{where } E_{ij} = E\left[(X_1 - \mu_1)^i\ \ (X_2 - \mu_2)^j\right]$$

This enables a few more situations to be considered analytically. For example, if X_1 and X_2 are independent of each other and of all other variables and if each net cash flow is a linear function of

$$X_1X_2,\ X_3,\ X_4 \ldots . X_m$$

then (providing NPV is approximately normal) the X_js will be approximately type I. Unfortunately however no general results applicable to situations where variables are multiplied together in a cash flow model are available. To illustrate the reason why this is so, suppose that the X_j are independent and that:

$$NPV = \sum_k \gamma_k X_1^{q_{1k}} X_2^{q_{2k}} \ldots X_m^{q_{mk}} \qquad 4.7$$

where the γ_k are constants and the q_{jk} are non-negative integers. Although in certain circumstances it may be possible to use equations 4.3 and 4.4 to calculate the mean and standard deviation of any one term on the right hand side of 4.7 from the means and standard deviations of the X_js, there is in general no way in which the coefficient of correlation between two different terms on the right hand side of 4.7 can be calculated. (It is however worth noting that Wagle (1967) does provide some useful results for tackling models such as the one in equation 4.7 when the X_js have a multivariate normal distribution).

Finally it should be pointed out that since the distribution of IRR can be obtained by using the relationship:

Prob. (IRR < R) = Prob. (NPV < 0/Disc. Rate = R)

it follows that, if a variable or a dependence is approximately type I with respect to NPV for all discount rates, it is also approximately type I with respect to IRR.

4.6 EMPIRICAL RESULTS CONCERNING TYPE I VARIABLES

The parameters of the uniform distributions used in simulations No. 2 and of the ◺ shaped distributions used in simulation No. 3 were chosen so that the distributions had the same means and standard deviations as the corresponding triangular distributions used in simulation No. 1. It follows that an overall indication of the extent to which the variables in case studies A, B, C, D and E are approximately type I can be obtained by comparing the outputs from the three sets of simulations.

Table 4.8 shows the values obtained for the chi-square statistic:

$$\sum_{i=1}^{n} \left[\frac{(O_{i1} - E_i)^2}{E_i} + \frac{(O_{i2} - E_i)^2}{E_i} + \frac{(O_{i3} - E_i)^2}{E_i} \right]$$

when n is the number of class intervals, O_{ij} is the number of observations in the i-th class interval in simulation No. j and:

$$E_i = \sum_{j=1}^{3} \frac{O_{ij}}{3}$$

The number of degrees of freedom of this statistic is 2 (n-1) and the class intervals used in its calculation coincide with those in appendix D. Two points should however be born in mind when table 4.9 is being interpreted:

(i) The hypothesis is that the distributions are approximately the same, not that they are exactly the same. The chi-square statistic cannot therefore be used in the usual way to accept or reject the hypothesis.

(ii) The values of the chi-square statistic for the distributions of IRR in case studies B and C are to a large extent meaningless because - as pointed out in section 4.4 - the equation for IRR frequently had no solution in these case studies.

Table 4.8 Comparison of Distributions obtained for NPV and IRR by Means of Chi-Square Statistic

	Value of chi-square Statistic		Degrees of Freedom	
	NPV	IRR	NPV	IRR
Case A	61	50	60	54
Case B	66	26	68	70
Case C	87	4	56	58
Case D	62	118	66	78
Case E	101	178	50	88

Tables 4.9 - 4.18 compare the tails of the distributions output from simulation nos. 1, 2 and 3 in case studies A, B, C, D and E by displaying:

Prob. (Perf. Meas > X_1); Prob. (Perf. Meas > X_2)
and Prob. (Perf. Meas < Y_1); Prob. (Perf. Meas < Y_2)

where X_1 and X_2 are convenient values approximately 1 and 2 standard deviations above the mean of the performance measure and Y_1 and Y_2 are convenient values approximately 1 and 2 standard deviations below the mean of the performance measure. (The distributions obtained for IRR in case studies B and C have not been compared in tables 4.9 - 4.18 for the reasons already mentioned).

Table 4.9 Comparison of Tails of Distributions obtained for NPV in Case A

	Prob. NPV > 18000	Prob. NPV > 8000	Prob. NPV < -12000	Prob. NPV < -22000
Simulation No. 1	.026	.125	.148	.026
Simulation No. 2	.030	.130	.156	.017
Simulation No. 3	.029	.118	.150	.017

Table 4.10 Comparison of Tails of Distributions obtained for IRR in Case A

	Prob. IRR > 45	Prob. IRR > 25	Prob. IRR < -20	Prob. IRR < -45
Simulation No. 1	.031	.151	.135	.020
Simulation No. 2	.023	.158	.140	.024
Simulation No. 3	.021	.147	.124	.020

Table 4.11 Comparison of Tails of Distributions obtained for NPV in Case B

	Prob. NPV > 13000	Prob. NPV > 7000	Prob. NPV < -4000	Prob. NPV < -10000
Simulation No. 1	.028	0.161	.128	.023
Simulation No. 2	.032	0.164	.132	.013
Simulation No. 3	.030	0.148	.133	.028

Table 4.12 Comparison of Tails of Distributions obtained for NPV in Case C

	Prob. NPV > 260	Prob. NPV > 160	Prob. NPV < -20	Prob. NPV < -120
Simulation No. 1	.034	.186	.127	0
Simulation No. 2	.031	.199	.132	0
Simulation No. 3	.046	.186	.105	0

Table 4.13 Comparison of Tails of Distributions obtained for NPV in Case D

	Prob. NPV > 9900	Prob. NPV > 9400	Prob. NPV < 8300	Prob. NPV < 7800
Simulation No. 1	.025	.151	.153	.027
Simulation No. 2	.022	.151	.165	.022
Simulation No. 3	.028	.148	.153	.013

Table 4.14 Comparison of Tails of Distributions obtained for IRR in Case D

	Prob. IRR > 87	Prob. IRR > 81	Prob. IRR < 68	Prob. IRR < 62
Simulation No. 1	.032	.171	.121	. 0
Simulation No. 2	.024	.188	.137	0.001
Simulation No. 3	.009	.152	.115	0

Table 4.15 Comparison of Tails of Distributions obtained for NPV in case E

	Prob. NPV > 7000	Prob. NPV > 5000	Prob. NPV < 1000	Prob. NPV < -1000
Simulation No. 1	.036	.192	.189	0
Simulation No. 2	.019	.197	.189	0
Simulation No. 3	.043	.192	.226	0

Table 4.16 Comparison of Tails of Distributions obtained for IRR in Case E

	Prob. IRR > 15.2	Prob. IRR > 13.8	Prob. IRR < 10.8	Prob. IRR < 9.4
Simulation No. 1	.031	.190	.193	.002
Simulation No. 2	.017	.209	.192	.002
Simulation No. 3	.037	.190	.232	0

Tables 4.17 and 4.18 compare the means and standard deviations of the performance measures obtained for case studies A, B, C, D and E in simulations Nos. 1, 2 and 3.

Table 4.17 Comparison of Means and S.D.s of NPV

	Case A		Case B		Case C		Case D		Case E	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Simulation No. 1	-2719	9922	1680	5669	71	92	8835	535	3055	2025
Simulation No. 2	-2714	9884	1711	5453	72	92	8829	531	3058	1994
Simulation No. 3	-2697	9869	1671	5853	70	96	8830	521	2977	2110

Table 4.18 Comparison of Means and S.D.s of IRR

	Case A		Case B		Case C		Case D		Case E	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Simulation No. 1	1.8	22.5	N.A.	N.A.	N.A.	N.A.	74.7	6.3	12.3	1.5
Simulation No. 2	1.8	22.1	N.A.	N.A.	N.A.	N.A.	74.6	6.2	12.3	1.4
Simulation No. 3	1.9	22.2	N.A.	N.A.	N.A.	N.A.	74.7	5.6	12.2	1.5

Additional investigations were carried out to determine whether particular variables were approximately type I with respect to NPV. For each of the variables chosen, these investigations involved comparing the output obtained from the following four simulations:

Simulation (i) All variables have the triangular distributions used in simulation no. 1.

Simulation (ii) All variables except variable under consideration have the triangular distributions in simulation no. 1; the variable under consideration has a uniform distribution with the same mean and standard deviation as the corresponding triangular distribution in simulation no. 1.

Simulation (iii) All variables except variable under consideration have the triangular distributions used in simulation no. 1; the variable under consideration has a ◺-shaped distribution with the same mean and standard deviation as the corresponding triangular distribution in simulation no. 1.

Simulation (iv) All variables except variable under consideration have the triangular distributions used in simulation no. 1; the variable under consideration has a triangular distribution with the same mean as the corresponding triangular distribution in simulation no. 1, but with a standard deviation which is 10% greater.

The results obtained are summarised in tables 4.19 and 4.20. In each case the performance measure was NPV with a discount rate of 10% and 2000 runs were carried out.

Table 4.19 Comparison of Means of Performance Measure in the Simulations which were carried out to test whether particular variables were type I

Variable	Mean of NPV in simulation (i)	Mean of NPV in simulation (ii)	Mean of NPV in simulation (iii)	Mean of NPV in simulation (iv)
Selling Price: Case A	-2719	-2712	-2726	-2733
Cost Growth Rate: Case B	1680	1709	1656	1562
Start Year: Case D	8835	8835	8836	8837
Life of Plant: Case D	8835	8834	8839	8831

Table 4.20 Comparison of Standard Deviations of Performance Measure in the Simulations which were carried out to test whether particular variables were type I

Variable	S.D. of NPV in simulation (i)	S.D. of NPV in simulation (ii)	S.D. of NPV in simulation (iii)	S.D. of NPV in simulation (iv)
Selling Price: Case A	9922	9897	9788	10431
Cost Growth Rate: Case B	5669	5450	5779	6075
Start Year: Case D	535	535	532	541
Life of Plant: Case D	535	534	527	563

4.7 DISCUSSION OF RESULTS CONCERNING TYPE I VARIABLES

Consider first table 4.8. This table can be interpreted with reference to the percentage points of the chi-square distribution shown in table 4.21. If the hypothesis had been that the variables in the case studies were exactly type I then this hypothesis could not have been rejected using a level of significance of 0.05 on the basis of:

the distributions for NPV in case A
the distributions for IRR in case A
the distributions for NPV in case B
the distributions for IRR in case B
the distributions for IRR in case C
the distributions for NPV in case D

Also the hypothesis could not have been rejected using a level of significance of 0.001 on the basis of

the distributions for NPV in case C
the distributions for IRR in case D

Table 4.21 Percentage Points for Chi-Square Statistic

No. of degrees of freedom	$\chi^2_{.05}$	$\chi^2_{.01}$	$\chi^2_{.001}$
50	68	76	87
60	79	88	100
70	91	100	112
80	102	1112	125

If the term 'the variable is approximately type I' is taken to mean:

the hypothesis that the variable is exactly type I cannot be rejected using a level of significance of 0.001.

then it can be concluded that the results which have been produced suggest that the variables in case studies A, B, C and D are approximately type I with respect to both NPV and IRR for the range of distributions considered. The variables in case E are not approximately type I and this is probably in part due to the fact that there are only 4 variables in case E.

From tables 4.9 to 4.16 it can be seen that the tails of the distributions obtained for NPV and IRR in simulation nos. 1, 2 and 3 are fairly similar. (Note that the standard error of the numbers in columns 1 and 4 of the tables is approximately 0.003 and that the standard error of the numbers in columns 2 and 3 is approximately 0.008). The only notable differences occur in:

(a) the right hand tails of the distributions of IRR in case D

and (b) the right hand tails of the distributions of both NPV and IRR in case E.

(a) is due to the fact that the relationship between IRR and the variable 'start year' in case D is 'highly non-linear' (This was demonstrated by an extra run where 'start year' was held fixed at its best estimate - see appendix E).

From tables 4.17 and 4.18 it can be seen that the means and standard deviations obtained for NPV and IRR in simulations nos. 1, 2 and 3 are similar.

Whereas tables 4.9 to 4.18 show the effect of making changes to all the distributions in the case studies simultaneously, tables 4.19 and 4.20 show the effect of changing the distribution of one of the variables while keeping all the other distributions fixed. It is clear from tables 4.19 and 4.20 that the effect on the distribution of NPV of a 10% increase in the standard deviation of a variable is considerably greater than that of an alteration in the shape of the distribution of the variable. Define:

Φ_1 = Change in mean of NPV resulting from changing the distribution of the variable from triangular to uniform (i.e. col. 2 - col. 1 in table 4.19).

Φ_2 = Change in mean of NPV resulting from changing the distribution of the variable from triangular to $\triangle$ shaped (i.e. col. 3 - col. 1 in table 4.19).

Θ = Change in mean of NPV resulting from a 10% increase in the S.D. of the variable (i.e. col. 4 - col. 1 in table 4.19).

Ψ_1 = Change in S.D. of NPV resulting from changing the distribution of the variable from triangular to uniform (i.e. col. 2 - col. 1 in table 4.20).

Ψ_2 = Change in S.D. of NPV resulting from changing the distribution of the variable from triangular to $\triangle$ shaped (i.e. col. 3 - col. 1 in table 4.20)

ω = Change in S.D. of NPV resulting from 10% increase in S.D. of the variable (i.e. col. 4 - col. 1 in table 4.20).

Define:

$$\xi = \frac{\Phi_1 + \Phi_2}{2} \quad ; \quad \eta = \frac{\Psi_1 + \Psi_2}{2}$$

$$\alpha = \frac{\Theta}{\xi} \quad ; \quad \beta = \frac{\omega}{\eta}$$

α and β are rough measures of the extent to which a variable is type I. Their values for the variables in tables 4.19 and 4.20 are shown in table 4.22.

Table 4.22 Values of α and β

Variable	α	β
Selling Price: Case A	2.0	6.4
Cost Growth Rate: Case B	4.5	2.5
Start Year: Case D	4.0	4.0
Life of Plant: Case D	1.6	6.2

4.8 EXTENSION OF ANALYSIS TO COVER NON-UNIMODAL DISTRIBUTIONS

The triangular, uniform and ◺shaped distributions which have been considered so far in this chapter are fairly extreme examples of what might be termed 'pointed', 'flat' and 'skewed' distributions. The one thing which they do all have in common however is that they are unimodal. (A uniform distribution can be regarded as the limiting case of a unimodal distribution). In order to provide an indication of the extent to which the results which have been produced can be extended to cover non-unimodal distributions, further simulations were carried out with each variable being described by a symmetrical V-shaped distribution (see figure 4.4). If μ and σ are the mean and standard deviation of the triangular distribution used for a variable then the bounds of the V-shaped distribution used for the variable are:

$$\mu - \sqrt{2}\,\sigma$$

$$\text{and } \mu + \sqrt{2}\,\sigma$$

(This ensures that the V-shaped distributions have the same means and standard deviations as the corresponding triangular, uniform and ◺shaped distributions).

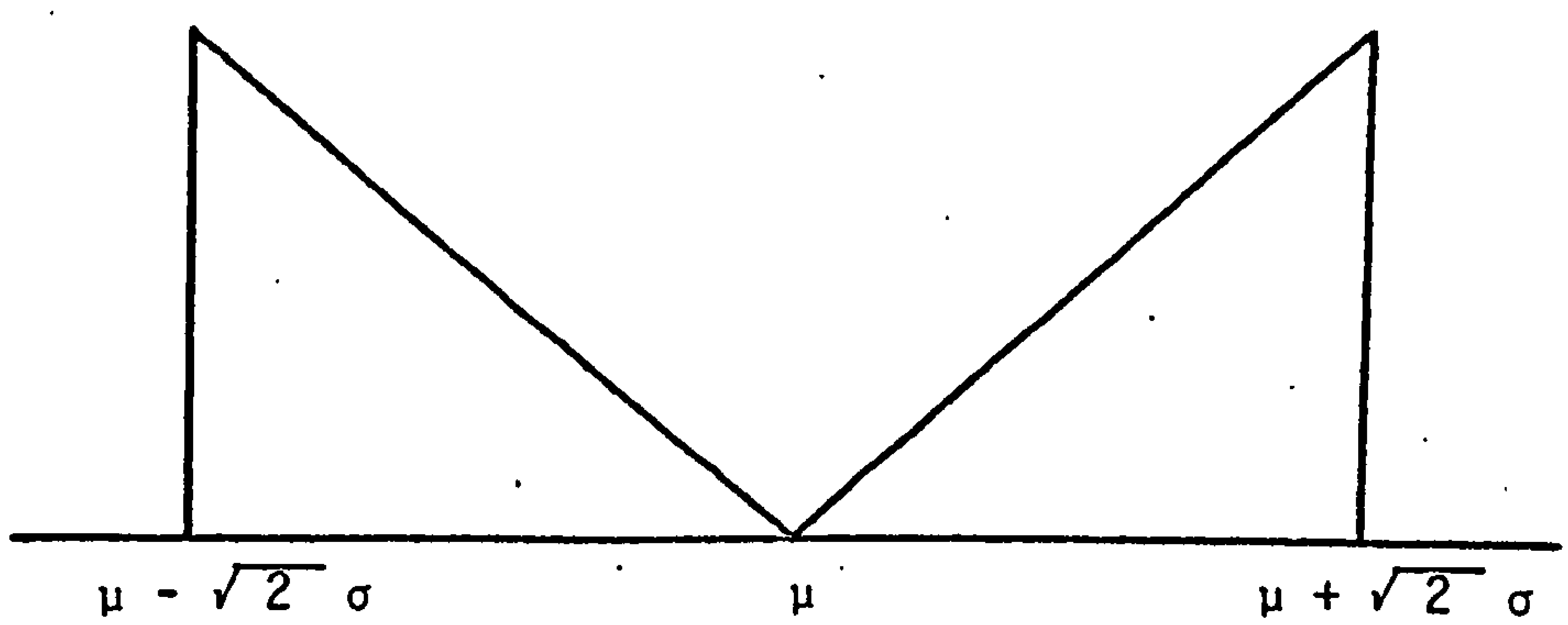

Figure 4.4 A V-Shaped Distribution

The distributions output from simulations using V-shaped distributions are displayed and analysed in appendix E. The appendix shows that some of the results obtained in this chapter for case studies A-E can in fact be extended to cover the use of V-shaped distributions.

4.9 SUMMARY AND CONCLUSIONS

This chapter set out to consider three questions. The first of these questions was concerned with the normality of the distributions of NPV and IRR; the second was concerned with type I variables; the third was concerned with type I dependencies.

As far as the first question is concerned it has been shown that:

(a) the distribution of NPV is approximately normal in 3 out of 5 of the case studies for a wide range of shapes of input distributions.

and (b) the distribution of IRR is approximately normal in 1 out of 5 of the case studies for a wide range of shapes of input distributions.

(For the sense in which the term 'approximately normal' is used, see section 4.4).

Three main reasons why NPV might not be approximately normal in a given situation have been identified. These are:

(i) the investor might have options (e.g. abandonment or expansion options) open to him at stages during the project's life.

(ii) there might be non-linearities in the cash flow model. (These could be caused either by the presence of variables such as 'growth rate' and 'life of project' or by conditions within the model itself e.g. the condition: supply the export market only if there is sufficient capacity after the home market has been satisfied).

and (iii) there might be insufficient uncertain variables.

The shapes of the distributions which are input would appear to be a relatively unimportant consideration.

Table 4.23 summarises the results obtained in section 4.3. It can be seen that non-linearities do not always cause the distribution of NPV to be markedly different from normal and that 9 uncertain variables were sufficient in case A to produce an approximately normal distribution. (It is of course difficult to come up with a general rule concerning the number of uncertain variables necessary to produce a normal distribution; this must depend to some extent on the cash flow model and on the relative sensitivities of the different variables).

Table 4.23 Summary of Results concerning the Normality of NPV

	Total No. of Variables	No. of non-linearities	Did investor have options	Was NPV approximately normal
Case A	9	2	No	Yes
Case B	12	3	No	Yes
Case C	7	1	Yes	No
Case D	17	2	No	Yes
Case E	4	4	No	No

Hillier (1963, 1969) and Wagle (1967) assume that NPV is approximately normal in the models which they consider. As these models do not in general deal with situations where there are options open to the investor or with situations where there are non-linearities in the cash flow model, this assumption may be reasonable. Indeed on the basis of the results produced in this chapter it seems likely that, if a cash flow model is sufficiently 'well behaved' for the Hillier - Wagle analytic approach to be applicable, then the distribution of NPV will be approximately normal providing that there are a sufficient number of uncertain variables in the model.

The theoretical arguments suggesting that the distribution of IRR is approximately normal are in any given situation likely to be weaker than those for NPV (see section 4.2). Furthermore, one reason in addition to (i), (ii) and (iii) above, to explain why IRR might not be normal in a given situation, has been identified: there could be a significant probability that all cash flows have the same sign and that the equation for IRR does not in consequence have a real finite root.

In the case of the second of the questions discussed in this chapter it has been shown that the hypotheses:

all variables in case A are exactly type I with respect to NPV

all variables in case A are exactly type I with respect to IRR

all variables in case B are exactly type I with respect to NPV

all variables in case B are exactly type I with respect to IRR

all variables in case C are exactly type I with respect to IRR

all variables in case D are exactly type I with respect to NPV

could not be rejected using a significance level of 0.05 on the basis of tests carried out using triangular, uniform and $\triangle$ shaped distributions and that the hypotheses:

all variables in case C are exactly type I with respect to NPV

all variables in case D are exactly type I with respect to IRR could not be rejected using a significance level of 0.001 on the basis of the tests. The distributions which were produced for case E using differently shaped distributions could not be regarded as 'approximately type I' on the basis of the criterion which was used (see section 4.7).

It is particularly notable that the probability distribution of NPV being approximately normal has been shown <u>not</u> to be a necessary pre-requisite for the variables in the cash flow model being type I. (Case C provides the evidence here).

There are a number of reasons why a particular variable X, might not be approximately type I in a given situation. For example:

(a) the decision on whether to abandon the investment might depend on whether X is less than some pre-determined level. (Often however abandonment decisions will - as in case C - be based on the values of variables which are calculated from input variables not on the values of input variables themselves).

(b) the uncertainty in the performance measure might depend almost entirely on the uncertainty in X.

and (c) there might be very few uncertain variables apart from X in the cash flow model.

Nevertheless the results which have been obtained do suggest that variables are approximately type I in a wide variety of situations.

As far as the third question mentioned in section 4.1 is concerned it has only been possible to present a number of theoretical arguments. If NPV can be assumed to be normal, these arguments show that the dependencies between variables which are added together in a cash flow model are type I and that dependencies between variables which are multiplied together are in certain circumstances approximately type I.

CHAPTER 5

A COMPARISON OF METHODS FOR ASSESSING SUBJECTIVE PROBABILITY DISTRIBUTIONS

5.1 INTRODUCTION

The analyses carried out in chapter 4 strongly suggest that the most important parameters of most of the variables in risk evaluation models are their means and their standard deviations. This chapter considers how accurately the different methods available for assessing subjective probability distributions are capable of providing estimates for these parameters.

There is not a great deal of literature in this area. A number of authors have - mainly in connection with the use of PERT - investigated methods for estimating the mean and standard deviation of a variable from optimistic, pessimistic and best estimates of its value. Malcolm et al (1959) were the first to suggest the following well known formulae (which are based on the beta distribution).

$$\mu = \frac{p_0 + 4m + p_{100}}{6} \qquad 5.1$$

$$\sigma = \frac{p_{100} - p_0}{6} \qquad 5.2$$

where p_n is the n-th percentile of the variable, m is its mode, μ is its mean and σ is its standard deviation. Later Moder and Rogers (1968) suggested the use of 5 and 95 percentiles and derived the following formula for the standard deviation:

$$\sigma = \frac{p_{95} - p_5}{3.2} \qquad 5.3$$

Perry and Grieg (1975) suggest

$$\sigma = \frac{p_{95} - p_5}{3.25} \qquad 5.4$$

if the distribution is known to be 'rounded' in shape rather than 'peaked' and these authors also show that:

$$\mu = \frac{p_5 - 0.95m + p_{95}}{2.95} \qquad 5.5$$

provides good estimates of the mean for a range of beta, gamma and lognormal distributions. (For distributions not extremely skewed the estimation error using equation 5.5 was found to be less than 5% of the standard deviation and usually in the region of 1% or 2% of the standard deviation).

Pearson and Tukey (1965) have investigated the use of other percentiles. They found that when the median is assessed instead of the mode better estimates can be obtained. For example their estimate for the mean of a distribution using the median, 5 percentile and 95 percentile is:

$$\mu = p_{50} + 0.185 \; (p_{95} + p_5 - 2p_{50}) \qquad 5.6$$

and this usually gives errors equal to less than 0.1% of the standard deviation.

The formulae in equations 5.1 to 5.6 could be used directly in the analytic approaches to risk analysis suggested by Hillier (1963) and Wagle (1967). They could also be used in a risk simulation study (although complete distributions would then have to be constructed with the calculated means and standard deviations for the purposes of sampling). Three points should, however, be born in mind:

(i) the formulae in equations 5.1 to 5.6 have been constructed with reference to standard distributions such as the beta, the lognormal and the gamma.

(ii) the estimation errors which are quoted take no account of biases inherent in the estimates made by management (For a discussion of biases, see section 2.4)

(iii) the estimation errors take no account of the extent to which management are capable of discriminating between different values of the variable and different probabilities when making assessments (For example, if management are only capable of providing estimates to the nearest 0.1 then the errors quoted above are liable too small).

As far as (ii) is concerned, Wallace (1975) provides an interesting discussion of adjustments which can be made to allow for a central bias. He suggests that:

$$\frac{\sigma'}{\sigma} = \kappa$$

for a value of κ between 0.35 and 0.8 where σ' is the value of the standard deviation calculated from estimates and σ is what might be termed a 'realistic standard deviation'.

The research which is described in this chapter involves defining a number of different 'true' probability distributions and investigating the accuracy of the means and standard deviations which would be calculated using a number of different assessment procedures. The 'true' probability distributions represent the distributions which the assessor would produce if he were capable of making an infinitely large number of infinitely accurate individual probability assessments.

Two separate sets of analyses are carried out. The first assumes that the assessor is capable of making a finite number of infinitely accurate probability assessments; the second assumes that the assessor makes a finite number of assessments, but that his ability to discriminate between different probabilities and different values of the variable is limited.

In connection with the second set of analyses it is interesting to note that theoretical discussions of likelihood discrimination are provided by

Luce and Raiffa (1967) and Suppes (1974). Luce and Raiffa suppose that given two events a and b there is a probability P (a, b) that a subject will prefer a to b and P (b, a) that the subject will prefer b to a and that:

$$P(a, b) + P(b, a) = 1$$

Suppes provides an axiomatic treatment of the situation where a subject can only distinguish a finite number of different types of events e.g. 'Certain' events, 'more likely than not' events, 'as likely as not' events, 'less likely than not' events and 'impossible' events.

5.2 ASSESSMENT PROCEDURES CONSIDERED

The following is a list of the procedures for assessing subjective probability distributions which have been investigated. The procedures are all described and discussed in detail in chapter 2. In each case the range of possible values of the variable is assumed to be known and finite and the distribution which is being assessed is assumed to be unimodal. n equals the total number of assessments made (excluding the end points of the range). For procedures 5 and 6 n must be odd.

Procedure 1: Divide the range of possible values of the variable into n+1 intervals of equal width. Assess the probability of variable lying in each interval. Fit a piecewise linear cumulative distribution function to the assessments.

Procedure 2: Divide the range of possible values of the variable into n intervals of equal width. Assess the probability of the variable lying in each interval. Fit a piecewise quadratic cumulative distribution function to the assessments.

Procedure 3: Assess the $\frac{1}{n+1}, \frac{2}{n+1}, \frac{3}{n+1} \ldots\ldots \frac{n}{n+1}$ fractiles of the distribution. Fit a piecewise linear cumulative distribution function to the assessments.

Procedure 4: Assess the $\frac{1}{n+1}, \frac{2}{n+1}, \frac{3}{n+1} \ldots\ldots \frac{n}{n+1}$ fractiles of the distribution. Fit a piecewise quadratic cumulative function to the assessments.

Procedure 5: Assess the mode. Calculate the $\frac{n-1}{2}$ values of the variable which divide the range of possible values above the mode into intervals of equal width and assess the chance of each occurring relative to the chance of the mode occurring. Fit a piecewise quadratic cumulative distribution function to the assessments.

Procedure 6: Define $N = \frac{n+1}{2}$. Assess the mode. Assess the two values of the variable which are $\frac{1}{N}$ times as likely as the mode. Assess the two values of the variable which are $\frac{2}{N}$ times as likely as the mode etc. Fit a piecewise quadratic cumulative distribution function to the assessments.

The probability density functions corresponding to piecewise linear and piecewise quadratic cumulative distribution functions are illustrated in figures 5.1 and 5.2 respectively. The way in which the distributions are fitted to the assessments is straightforward in procedures 1, 3, 5 and 6. In procedures 2 and 4 the method recommended by Schaifer (1971) and described in section 2.6 for fitting piecewise quadratic cumulative distribution functions to fractile assessments was used.

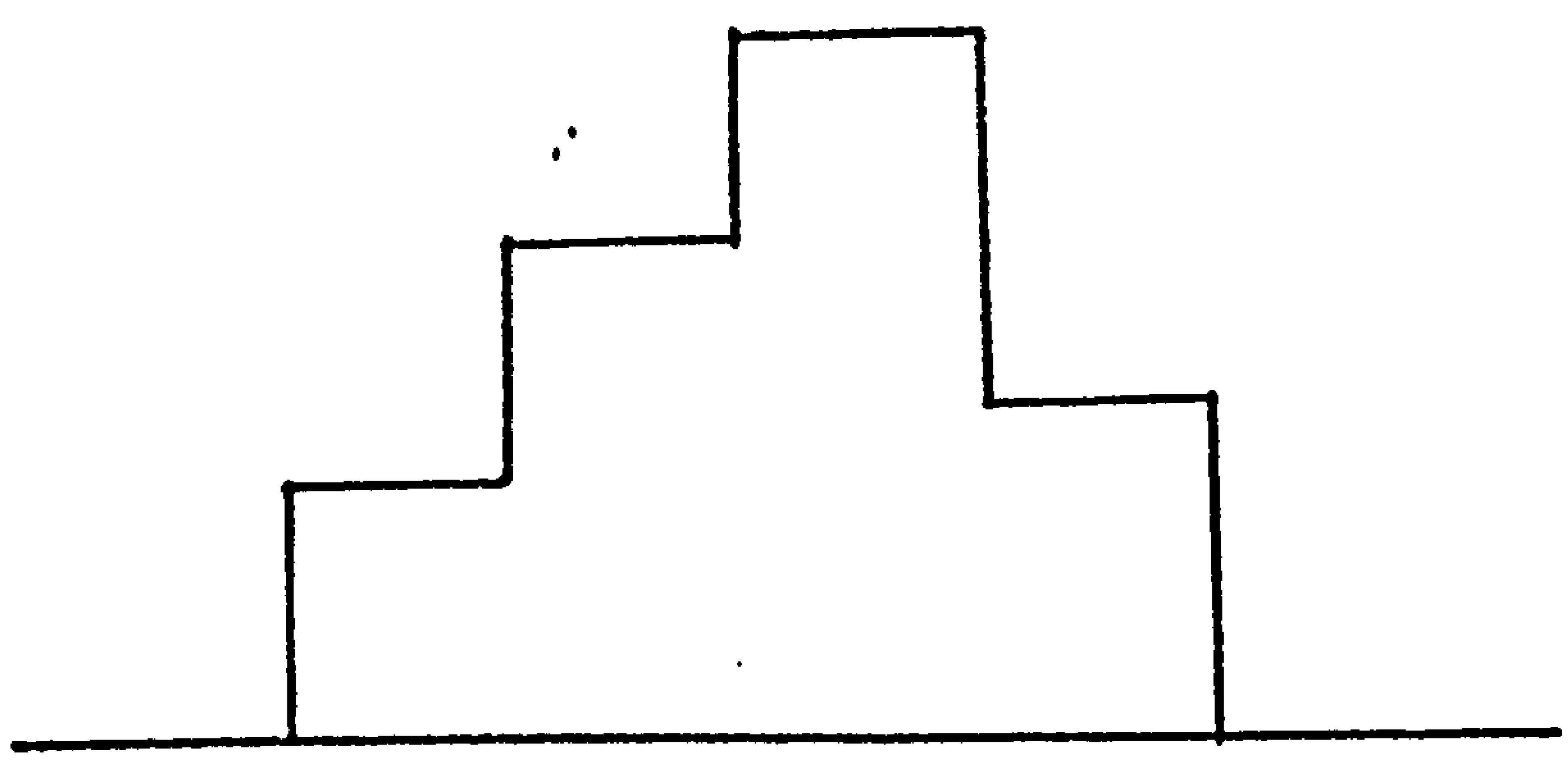

Figure 5.1 Probability Distribution Function Corresponding to a Piecewise Linear Cumulative Distribution

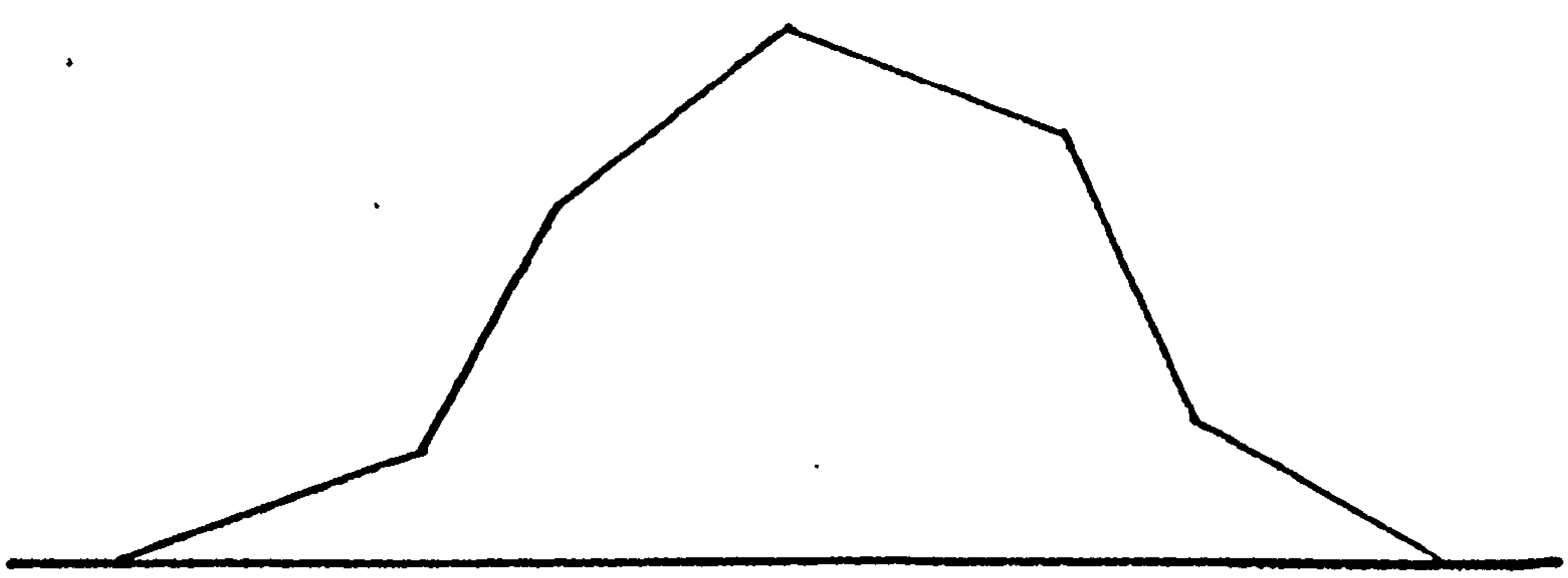

Figure 5.2 Probability Distribution Function Corresponding to a Piecewise Quadratic Cumulative Distribution

5.3 DISTRIBUTION USED

It was considered that the 'true' distributions used in the analysis should have the following properties:

(a) It should be possible to calculate the mean and the standard deviation of each of the distributions exactly.

(b) Each of the distributions should have a known finite range.

(c) Each of the distributions should be unimodal and it should be possible to determine the position of the mode exactly.

(d) It should be possible to calculate ordinates of the distributions exactly.

(e) It should be possible to calculate cumulative probabilities and fractiles exactly.

(f) The distributions should not have a cumulative which is piecewise linear or piecewise quadratic; nor should it have any other properties which make it particularly easy to assess the distribution accurately using the procedures mentioned in section 5.2.

The only family of distributions found with all of these properties was the beta family and 55 distributions were therefore selected from the beta family to serve as 'true' distributions in the analyses.

The beta distribution has the following probability density function.:

$$f(x) = \frac{(a + b - 1)!}{(a - 1)!\,(b - 1)!}\, x^{a-1} (1 - x)^{b-1}$$

where a and b are positive. If a and b are both greater than 2 it has the general form shown in figure 5.3. The mean of the distribution is:

$$\frac{a}{a+b}$$

The variance is:

$$\frac{ab}{(a + b + 1)\,(a + b)^2}$$

and the mode is:

$$\frac{a-1}{a + b - 2}$$

the range of the distribution is 0 to 1 (although this can be altered by applying a linear transformation to the variable). Cumulative probabilities can be calculated when a and b are integral from the relationship which exists between the beta distribution and the binomial distribution.

$$\sum_{s=r}^{n} {}^{n}C_{s} \; p^{s} \; (1-p)^{n-s} = \frac{\int_{0}^{p} x^{r-1} (1-x)^{n-r} \, dx}{\int_{0}^{1} x^{r-1} (1-x)^{n-r} \, dx}$$

(See for example Fletcher et al (1962, page 307)).

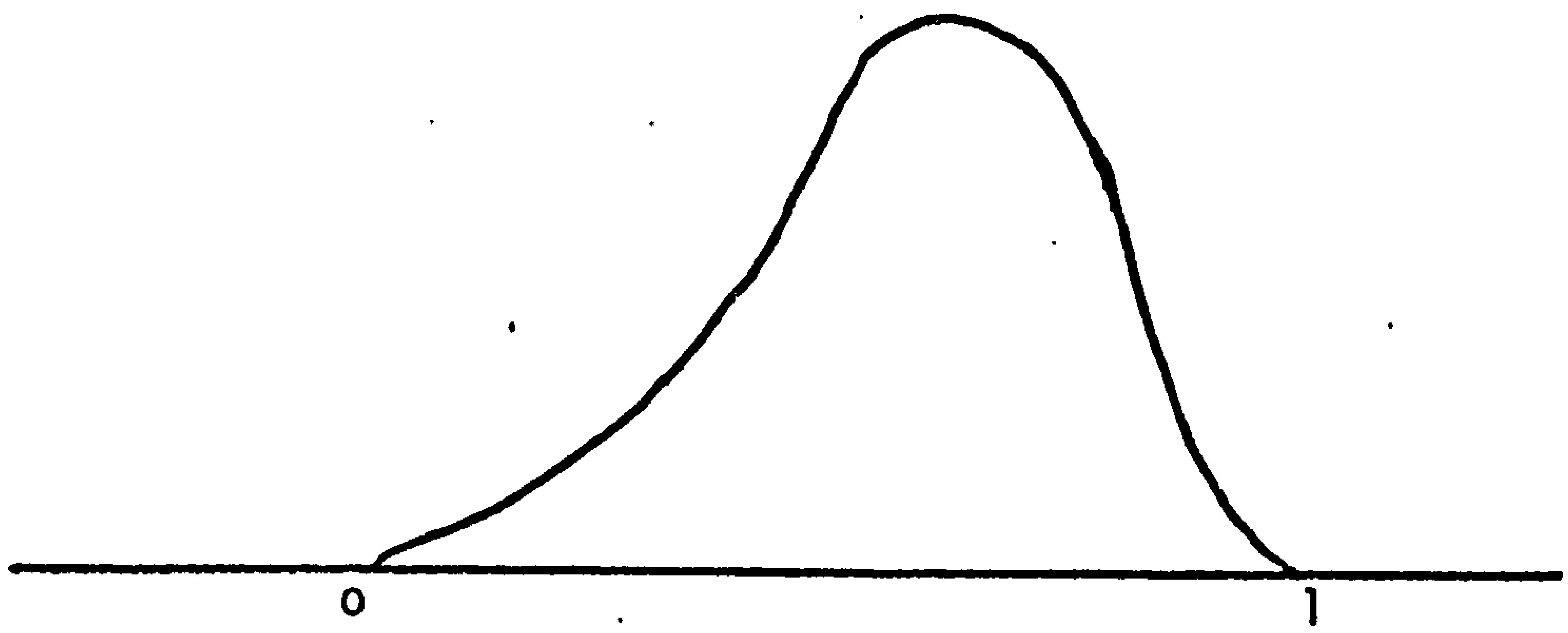

Figure 5.3 A Beta Distribution

The 55 beta distributions which were selected were those where the parameters a and b satisfied the following conditions:

$3 \leqslant a \leqslant 12$

$3 \leqslant b \leqslant 12$

$b \geqslant a$

a, b integral

5.4 RESULTS ASSUMING PERFECT ASSESSMENTS

. The first set of analyses assumed that the assessor was capable of making a finite number of perfect (i.e. infinitely accurate) assessments. The 'true' distributions used were the beta distributions described in section 5.3 and the assessment procedures which were simulated were those described in section 5.2. For the purposes of summarising the results the distributions were divided into two groups:

Highly skewed distributions

and Not highly skewed distributions

A highly skewed distribution was defined as a distribution where the mode lay outside the range 0.4 to 0.6 i.e. as one where the parameters a and b satisfied either:

$$\frac{a-1}{a+b-2} > 0.6$$

or:

$$\frac{a-1}{a+b-2} < 0.4$$

(i.e. where: 4a > 6b - 2
or: 4b > 6a - 2)

27 of the distributions were 'highly skewed'; 28 were 'not highly skewed'.

The parameter n (equal to the total number of individual assessments assumed to be made in addition to the end points of the range) was allowed to take three values: 3, 7 and 15.

A computer program PROBSIM which is described in appendix B was written to carry out the analyses. The results are shown in full in appendix F and summarised in tables 5.1 and 5.2. Table 5.1 shows the average absolute errors in the estimates calculated for the means as a percentage of the range of the distribution; table 5.2 shows the average absolute values of the percentage errors in the estimates calculated for the standard deviations.

Table 5.1 Average absolute errors in estimates calculated for means as a percentage of the range of the distribution assuming perfect assessments.

	Average absolute error in means as a percentage of range					
	n=3		n=7		n=15	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	0.2	0.0	0.0	0.0	0.0	0.0
Procedure 2	0.6	0.3	0.0	0.0	0.0	0.0
Procedure 3	4.0	0.8	1.9	0.4	0.9	0.2
Procedure 4	1.1	0.3	0.5	0.1	0.2	0.0
Procedure 5	1.6	0.2	0.6	0.1	0.2	0.0
Procedure 6	7.7	1.4	4.5	0.8	2.4	0.4

Table 5.2 Average absolute percentage errors in estimates calculated for standard deviations assuming perfect assessments.

	Average absolute percentage error in standard deviations					
	n=3		n=7		n=15	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	24.5	23.2	8.2	7.8	2.2	2.0
Procedure 2	14.1	15.4	1.8	2.3	0.2	0.2
Procedure 3	48.5	44.8	36.0	32.1	23.9	20.5
Procedure 4	22.2	19.0	13.4	11.0	7.4	5.9
Procedure 5	19.3	19.3	8.9	7.8	2.5	2.2
Procedure 6	41.3	35.7	33.9	27.1	24.8	18.2

5.5 DISCUSSION OF RESULTS ASSUMING PERFECT ASSESSMENTS

An examination of table 5.1, table 5.2 and appendix F reveals that for the beta distributions which have been considered:

(i) Assessment procedure 1 is consistently better than assessment procedure 3.

(ii) Assessment procedure 2 is consistently better than assessment procedure 4.

(iii) Assessment procedure 5 is consistently better than assessment procedure 6.

It will now be shown that these results have an intuitively appealing explanation.

Consider the result in (i) first. In both assessment procedure 1 and assessment procedure 3 a piecewise linear cumulative distribution function is fitted to the assessments. The only difference between the two procedures is that in procedure 1 cumulative probabilities are determined at equally spaced values of the variables whereas in procedure 3 values of the variable are determined at equally spaced cumulative probabilities. (If a distribution such as that shown in figure 5.4 were being assessed procedure 1 (with n=3) would produce a distribution similar to the one in Figure 5.5, whereas procedure 3 (again with n=3) would produce a distribution similar to the one in figure 5.6) The result in (i) does therefore suggest the following:

Estimates calculated for the mean and standard deviation are most accurate when the values of the variable at which cumulative probabilities are known are equally spaced. 5.1

This conforms with intuition.

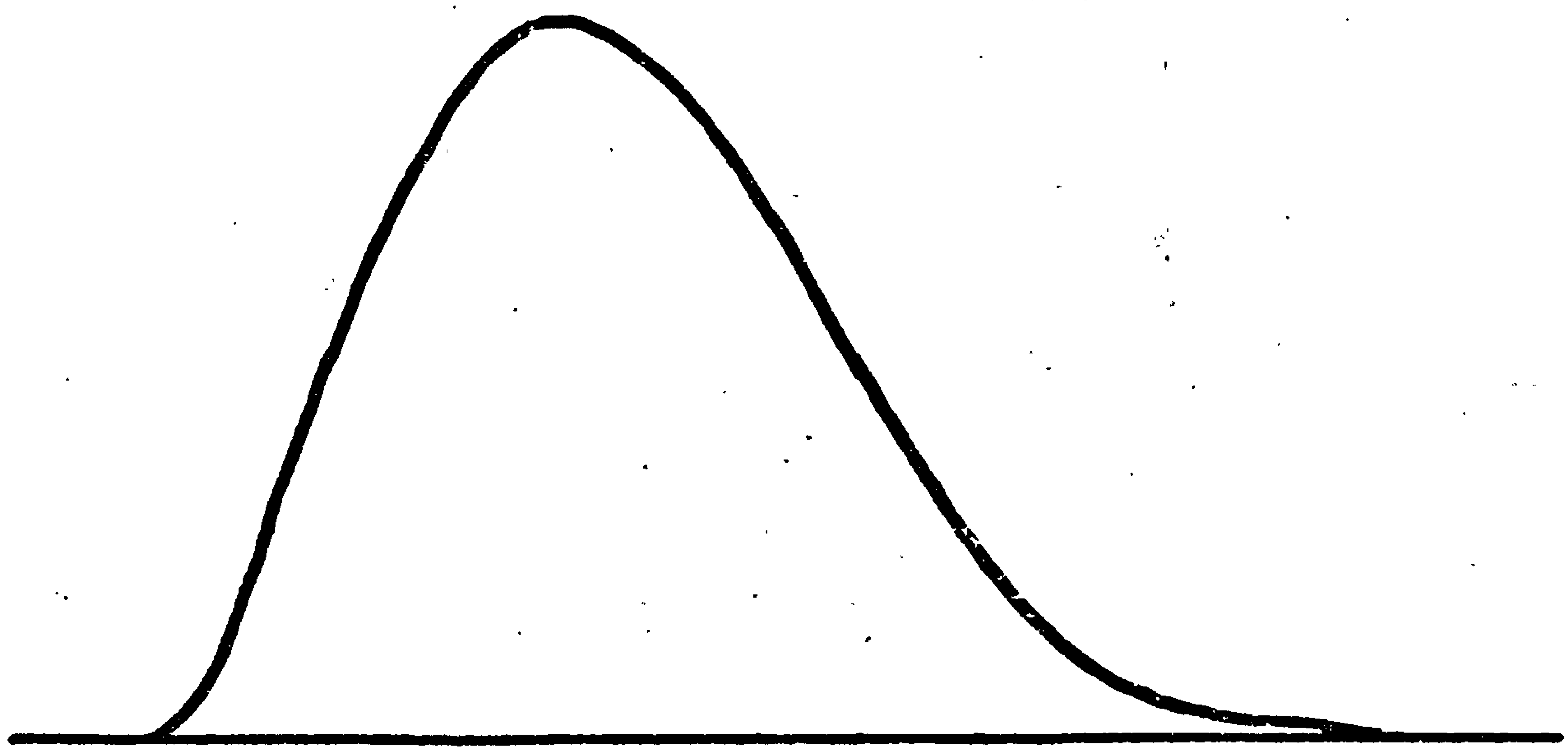

Figure 5.4 True Distribution to be Assessed

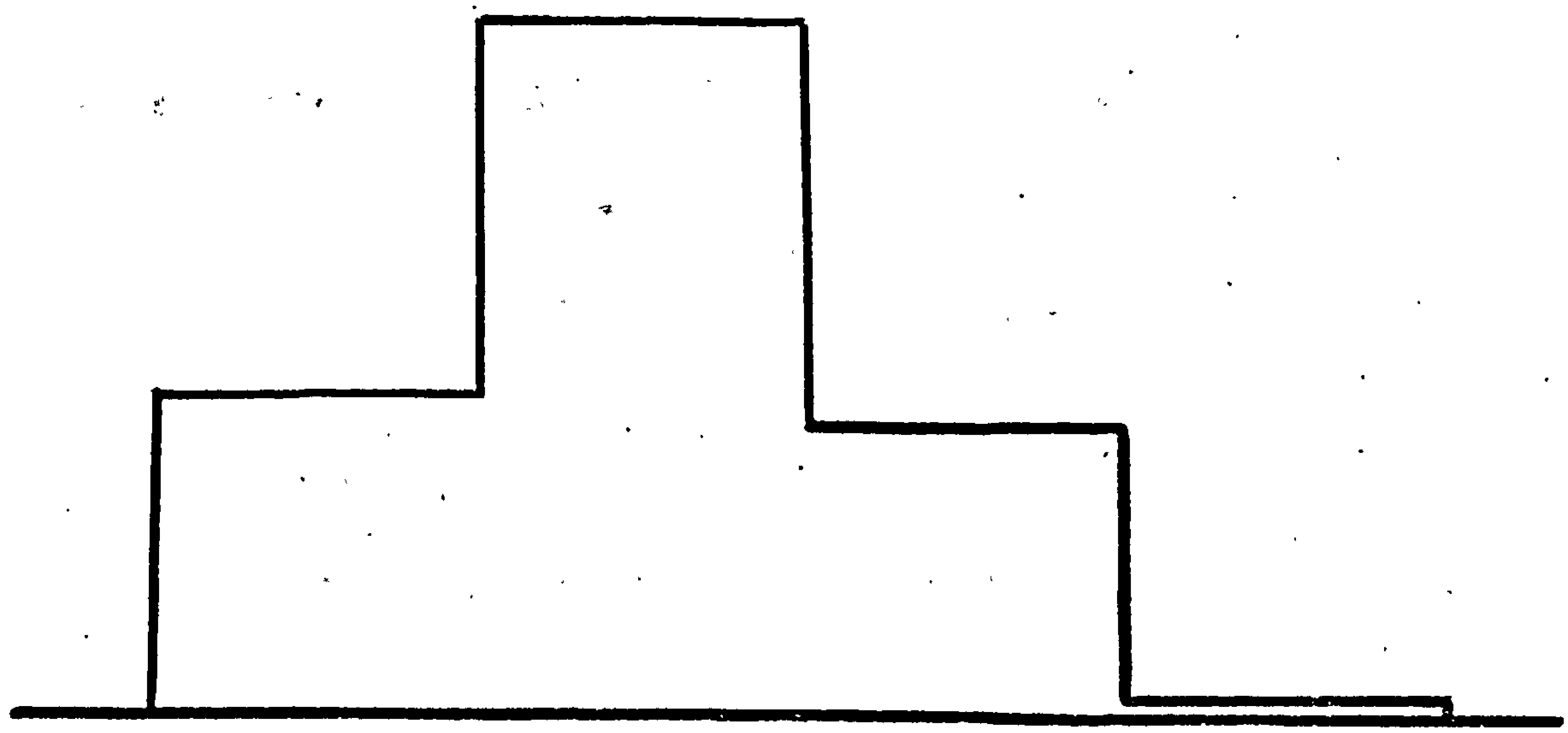

Figure 5.5 Results Obtained Using Assessment Procedure 1

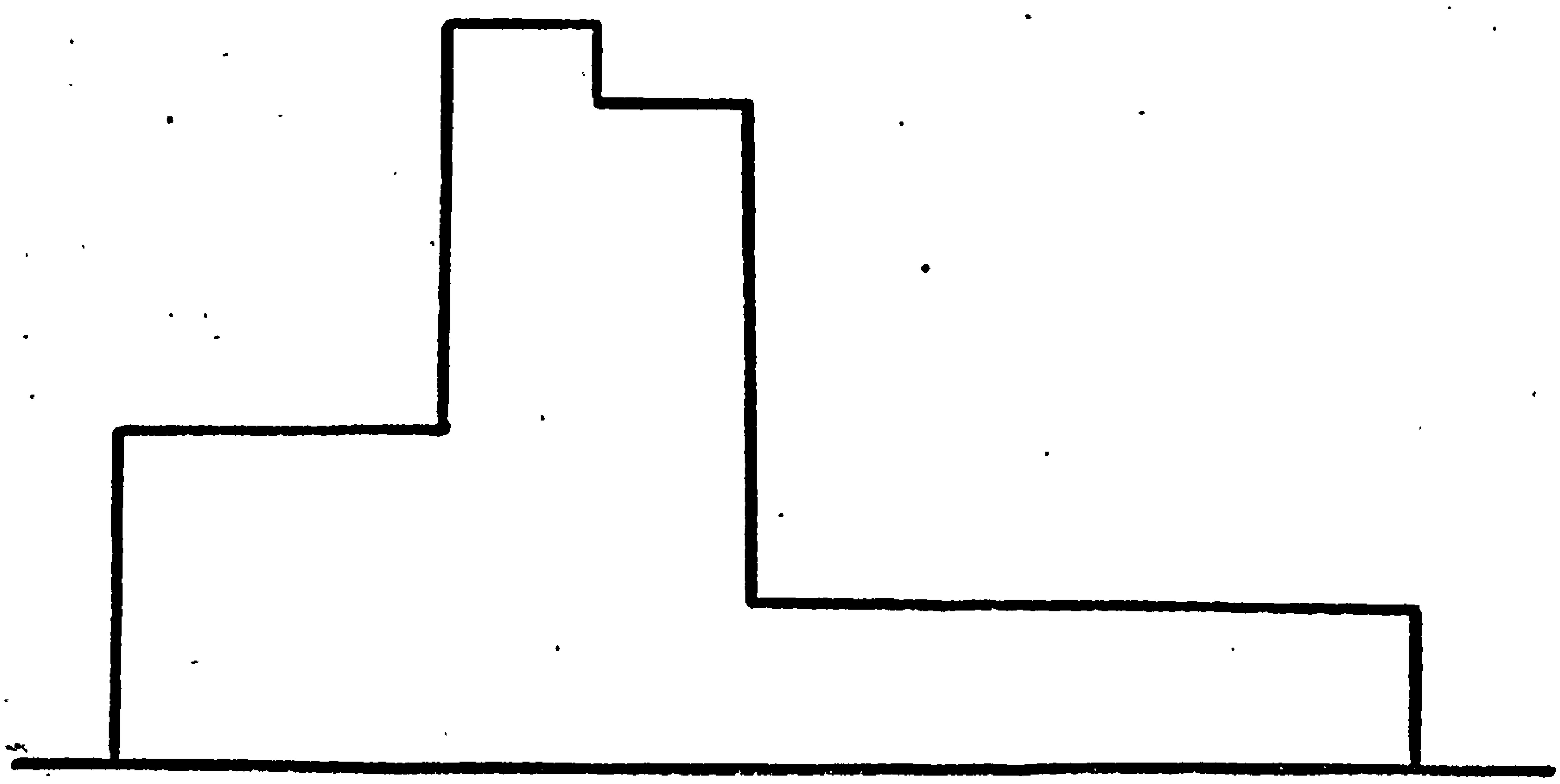

Figure 5.6 Results Obtained Using Assessment Procedure 3

The result in (ii) above is precisely analogous to the result in (i) and provides further evidence to support the hypothesis in 5.1.

Finally, consider result (iii). In procedure 5 the values of the variable at which relative likelihoods are determined are, on each side of mode, equally spaced (see figure 5.7) whereas in procedure 6 this is not so (see figure 5.8). This suggests the following:

> The accuracy of estimates made for the mean and standard deviation of a variable increases as the values of the variable at which relative likelihoods are known become 'more equally spaced'.

Again this conforms with intuition.

Figure 5.7 Values of the variable at which relative likelihoods would be known if procedure 5 were used to assess the distribution in figure 5.4

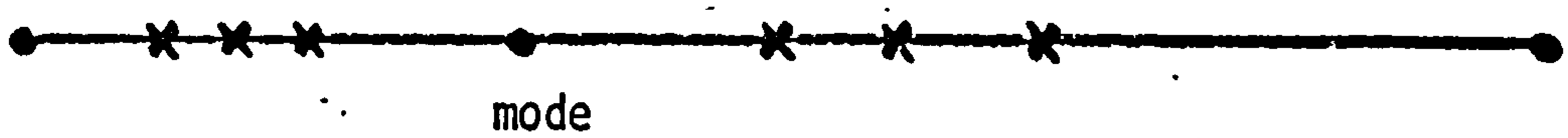

Figure 5.8 Values of the variable at which relative likelihoods would be known if procedure 6 were used to assess the distribution in figure 5.4

A more general hypothesis to replace 5.1 and 5.2 is the following:

> The accuracy of estimates for the mean and standard deviation of a variable increases as the values of the variable at which cumulative probabilities/relative likelihoods are known become 'more equally spaced'.

Extra evidence in support of this is provided by the fact that the differences between the performances of:

(a) Procedures 1 and 3

(b) Procedures 2 and 4

and (c) Procedures 5 and 6

are all greater for highly skewed than for not highly skewed distributions.

A comparison of the accuracies of procedures 1, 2 and 5 in tables 5.1 and 5.2 reveals that procedure 2 produces on average the best estimates for the standard deviation and that procedure 1 produces on average the best estimates for the mean. The tables also show that the accuracy of estimates made for the mean of highly skewed distributions are on average considerably worse than those made for the mean of not highly skewed distributions and that the accuracy of estimates made for the standard deviation of highly skewed distribution are on average only marginally worse than those made for the standard deviation of not highly skewed distributions. Define the skewness of a distribution, s, as follows:

$$s = \left| \frac{\text{Mode} - \text{Lower Bound}}{\text{Range}} - 0.5 \right|$$

Best fit relationships of the form:

Error in Mean $= a_1 + b_1 s$

Error in Standard Deviation $= a_2 + b_2 s$

where a_1, a_2, b_1, b_2 are constants are derived in appendix F. Errors in the mean are in most cases far more highly correlated with s than errors in the standard deviation and in some cases there was found to be no significant relationship between errors in the standard deviation and s.

5.6 RESULTS ASSUMING IMPERFECT ASSESSMENTS

The second set of analyses assumed the existence of a number e (which will be termed the assessor's 'accuracy parameter') with the property that:

0, e, 2e, 3e.....

were the only numbers normally used by the assessor when providing assessments. (To illustrate the basic idea here suppose that for a certain individual e = 0.1. This would mean that probabilities, values of the variable and relative likelihoods could all only be assessed by the individual to the nearest 0.1).

Three different values of e were considered:

0.2, 0.1, 0.01

All six of the assessment procedures described earlier in this chapter were investigated for n=3 and n=7. The assumption which was generally made was, as indicated above, that probabilities values of the variable and relative likelihoods could only be assessed to the nearest e. However, occasionally this assumption had to be modified because it led to either:

(i) two different cumulative probabilities corresponding to the same value of the variable (in procedures 3 and 4).

or (ii) two different relative likelihoods corresponding to the same value of the variable (in procedure 6).

or (iii) the probabilities assessed for different intervals adding up to same number X other than 1.0 (in procedures 1 and 2).

In the case of (i) and (ii) the modification involved judiciously reducing the accuracy parameter for particular assessments. When (iii) occurred the difficulty was overcome by dividing the probabilities for each interval by the number X.

The results are shown in full in appendix F and summarised in tables 5.3 to 5.8. Appendix F shows the standard deviations of the errors. Best fit relationships of the form:

$$\text{Error in Mean} = a_1 + b_1 s$$

$$\text{Error in Standard Deviation} = a_2 + b_2 s$$

where a_1, a_2, b_1, b_2 are constants are also derived in appendix F.

Table 5.3 Average absolute errors in estimates calculated for mean as a percentage of the range of the distribution assuming accuracy parameter = 0.2

	Average absolute error in mean as a percentage of range (Acc. Par. = 0.2)			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	1.8	0.9	1.4	0.9
Procedure 2	1.6	0.8	1.4	0.9
Procedure 3	4.4	2.7	3.4	2.1
Procedure 4	3.5	2.7	2.9	2.4
Procedure 5	1.7	1.3	0.8	0.7
Procedure 6	9.2	1.4	6.2	1.8

Table 5.4 Average absolute errors in estimates calculated for mean as a percentage of the range of the distribution assuming accuracy parameter = 0.1

	Average absolute error in mean as a percentage of range (Acc. Par. = 0.1)			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	1.0	0.7	1.0	0.7
Procedure 2	1.1	0.8	1.0	0.7
Procedure 3	4.7	1.4	2.2	1.1
Procedure 4	2.1	1.3	1.6	1.0
Procedure 5	1.5	0.4	0.5	0.3
Procedure 6	7.6	1.6	5.3	1.1

Table 5.5 Average absolute errors in estimates calculated for mean as a percentage of the range of the distribution assuming accuracy parameter = 0.01

	Average absolute error in mean as a percentage of range (Acc. Par. = 0.01)			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	0.2	0.1	0.2	0.1
Procedure 2	0.5	0.5	0.2	0.1
Procedure 3	4.0	0.8	1.9	0.4
Procedure 4	1.0	0.2	0.5	0.1
Procedure 5	1.6	0.2	0.6	0.1
Procedure 6	7.7	1.4	4.5	0.9

Table 5.6 Average absolute percentage errors in estimates calculated for standard deviation assuming accuracy parameter = 0.2

	Average absolute percentage error in S.D. Acc. Par. = 0.2			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	20.4	16.3	12.9	13.0
Procedure 2	9.9	12.7	20.9	20.5
Procedure 3	49.4	46.0	35.8	32.4
Procedure 4	25.9	23.8	13.3	11.6
Procedure 5	14.4	19.7	4.7	5.7
Procedure 6	40.7	36.2	33.3	28.8

Table 5.7 Average absolute percentage errors in estimates calculated for standard deviation assuming accuracy parameter = 0.1

	Average absolute percentage error in S.D. assuming acc. par. = 0.1			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	22.8	19.3	6.0	6.8
Procedure 2	10.0	11.4	10.4	8.2
Procedure 3	49.1	45.3	38.6	35.6
Procedure 4	26.3	23.5	21.6	21.0
Procedure 5	16.6	18.3	5.8	5.8
Procedure 6	41.5	35.9	34.9	29.8

Table 5.8 Average absolute percentage errors in estimates calculated for standard deviation assuming accuracy parameter = 0.01

	Average absolute percentage error in S.D. assuming acc. par. = 0.01			
	n=3		n=7	
	Highly Skew	Not Highly Skew	Highly Skew	Not Highly Skew
Procedure 1	24.2	23.0	7.7	7.2
Procedure 2	10.5	13.6	1.3	1.7
Procedure 3	48.4	44.8	36.0	32.1
Procedure 4	22.1	19.0	13.6	11.0
Procedure 5	19.3	19.3	8.8	7.6
Procedure 6	41.2	35.7	33.9	27.1

It can be seen from tables 5.1, 5.3, 5.4 and 5.5 that the average error in the mean almost invariably increases as the accuracy parameter increases. Some of the increases(particularly those corresponding to procedures 1 and 2) can be shown to be statistically significant from the standard deviations of the absolute errors in Appendix F.

It can be seen from tables 5.2, 5.6, 5.7 and 5.8 that for some procedures the average absolute percentage error in the standard deviation actually decreases as the standard deviation increases(and appendix F shows that some of the decreases are statistically significant). The explanation for this result is as follows: all of the procedures naturally tend to over - estimate the standard deviation of a distribution which is shaped like the beta and increasing the accuracy parameter tends to reduce the assessed standard deviation by 'chopping off' tails of the distribution.

5.7 MAXIMUM ERRORS

The results in appendix F can be used to suggest rough estimates for the maximum error to be expected when distributions similar in shape to the beta are being assessed. As an example suppose that procedure 1 with n=3 is being used for a variable for which s=0.1 and that the assessor's accuracy parameter is 0.1

The best fit relationship between errors in the mean and s is:

$$\text{Error in Mean (\% of range)} = 0.5 + 2.6s$$

and the standard error of the estimate is 0.8

A '95% confident' maximum error for the mean (as a % of range) is therefore:

$$0.5 + 2.6 \times 0.1 + 1.64 \times 0.8 = 2.1$$

Errors in the standard deviation are not (at the 0.05 significance level) related to s. Since:

Average of errors in standard deviation = 21.0%

Standard deviation of errors in standard deviation = 6.6%

a '95% confident' maximum error for the standard deviation is 32%.

(It is interesting to note that increasing n from 3 to 7 has virtually no effect on the maximum error in the mean while decreasing the maximum error in the standard deviation from 32% to 13%).

It should be noted that no account has been taken in the above of the fact that all six procedures tend to over-estimate the standard deviation (see computer output in appendix F). Strictly speaking the 95% confidence interval for the standard deviation should be symmetrical.

5.8 SUMMARY AND CONCLUSIONS

This chapter has studied the way in which 55 beta distributions would be assessed using six different procedures. The results obtained assuming that management are capable of making infinitely accurate assessments clearly demonstrate:

(i) For the same number of assessments, fixed interval methods provide better estimates of both the mean and the standard deviation than variable interval methods.

(ii) For the same number of assessments, procedure 5 provides better estimates of both the mean and the standard deviation than procedure 6. (See section 5.2 for a description of the assessment procedures).

(iii) The accuracy of an estimate for the mean of a distribution tends to depend more heavily on the distribution's skewness than the accuracy of an estimate for its standard deviation.

Assuming the existence of an accuracy parameter equal to 0.1 or 0.2 had a particularly significant effect on average absolute errors in estimates of the mean in the case of procedures 1 and 2. The effect of the accuracy parameter on estimates of the standard deviation was in some cases the reverse of what was expected. The reasons for this are given in section 5.6

If the assumption can be made that the 55 beta distributions are representative of a far wider class of distributions, the results can be used to provide estimates of the maximum errors in assessed means and assessed standard deviations for all distributions in the class.

It should be emphasised that the conclusions in this chapter are based on the assumption that the 'true' distribution is the same for all probability assessment procedures. In view of the experimental evidence which is available on the assessment of subjective probability distributions (see section 2.3) this assumption is questionable and further work aimed at developing a more detailed model of the process by which probability assessments are made could usefully be carried out.

CHAPTER 6

DEALING WITH DEPENDENCE IN RISK EVALUATION MODELS

6.1 INTRODUCTION

This chapter starts with a brief consideration of the importance of dependencies between the variables in a risk analysis study. The methods which have been suggested in the literature for assessing dependence are then critically examined and a new method, particularly suitable for risk simulation, is suggested and evaluated. The chapter concludes with a discussion of methods for dealing with the situation where the value of a variable in one time period is dependent on its value in one or more previous time periods. (This might be referred to as the 'growth rate problem').

6.2 THE IMPORTANCE OF DEPENDENCIES

To provide an indication of the potential importance of dependencies in a risk analysis study, the effect on NPV of assuming total positive dependence between each pair of variables in case A (the Hertz model) was investigated. (For a precise definition of 'total positive dependence' see section 6.3). The results obtained are summarised in tables 6.1 and 6.2.

Table 6.3 shows, for the purposes of comparison:

(i) the effect on the mean of NPV of increasing the mean of each variable by an amount equal to 5% of the variable's range.

(ii) the effect on the standard deviation of NPV increasing the standard deviation of each variable by 30%.

Triangular distributions were used for the variables with 1000 simulation runs being carried out. (The distribution of NPV assuming no dependence and using the original triangular distributions has, in \$M, a mean of -2.7 and a standard deviation of 9.5).

Table 6.1 Effect on mean of NPV in Case A of total positive dependence between two variables ($M)

	Initial Market Size	Market Growth	Selling Price	Market Share	Initial Investment	Life of Investment	Residual Value	Operating Costs	Fixed Costs
Initial Market Size		0	1.6	0.7	0	0.2	0	-1.4	0
Market Growth			1.0	0.6	0	0.2	0	-1.0	0
Selling Price				1.9	0	0.9	0	0	0
Market Share					0	0.2	0	-1.2	0
Initial Investment						0	0	-0.2	0
Life of Investment							0	-1.6	0
Residual Value								-0.1	0
Operating Costs									0
Fixed Costs									

Table 6.2 Effect on S.D. of NPV in Case A of total positive dependence between two variables ($M)

	Initial Market Size	Market Growth	Selling Price	Market Share	Initial Investment	Life of Investment	Residual Value	Operating Costs	Fixed Costs
Initial Market Size		0.2	1.5	1.4	-0.1	0.4	0	-0.4	0
Market Growth			1.0	0.9	0	0.2	0	0.4	0
Selling Price				1.4	-0.5	0.2	0.1	-6.8	-0.1
Market Share					-0.1	0.4	0	0	0
Initial Investment						0	0	0.9	0
Life of Investment							0	0.7	0
Residual Value								0.2	0
Operating Costs									0.1
Fixed Costs									

Table 6.3 Effect on Mean and S.D. of NPV of errors in the mean and S.D. of variables ($M)

	Effect of increasing mean by 5% of range on		Effect of increasing S.D. ny 30% on	
	Mean of NPV	S.D. of NPV	Mean of NPV	S.D. of NPV
Init. Mkt. Size	+0.3	+0.5	+0.2	0
Mkt. Growth	+0.1	+0.1	+0	0
Selling Price	+1.6	+0.2	+1.6	0
Mkt. Share	+0.5	+0.6	+0.3	0
Init. Invest.	-0.2	0	0	0
Life of Invest.	+0.1	+0.3	0	0
Residual Value	0	0	0	0
Op. Costs	-1.5	-0.1	+1.3	+0.1
Fixed Costs.	0	0	0	0

Tables 6.1 to 6.3 show that the effect on a performance measure of a dependence between two variables is liable to be just as large - if not larger - than the effect on the performance measure of:

(a) a 30% error in the standard deviation of one of the variables

or: (b) an error in the mean of one of the variables equal to 5% of the variable's range.

In some cases the effect of a dependence can be quite large. For example, perfect dependence between selling price and operating costs would in the Hertz model reduce NPV by 72%.

Of course it is still true that in any given situation many of the dependencies will be unimportant. The problem for the analyst is one of separating the important dependencies from the unimportant ones before the assessment procedure begins so that the total number of probability assessments to be provided by management is kept to a minimum. (This problem is discussed in chapter 7).

6.3 METHODS SUGGESTED IN THE LITERATURE FOR ASSESSING DEPENDENCE

The procedures which have been developed for assessing dependence fall into two general categories:

(A) those which are suitable for analytic approaches to risk analysis because they produce a value for the coefficient of correlation.

and (B) those which are suitable for risk simulation because they provide a complete pattern for the dependence (i.e. a sampling scheme).

The most important work as far as category (A) is concerned has been carried out by Hillier (1969). He suggests that for a given value, say x, of the independent variable V_1 the assessor be asked to make optimistic, pessimistic and best estimates for the dependent variable V_2. These yield (using, for example, equation 5.1) the conditional expectation:

$$E\left[V_2/V_1 = x\right]$$

If V_1 and V_2 have a bivariate normal distribution and if σ_1 and σ_2 are the standard deviations of V_1 and V_2 then the relationship:

$$E\left[V_2/V_1 = x\right] = E(V_2) + \rho\frac{\sigma_2}{\sigma_1}(x - E(V_1)) \qquad 6.1$$

can be used to estimate the coefficient of correlation ρ. Hillier suggests that two different values of x (equal to the optimistic and pessimistic estimates of V_1) should be used in the estimating process and that the values of ρ which are calculated be averaged. Furthermore Hillier puts forward arguments to suggest that since equation 6.1 is the mean square regression line of V_2 on V_1 the same equation can be used when V_1 and V_2 do not have normal distributions.

The simplest approach as far as category (B) is concerned is undoubtedly to assume either:

(a) No dependence

or (b) Total dependence (positive or negative)

However, total dependence between two variables V_1 and V_2 needs careful defining in this context. It is not useful to define it as the situation where ρ, the coefficient of correlation, equals either + 1 or - 1 as this implies that:

$$V_2 = a + b V_1 \quad a, b \text{ constant } b \neq 0$$

and means that V_1 and V_2 must have unconditional distributions which are of the same type. A more useful definition would seem to be as follows:

> x and y are totally positively dependent if when x takes a value equal to its k-th fractile y also takes a value equal to its k-th fractile. x and y are totally negatively dependent if when x takes a value equal to its k-th fractile, y takes a value equal to its (1-k) -th fractile.

(This allows for example, a triangular distribution to be totally dependent on a normal distribution etc).

As far as the assessment of partial dependence for a risk simulation is concerned, Hertz (1964) suggests that a single subjective probability distribution be assessed for the independent variable and that several conditional subjective probability distributions be assessed for the dependent variable (each of these being conditional on the independent variable lying within a different interval). On each run of the simulation the value sampled for the independent variable would then determine the particular conditional subjective probability distribution used. The major drawback of this procedure is that it involves management in making an unreasonably large number of individual probability assessments.

Eilon and Fowkes (1973) consider the problem of dependence in some detail and suggest a number of different discriminant sampling procedures where the range of possible values of the dependent variable is restricted in some way according to the value sampled for the independent variable. This is a sensible idea. It has the disadvantage that a particular discriminant sampling scheme cannot easily be related to a judgement by management as to 'the extent of the dependence'. Also if the unconditional distribution of the dependent variable has been assessed in advance it is sometimes difficult to choose the scheme so as to satisfy the consistency condition:

$$g(V_2) = \int_{-\infty}^{+\infty} h(V_2/V_1)\, f(V_1)\, dV_1 \qquad 6.2$$

where g is the unconditional distribution of the dependent variable (V_2), f is the distribution of the independent variable (V_1) and h is the conditional distribution of V_2 given the value of V_1.

Kryzanowski et al (1973) suggest the following procedure when the value of the independent variable is less than its 0.50 fractile:

(i) if the intensity of the dependence is 'slight' sample in such a way that the probability of the dependent variable being less than its 0.50 fractile is 0.60 for positive dependence and 0.40 for negative dependence.

(ii) if the intensity of the dependence is 'moderate', sample in such a way that the probability of the dependent variable being less than its 0.50 fractile is 0.75 for positive dependence and 0.25 for negative dependence.

(iii) if the intensity of the dependence is 'high', sample in such a way that the probability of the dependent variable being less than its 0.50 fractile is 0.90 for positive dependence and 0.10 for negative dependence.

Similar rules are suggested when the independent variable is greater than its 0.50 fractile. The procedures satisfy the condition in 6.2. Their main disadvantage would seem to be that the terms 'slight', 'moderate' and 'high' are imprecise.

Finally, it is worth noting that Pouliquen (1970) suggests four methods for dealing with dependence. Briefly they are:

(i) limit disaggregation in the cash flow model

(ii) isolate the sources of uncertainty and redefine the variables accordingly.

(iii) the pessimistic-optimistic approach - i.e. assume total dependence and see what effect it has on the results.

(iv) collect more data (This is only applicable to those situations where data exists).

6.4 A NEW METHOD FOR ASSESSING DEPENDENCE: BASIC IDEAS

Suppose that variable V_2 is considered to depend on variable V_1 in a risk simulation and that unconditional distributions for V_1 and V_2 have already been assessed by management. One of the simplest single assessments of the extent of the dependence of V_2 on V_1 which can be made by management is the following:

'assuming $V_1 = Q$ my median estimate for V_2 is P' 6.3

where Q is a value in one of the tails of the distribution of V_1.

The two sections which follow will show that if an assessment such as that in 6.3 is made then it is possible to find a set of conditional distributions for V_2 which are both consistent with the assessment and consistent with the unconditional distributions of V_2 and V_1. This gives rise to a new (category B) method for assessing dependence which overcomes some of the drawbacks of existing methods because:

(a) it takes account of a numerical estimate of the extent of the dependence.

and (b) it gives rise to conditional distributions which satisfy the consistency condition in equation 6.2.

6.5 THE NEW METHOD FOR ASSESSING DEPENDENCE: UNDERLYING THEORY

As already noted, if g is the unconditional distribution of the dependent variable (V_2), if f is the distribution of the independent variable (V_1) and if h is the conditional distribution of V_2 given the value of V_1 then for consistency:

$$g\ (V_2) = \int_{-\infty}^{+\infty} h\ (V_2/V_1)\ f\ (V_1)\ d\ V_1 \qquad 6.2$$

The analysis which now follows finds functional forms for h satisfying 6.2 in situations where f and g are either (a) normal distributions or (b) distributions which become normal when a transformation is applied to the variable.

(a) f and g Normal

Suppose $f\ (V_1) \sim N(\mu_1, \sigma_1^2)$

$g\ (V_2) \sim N(\mu_2, \sigma_2^2)$

then from the theory of the bivariate normal distribution 6.2 is satisfied if:

$$h\ (V_2/V_1 = x) \sim N\left\{\mu_2 + \rho\ \frac{\sigma_2}{\sigma_1}\ (x - \mu_1), \sigma_2^2\ (1-\rho^2)\right\} \qquad 6.4$$

where ρ is the correlation coefficient between V_1 and V_2.

This result is used by Hillier (1969) and Wagle (1967) in their analytic approaches to risk evaluation.

(b) f and g become Normal after a transformation

Suppose now that there exist functions z and w such that z (V_1) and w (V_2) have distributions N (μ_1, σ_1^2) and N (μ_2, σ_2^2) respectively, i.e. Suppose that:

$$f\ (V_1) = \frac{dz}{dV_1}\ F\left\{z\ (V_1)\right\}$$

and
$$g\ (V_2) = \frac{dw}{dV_2}\ G\left\{w\ (V_2)\right\}$$

where F and G are the p.d.f.s. of variables with distributions N (μ_1, σ_1^2) and N (μ_2, σ_2^2) respectively. Then it can be deduced from the results in (a) that 6.2 is satisfied if:

$$h\ (V_2/V_1 = x) = \frac{dw}{dV_2}\quad H\left\{w\ (V_2)\right\} \tag{6.5}$$

where H is the p.d.f. of a variable with distribution:

$$N\left\{(\mu_2 + \rho^*\frac{\sigma_2}{\sigma_1}\ (z\ (x) - \mu_1)\ ,\ \sigma_2^2\ (1 - \rho^{*2})\right\} \tag{6.6}$$

ρ^* $(-1 \leq \rho^* \leq 1)$ is the correlation coefficient between $z(V_1)$ and $w(V_2)$ and not that between V_1 and V_2. However providing z and w are monotonic ρ^* is still a measure of the extent of the dependence between V_1 and V_2.

These results can be summarised as follows: Suppose V_2 depends on V_1 in a risk simulation. Then, providing the unconditional distributions for V_1 and V_2 can be chosen as distributions which become normal after a known transformation is applied to the variable, conditional distributions can be chosen for V_2. The conditional and unconditional distributions satisfy the convolution property 6.2 and a parameter, ρ^*, in the model enables the extent of the dependence between V_1 and V_2 to be varied. The conditional distributions for V_2 are of the same general type as the unconditional distributions in the sense that the same transformation (denoted by w above) converts the distributions into normal distributions.

The transformations z and w which are chosen in a particular situation will depend on the properties which management judge the variables to have. Of the distributions which Schlaifer (1971) recommends for the assessment of subjective probability distributions, a number are simple transformations of the normal distribution and do therefore give rise immediately to possible functional forms z and w. Table 6.4 presents a list of the characteristics of these distributions and includes the fractiles which Schlaifer suggests should be assessed. (For completeness two 3-parameter lognormal distributions have been added to Schlaifer's list).

Table 6.4 Selection of the Distribution Recommended by Schlaifer

Type of Distribution	Transformation t which must be applied to obtain a Normal Dist^n. $N(\mu, \sigma^2)$	Fractiles which must be assessed	Range	Other Properties
Normal	$t(x) = x$	.25, .75	$-\infty$ to $+\infty$	Symmetrical
Lognormal	$t(x) = \log x$	.25, .75	0 to ∞	Positively Skew
3-parameter Lognormal	$t(x) = \log(x-a)$	0, .25, .75	a to ∞	Positively Skew
3-parameter Lognormal	$t(x) = \log(a-x)$	.25, .75, 1	$-\infty$ to a	Negatively Skew
4-parameter Lognormal	$t(x) = \log \frac{x-a}{b-x}$	0, .25, .75, 1	a to b	Symmetrical or pos/neg Skew
Arc Sinh Normal	$t(x) = \sinh^{-1} \frac{x-m}{s}$	.25, .5, .75 & either .875 or .125	$-\infty$ to $+\infty$	Symmetrical or pos/neg Skew

As far as distributions not included in table 6.4 are concerned, it should be noted that <u>any</u> distribution can be transformed into the normal distribution on a 'fractile to fractile' basis i.e. by using transformation.

$$t(p) = q \qquad 6.7$$

where, if p is the k-th fractile of the distribution, q is the k-th fractile of the normal distribution, N (0, 1).

6.6 THE NEW METHOD: DETERMINING PARAMETERS AND SAMPLING

We assume here that unconditional distributions have already been assessed for variables V_1 and V_2. These unconditional distributions may or may not be members of one of the families in table 6.4.

An estimate for the parameter ρ^* which measures the extent of the dependence of V_2 on V_1 can be calculated if management provide a single assessment of the form:

'assuming that $V_1 = Q$ my median estimate for V_2 is P' 6.3

To show why this is so we first note that all the transformations, t, considered in the previous section transformed fractiles of a given distribution into the corresponding fractiles of the normal distribution. It follows (using the notation introduced under (b) in the previous section that since P is the 0.50 fractile of:

$$h(V_2/V_1 = Q)$$

w (P) must be the 0.50 fractile of the normal distribution in 6.6 when x = Q. This means that:

$$w\ (P) = \mu_2 + \rho^* \ \frac{\sigma_2}{\sigma_1}\ (z\ (Q) - \mu_1) \tag{6.8}$$

i.e. that

$$\rho^* = \frac{(w\ (P)\ -\ \mu_2)\ \sigma_1}{(z\ (Q)\ -\ \mu_1)\ \sigma_2} \tag{6.9}$$

In theory any value except the median of V_1 can be chosen for Q. However as the upper and lower quartile points in the distribution of V_1 will often have been assessed directly greater accuracy may well be achieved if Q is put equal to one of these. The rule which is adhered to in the example which follows involves putting Q equal to the quartile which lies in the longer tail of the distribution. (In practice it may sometimes be desirable to average two or three separate estimates for ρ^*).

When V_1 and V_2 have unconditional distributions taken from the families in table 6.3, calculating ρ^* and carrying out the sampling is relatively straightforward. This is because precise functional forms for w and z are known once the parameters of the distributions have been calculated from assessments made by management.

When V_1 and V_2 do not have unconditional distributions taken from the families in table 6.3, the situation is slightly more complicated. Suppose that Q is the k-th fractile of distribution f and P is the k_2-th fractile of distribution g. It follows from equation 6.9 using the transformations suggested in equations 6.7 that:

$$\rho^* = \frac{r_2}{r_1}$$

where r_2 is the k_2-th fractile of the normal distribution N (0, 1) and r_1 is the k_1-th fractile of the normal distribution N (0, 1).

The following sampling procedure is then applicable:

(i) Sample a value q_1 from N (0, 1)

(ii) Sample a value q_2 from N (0, 1) such that the coefficient of correlation between q_1 and q_2 is ρ^*.

(iii) Calculate the fractile (say j_1) of N (0, 1) to which q_1 corresponds and the fractile (say j_2) of N (0, 1) to which q_2 corresponds.

(iv) Put the V_1 sample equal to the j_1-th fractile of its distribution and the V_2 sample equal to the j_2-th fractile of its distribution.

Standard procedures (see, for example, Naylor et al (1968)) are available for sampling from a normal distribution and (ii) above can be accomplished using the result in equation 6.4.

6.7 THE NEW METHOD: AN EXAMPLE

The procedures outlined above will now be illustrated using the data of Eilon and Fowkes (1973) which is reproduced in tables 6.5, 6.6 and 6.7 . The data is based on case A as described in Hertz (1964). Table 6.5 shows Hertz's data. Tables 6.6 and 6.7 show the distributions which management are assumed to use when making assessments.

Table 6.5 Best Guess and Range for each Factor considered in the Case Study

		Best guess	Range
M	Initial market size (thousand tons)	250	100 - 340
G	Market growth rate (% p.a.)	3	0 - 6
P	Selling Price ($ per ton)	510	385 - 575
S	Share of market (%)	12	3 - 17
I	Initial investment (million $)	9.5	7.0 - 10.5
L	Useful life (years)	10	5 - 15
R	Residual value (million $)	4.5	3.5 - 5.0
V	Operating costs ($ per ton)	435	370 - 545
F	Fixed costs (thousand $)	300	250 - 375

Table 6.6 Independent Probability Distributions for the Factors listed in Table 6.5

M	100	0.05	145	0.12	190	0.23	240	0.40	290	0.20	340
G	0.0	0.15	1.5	0.20	2.5	0.30	3.5	0.20	4.5	0.15	6.0
P	385	0.03	420	0.07	460	0.30	500	0.55	540	0.05	575
S	3	0.05	5	0.15	8	0.25	11	0.35	14	0.20	17
I	7.0	0.08	8.0	0.30	9.0	0.30	9.5	0.20	10.0	0.12	10.5
L	5.0	0.04	7.5	0.26	9.5	0.40	11.5	0.26	13.5	0.04	15.0
R	3.50	0.15	4.00	0.15	4.25	0.33	4.50	0.23	4.75	0.14	5.00
V	370	0.05	405	0.55	440	0.30	475	0.07	510	0.03	545
F	250	0.15	275	0.30	300	0.30	325	0.15	350	0.10	375

Table 6.7 Conditional Probability Distributions for Dependent Factors in Table 6.5

G \ M	100–145	145–190	190–240	240–290	290–340
0.0–1.5	0.00	0.01	0.04	0.15	0.40
1.5–2.5	0.05	0.07	0.13	0.25	0.30
2.5–3.5	0.20	0.25	0.33	0.35	0.23
3.5–4.5	0.30	0.35	0.30	0.15	0.07
4.5–6.0	0.45	0.32	0.20	0.10	0.00

S \ M	100–145	145–190	190–240	240–290	290–340
3–5	0.01	0.02	0.03	0.03	0.16
5–8	0.02	0.03	0.04	0.05	0.58
8–11	0.07	0.08	0.13	0.40	0.22
11–14	0.20	0.37	0.50	0.43	0.04
14–17	0.70	0.50	0.30	0.09	0.00

L \ I	7.0–8.0	8.0–9.0	9.0–9.5	9.5–10.0	10.0–10.5
5.0–7.5	0.40	0.04	0.00	0.00	0.00
7.5–9.5	0.38	0.50	0.20	0.08	0.02
9.5–11.5	0.22	0.45	0.50	0.40	0.15
11.5–13.5	0.00	0.01	0.30	0.50	0.53
13.5–15.0	0.00	0.00	0.00	0.02	0.30

R \ I	7.0–8.0	8.0–9.0	9.0–9.5	9.5–10.0	10.0–10.5
3.50–4.00	0.60	0.25	0.08	0.01	0.00
4.00–4.25	0.35	0.27	0.11	0.03	0.02
4.25–4.50	0.05	0.37	0.50	0.30	0.05
4.50–4.75	0.00	0.09	0.20	0.50	0.35
4.75–5.00	0.00	0.02	0.11	0.16	0.58

F \ I	7.0–8.0	8.0–9.0	9.0–9.5	9.5–10.0	10.0–10.5
250–275	0.00	0.05	0.13	0.15	0.53
275–300	0.04	0.18	0.35	0.45	0.40
300–325	0.07	0.30	0.40	0.37	0.07
325–350	0.39	0.28	0.10	0.03	0.00
350–375	0.50	0.19	0.02	0.00	0.00

P \ V	370–405	405–440	440–475	475–510	510–545
385–420	0.60	0.00	0.00	0.00	0.00
420–460	0.30	0.10	0.00	0.00	0.00
460–500	0.10	0.40	0.25	0.00	0.00
500–540	0.00	0.50	0.70	0.85	0.20
540–575	0.00	0.00	0.05	0.15	0.80

The variables in this example are bounded both above and below and we assume that each can be represented by a 4-parameter lognormal distribution. (See table 6.4 for a definition of this distribution). The upper and lower quartiles which would be assessed by management have been calculated from table 6.6 on the basis of linear interpolation and are shown in table 6.8. The parameters μ and σ (equal to the means and standard deviations of the normal distributions into which the 4-parameter lognormal distributions can be transformed) are also shown in table 6.8. They are calculated by using the fact that if the 0.25 fractile of a 4-parameter lognormal distribution is F_1 and the 0.75 fractile is F_2 then the following relationships must hold.

$$\log \frac{F_1 - a}{b - F_1} = \mu - 0.6745\sigma$$

$$\log \frac{F_2 - a}{b - F_2} = \mu + 0.6745\sigma$$

Table 6.8 Calculation of Parameters of Unconditional Distributions

Variable	Lower Bound a	Upper Bound b	Lower Quartile	Upper Quartile	μ	σ
M	100	340	207.4	283.7	0.49	1.03
G	0	6	2.0	4.0	0.00	1.03
P	385	575	480.0	525.4	0.52	0.77
S	3	17	8.60	13.57	0.36	1.14
I	7	10.5	8.57	9.67	0.48	1.03
L	5	15	9.11	11.88	0.22	0.85
R	3.5	5	4.17	4.63	0.45	0.99
V	370	545	417.7	457.5	-0.49	0.73
F	250	375	283.3	325.0	-0.30	1.05

As mentioned in the previous section the values of ρ^* are assumed to be calculated from assessments of the form:

'assuming V_1 = Q my median estimate for V_2 is P'

where Q is the quartile in the longer tail of the distribution of V_1. In order to calculate the precise assessments which would be made by management it has been necessary to assume (a) that the probabilities in table 6.7 are evenly distributed over the intervals and (b) that a statement which can be deduced from the table concerning a whole interval can be replaced by one which concerns only a specific value within the interval. For example, because the following is true:

The probability that G is less than or equal to 3.5 given that M lies in the 190-240 interval is 0.50.

it has been assumed that management would make the assessment 'assuming M = 207.4, my median estimate for G is 3.5'. The other assessments which would be made by management can be deduced in a similar way from table 6.7. They are: 'assuming M = 207.4, my median estimate for S is 12.8', 'assuming I = 8.57, my median estimate for L is 9.34', 'assuming I = 8.57, my median estimate for R is 4.23', 'assuming I = 8.57, my median estimate for F is 322.5', 'assuming V = 457.5, my median estimate for P is 514.3'. (The accuracy here is of course unrealistic, but it helps to ensure that the results are comparable with those of Eilon and Fowkes (1973).

The values of ρ^* which would be calculated from equation 6.9 are shown in table 6.9. As an illustration of how the calculation of ρ^* can be carried out, consider the dependence of G on M. Using the notation of equation 6.9, Q = 207.4, P = 3.5 and the transformations are:

$$z(x) = \log \frac{x - 100}{340 - x} \quad ; \quad w(x) = \log \frac{x}{6 - x}$$

Furthermore, from table 6.8 it is known that $\mu_1 = 0.49$, $\mu_2 = 0$, $\sigma_1 = 1.03$, $\sigma_2 = 1.03$. From equation 6.9.

$$\rho^* = \frac{(\log 1.4 - 0)\ 1.03}{(\log 0.81 - 0.49)\ 1.03}$$

$$= -0.48$$

Table 6.9 Values Calculated for ρ^*

Dep/Indep Variable	G/M	S/M	L/I	R/I	F/I	P/V
ρ^*	-0.48	-0.63	0.86	0.76	-0.89	0.46

Two simulations each of 5000 runs were carried out. The first involved taking independent samples from each of the bounded lognormal distributions in table 6.8. The second involved sampling first from the unconditional distributions of M, I, V and then from the conditional distributions of the other variables as determined by 6.5 and 6.6 above. The results are compared with those of Eilon and Fowkes in table 6.10.

Table 6.10 Performance Measures Obtained Using Different Sampling Schemes

SCHEME	IRR (%)		NPV (10% discount rate) \$ million		NPV (20% discount rate) \$ million	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
A: "Best Estimate"	20.9		5.9		0.3	
B: Independent Sampling; True Distributions	12.0	22.3	2.2	10.2	-2.0	6.7
C: Independent Sampling; 4-parameter lognormal distributions	13.0	18.2	2.3	9.5	-2.0	6.2
D: Conditional Sampling; True Distributions	12.5	13.7	1.7	6.8	-2.3	4.5
E: Conditional Sampling; 4-parameter lognormal distributions	13.0	12.4	1.9	6.5	-2.3	4.3
F: Best Discriminant Sampling Scheme	10.7	14.0	0.9	6.2	-2.7	4.2

Scheme D involving conditional sampling using the true distributions (i.e. those in tables 6.6 and 6.7) gives results which are the most faithful reflexion of management's expectations. Hence table 6.10 shows that the procedure suggested by the analysis in this paper (scheme E) does produce some improvements over the best of the discriminant sampling schemes used by Eilon and Fowkes in this case. It would of course be dangerous to draw general conclusions from this one example. However, it is to be expected that any model which takes some account of the extent of the dependence will produce better results than one which does not.

6.8 THE NEW METHOD: ACCURACY OF COEFFICIENTS OF CORRELATION

The analyses in chapter 4 suggest that many of the dependencies in risk evaluation models are approximately type I. A small computer program was therefore written to test the accuracy of the coefficients of correlation under the proposed sampling scheme. The program calculated using simulation:

(i) 'true' coefficients of correlation for the dependencies in the Eilon and Fowkes model (based on the data in table 6.7).

and (ii) the coefficients of correlation which the sampling scheme described in this chapter would give rise to if the 'true' unconditional distributions in table 6.6 were used and if the following 12 extra managerial assessments were made.

if M = 167.5, the median value of G is 3.99
if M = 265, the median value of G is 2.76
if M = 167.5, the median value of S is 14
if M = 265, the median value of S is 11.14
if I = 8.5, the median value of L is 9.34
if I = 9.75, the median value of L is 11.58
if I = 8.5, the median value of R is 4.23
if I = 9.75, the median value of R is 4.58
if I = 8.5, the median value of F is 322.5
if I = 9.75, the median value of F is 294.4
if V = 422.5, the median value of P is 500.0
if V = 492.5, the median value of P is 523.5

The results which were obtained are shown in table 6.11. The average of the errors in the coefficients of correlation given by the sampling scheme is approximately 0.08. This is encouragingly small.

Table 6.11 Accuracy of Coefficients of Correlation

Dep/Indep.	True Coefficient of Correlation	Coefficient of Correlation by sampling scheme
G/M	-0.49	-0.58
S/M	-0.58	-0.49
L/I	0.63	0.75
R/I	0.63	0.71
F/I	-0.58	-0.56
P/V	0.50	0.45

6.9 THE GROWTH RATE PROBLEM

In risk analysis the value of a variable such as 'sales' or 'unit cost' in any one year is often considered to depend on its value in one or more previous years. It is customary to take account of this by describing the variable in terms of a distribution of its initial value and a distribution of its percentage annual growth rate. There are however, several difficulties as far as this practice is concerned. For example:

(a) The meaning of the term 'the distribution of the percentage annual growth rate' may be unclear to the assessor. Indeed, as has been pointed out elsewhere in this thesis, different writers on risk analysis have used the term in different ways. Wagle (1967) in an analysis of the Hertz model assumes that the distribution of the market growth rate is the same in each year but that the value of the growth rate in any one year in independent of that in any other year. Other authors who have analysed the same model using risk simulation assume that the market growth rate is constant over the life of the project (i.e. that it is only necessary to sample from the distribution once during each simulation run).

(b) If it is decided that the second of the two definitions of the 'distribution of the percentage annual growth rate' given in (a) is appropriate then it must be recognised that the growth rate will not be in practice exactly constant over the life of the project and that any assessments which management make will correspond to an 'average' growth rate. The question which then arises is:

do management calculate average growth rates from the scenarios of the future which they have in their minds in a sensible way. In this connection it is worth noting that, because of discounting, departures from the average growth curve early in a project's life are likely to be more important than departures later in the project's life.

(c) Management may not expect the average growth rate of a variable to remain constant over time. This is particularly likely to be the case if the variable is 'sales' or 'market size'. Bass (1969) has analysed data for 11 consumer durable products and shown that the model

$$S\ (T) \quad = \quad (m\ (p+q)^2/p) \quad e^{-(p+q)T}\ (q/p\ e^{-(p+q)T} + 1)^2 \qquad 6.10$$

gives a good fit where S (T) is the total sales at time T and m, p and q are constants. The model, which is illustrated in figure 6.1, is based on the assumption that the timing of a consumer's initial purchase is related to the number of previous buyers.

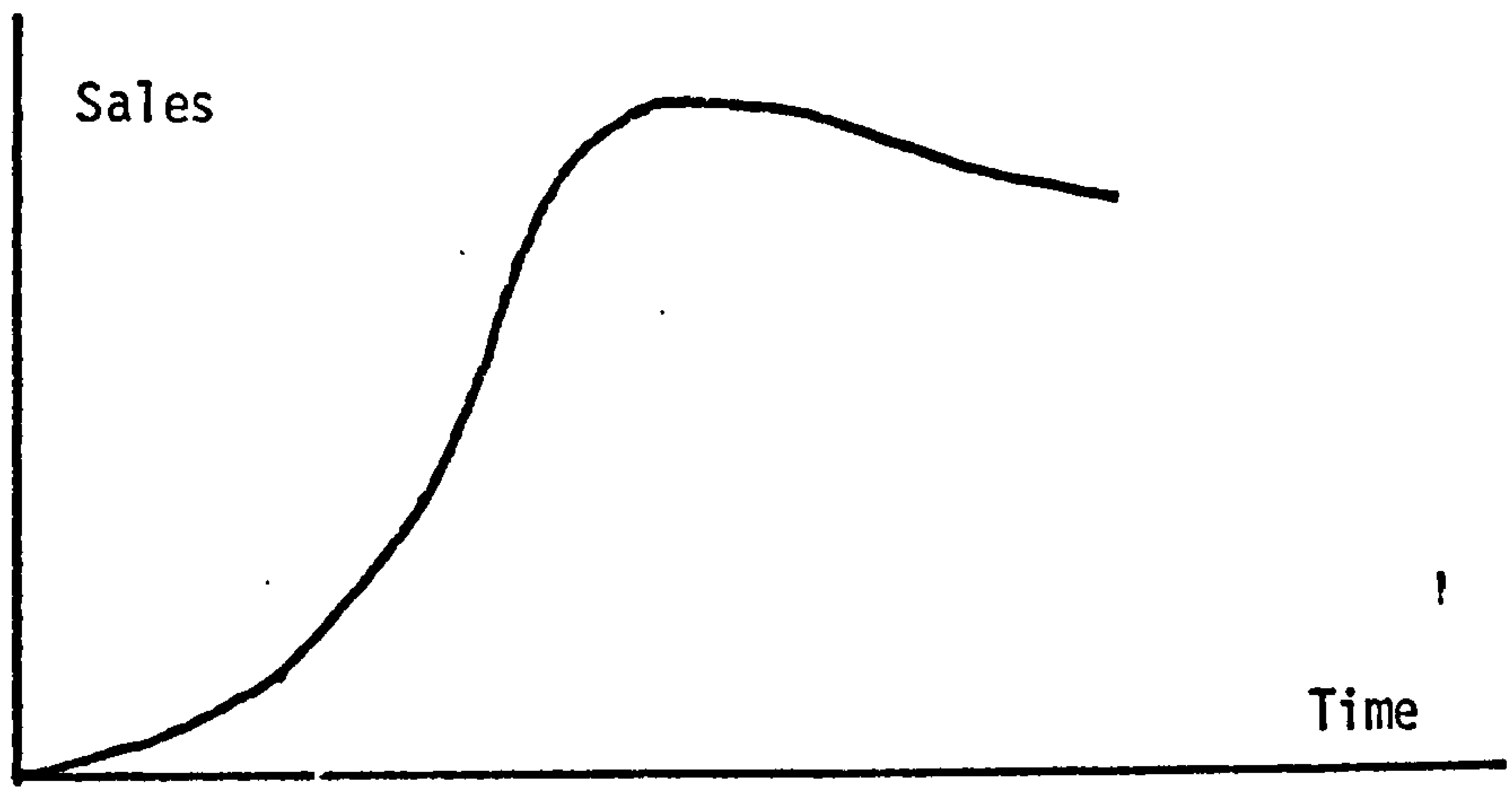

Figure 6.1 Growth of a New Product

In many situations therefore it might be felt desirable to abandon the 'initial value + constant growth rate' model. Three alternatives to the model applicable to situations where difficulty (c) above is encountered are listed below:

(i) Identify Growth Parameters

If growth is not expected to be exponential it may be possible for management to identify parameters describing the expected growth and to provide distributions for these parameters. With a new product the relevant parameters might be 'the initial growth rate' and 'the maximum sales'. Alternatively Bass's model (see equation 6.10) could be used with distributions being determined for p, q and m. (Bass defines p as the coefficient of innovation and q as the coefficient of imitation; m is related to the peak sales).

(ii) Use Several Different Growth Rates

If growth is not expected to be exponential throughout the whole of the life of a project it may be possible to divide the total life of the project into several parts and to obtain a probability distribution for the growth rate corresponding to each part. This does however have drawbacks as questions of the form: how does the growth rate in years 5-8 depend on that in years 1-4 immediately arise?

(iii) Use a Family of Growth Curves

If exponential growth curves are not appropriate it may be possible to define in conjunction with management, a special family of growth curves (see figure 6.2) for the variable under consideration. The 'median growth' curve, the 'upper quartile' growth curve and the 'lower quartile' growth curve could be assessed in a similar way to that for assessing the 0.5, 0.75, 0.25 fractiles of an ordinary distribution. Packages such as PROSPER it is interesting to note tend to encourage the use of families of growth curves. All growth curve methods do however have the disadvantage identified in (b) above.

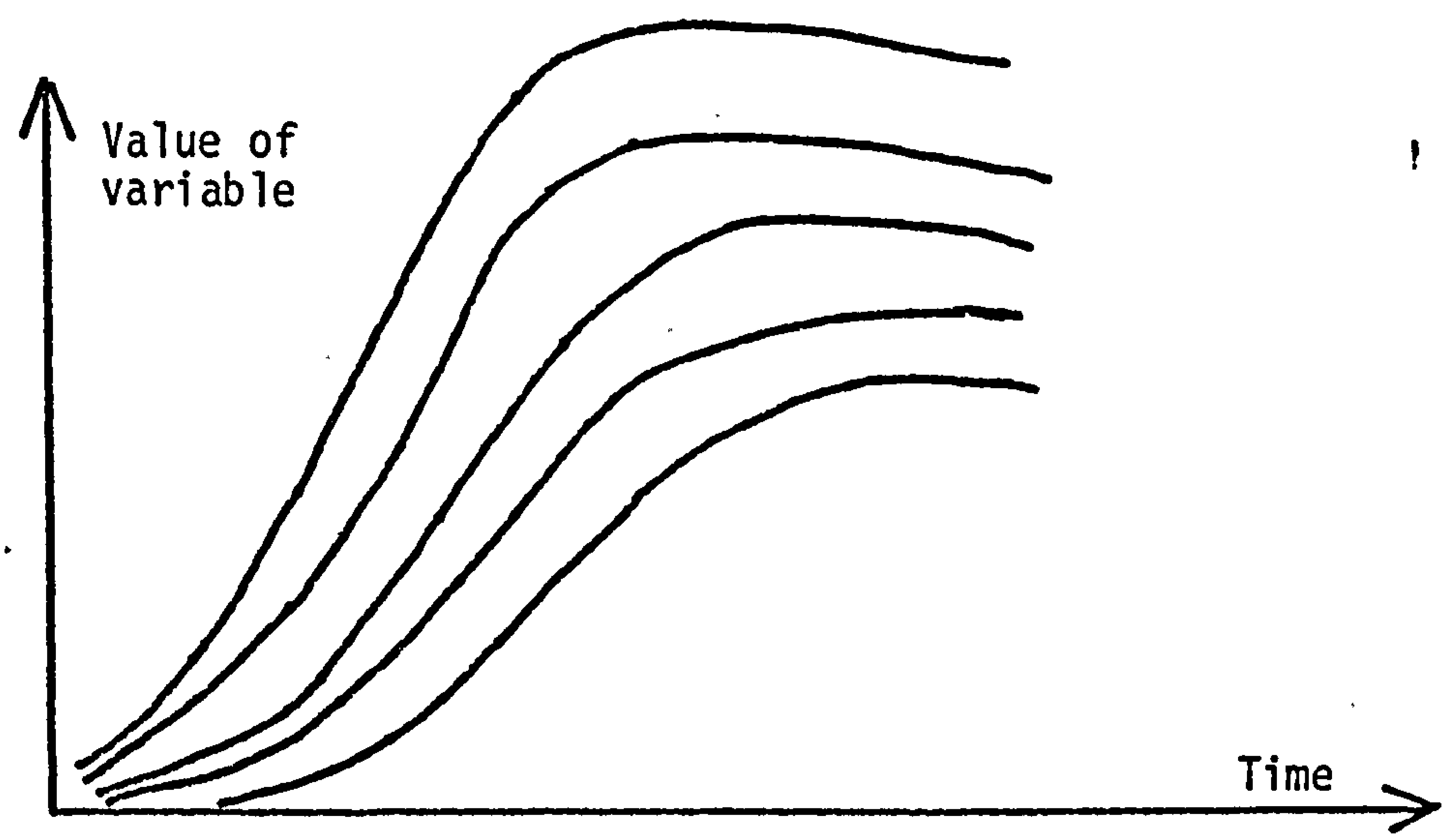

Figure 6.2 A Family of Growth Curves

Yet another alternative to the use of the 'initial value + constant growth rate model is the use of a model of probabilistic growth. A number of different models of probabilistic growth have been described in articles concerned with capacity expansion (see for example, Manne (1961) and

Giglio (1970). A further one can be based on the model of dependence which has been presented in this chapter. Define:

x_i the value of the variable in time period i

f_i the unconditional probability distribution of x_i

h_i (x_i/x_{i-1}) the conditional probability of x_i given the value of x_{i-1}

It has been shown in this chapter that the f_i and h_i are consistent if there exist transformations t_i such that:

applying t_i to f_i produces a normal distribution with mean μ_i and variance σ_i^2

and: applying t_i to g_i produces a normal distribution with mean:

$$\mu_i + \rho_i^* \frac{\sigma_i}{\sigma_{i-1}} \quad (t_{i-1}(x_{i-1}) - \mu_{i-1})$$

and variance:

$$\sigma_i^2 (1 - \rho_i^{*2})$$

where ρ_i^* is the coefficient of correlation between $t_{i-1}(x_{i-1})$ and $t_i(x_i)$.

Very often it will be possible to assume:

(a) that the f_i are all distributions of the same type (e.g. that they are all 4-parameters lognormal distributions)

and (b) that the ρ_i^* are all equal. (This corresponds approximately to constant auto-correlation).

It then becomes unnecessary to determine explicitly the parameters of f_i for every single value of i. Assessing the parameters directly for a few values of i and obtaining the rest by interpolation should be sufficient. In addition (because of (b) only one assessment of the form:

assuming the value of the variable in year i is Q, my median estimate for year i+1 is P.

is in theory necessary in order to determine the auto-correlation pattern.

6.10 SUMMARY AND CONCLUSIONS

This chapter has shown that the effect on a performance measure of a dependence between two variables is liable to be just as important as the effect of either:

(a) a 30% error in the standard deviation of one of the variables.

or (b) an error in the mean of one of the variables equal to 5% of the variable's range.

(One of the potential dependencies in case A was found to be particularly important in that it could decrease the standard deviation of NPV by up to 72%).

All of the methods suggested in the literature for assessing dependence have one or more of the following disadvantages:

(i) management are required to make an unreasonably large number of individual probability judgements.

(ii) management make no quantitative judgements at all concerning the extent of the dependence.

(iii) the conditional distributions used for sampling cannot in general be made consistent with pre-determined unconditional distributions.

(iv) a coefficient of correlation rather than a complete sampling scheme is produced. (This is not necessarily a disadvantage in analytic approaches to risk evaluation).

A new method, overcoming these disadvantages, which is particularly suitable for risk simulation, has been suggested. The method involves first asking management to assess unconditional distributions for all the variables and then asking management to make, for each pair of dependent variables X and Y, further assessments of the form:

if X = Q, my median estimate for Y is P.

All the assessments are fitted to a model which is obtained by applying monotonic transformations to the bivariate normal distribution.

The proposed method has been demonstrated on the data in Eilon and Fowkes (1973) and some encouraging results have been obtained concerning the accuracy of the coefficients of correlation between the distributions produced by the method.

It is worth noting that the method can be extended in an obvious way, using the properties of the multivariate normal distribution, to deal with situations where the value of a variable is dependent on the values of two or more other variables.

The discussion of 'the growth rate problem' in this chapter has in many ways raised more questions than it has answered. When a variable is expected to have a different value in each year of the life of the project there are a number of alternatives open to the analyst. These might be described as:

(a) assume an 'initial value + constant growth rate' model and obtain probability distributions for the initial value and the growth rate.

(b) identify growth parameters which are different from those in (a) and obtain probability distributions for them.

(c) assume that the variable has different growth rates at different stages during the project's life.

(d) use a family of growth curves.

(e) use a probabilistic growth model.

As far as (e) is concerned it is interesting to note that the model which is proposed for the assessment of the dependence between two variables can also be formulated as a model of probabilistic growth.

CHAPTER 7

DISTINGUISHING BETWEEN IMPORTANT AND UNIMPORTANT PROBABILITY ASSESSMENTS IN RISK EVALUATION

7.1 INTRODUCTION

In risk evaluation it is unreasonable to expect management to provide detailed probabilistic descriptions of every single variable and every single dependency. The analyst must make some attempt to distinguish the important probability assessments from the unimportant ones so that management attention can be directed towards the former.

Many authors have suggested that the importance of a particular variable can be ascertained by means of a sensitivity analysis. Pouliquen (1970) has also suggested that the importance of a particular dependence can be evaluated if two separate risk simulations are carried out, the first assuming no dependence, the second assuming total dependence (see section 6.3 for a discussion of the difficulties associated with the concept of total dependence). There has however been very little discussion in the literature of the precise criteria which the analyst should use when deciding whether it is necessary for management to make a particular probability assessment and it is this aspect of risk evaluation which the present chapter is concerned with. The chapter starts with a discussion of the way in which the output from a sensitivity analysis should be interpreted. It then uses the results in chapters 4, 5 and 6 to suggest an approach to risk simulation, which is designed to limit the total number of probability assessments which management are asked to make.

7.2 AN ANALYSIS OF THE OUTPUT FROM A SENSITIVITY ANALYSIS

This section considers analytically the interpretation which can be put on the results from a sensitivity analysis in two fairly simple situations. It is particularly concerned with the extent to which the results from sensitivity analyses provide indications as to:

(i) the relative importance of errors in the means of different variables.

(ii) the relative importance of errors in the standard deviations of different variables.

and (iii) the relative importance of dependencies between different pairs of variables.

Suppose that the investment involves n uncertain variables. Let X_i denote the value of the i-th variable and:

$$P = f\ (X_1, X_2 \ldots\ldots X_n)$$

where P is the value of the performance measure. Define:

μ_P Mean of P

σ_P Standard deviation of P

μ_i	Mean of X_i
σ_i	Standard deviation of X_i
ρ_{ij}	Coefficient of correlation between X_i and X_j
$\Delta\mu_i$	Small error in μ_i
$\Delta\sigma_i$	Small error in σ_i
$e_{i,1}$	Effect of the error $\Delta\mu_i$ on μ_P
$e_{i,2}$	Effect of the error $\Delta\sigma_i$ on μ_P
$e_{i,3}$	Effect of the error $\Delta\mu_i$ on σ_P
$e_{i,4}$	Effect of the error $\Delta\sigma_i$ on σ_P
g_{ij}	Increase in μ_P when ρ_{ij} changed from 0 to 1
h_{ij}	Increase in μ_P when ρ_{ij} changed from 0 to 1

It will be assumed that, for each variable i, management provide an upper bound U_i, a lower bound L_i and a best estimate E_i. The measure S_i of the importance of variable i which is provided by the sensitivity analysis will be assumed to be:

$$S_i = f(E_i, E_2 \ldots E_{i-1}, U_i, E_{i+1} \ldots E_n) - f(E_1, E_2 \ldots E_{i-1}, L_i, E_{i+1} \ldots E_n)$$

(See chapter 1 for a further discussion of the ways in which a sensitivity analysis can be carried out).

Define p_i and q_i ($1 \leqslant i \leqslant n$) as follows:

$$p_i = \frac{\Delta\mu_i}{U_i - L_i} \qquad 7.1$$

$$q_i = \frac{\Delta\sigma_i}{\sigma_i} \qquad 7.2$$

(p_i is the error in μ_i expressed as a fraction of the range; q_i is the error in σ_i expressed as a fraction of σ_i).

Also, define for i=1, 2....n:

$$k_i = \frac{\sigma_i}{U_i - L_i} \qquad 7.3$$

$$K_i = \frac{\mu_i}{E_i} \qquad 7.4$$

Situation No. 1

Suppose:

$$P = f(X_1, X_2 \ldots X_n) = \sum_{i=1}^{n} a_i X_i$$

where the a_is are constant. (This equation would be appropriate in the situation where P is Net Present Value and where each X_i represents either an inflow or an outflow of money).

Assume initially that the X_is are independent. It is easy to see that:

$$S_i = a_i \ (U_i - L_i) \qquad 7.5$$

Also

$$\mu_P = \sum_{i=1}^{n} a_i \mu_i \qquad 7.6$$

leading to:

$$\frac{\partial \mu_P}{\partial \mu_i} = a_i$$

and

$$e_{i,1} = a_i \Delta\mu_i$$

which using equations 7.1 and 7.5 reduces to:

$$e_{i,1} = p_i S_i \qquad 7.7$$

Returning to equation 7.3

$$\frac{\partial \mu_P}{\partial \sigma_i} = 0$$

and therefore:

$$e_{i,2} = 0 \qquad 7.8$$

It is also true that:

$$\sigma_P^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2 \qquad 7.9$$

and therefore:

$$\frac{\partial \sigma_P}{\partial \mu_i} = 0$$

leading to:

$$e_{i,3} = 0 \tag{7.10}$$

Also from equation 7.9

$$2\sigma_p \frac{\partial \sigma_p}{\partial \sigma_i} = 2\, a_i^2\ \sigma_i$$

i.e. $$\frac{\partial \sigma_p}{\partial \sigma_i} = \frac{a_i^2\ \sigma_i}{\sigma_p}$$

Hence, if $\Delta\sigma_i$ is small:

$$e_{i,4} \doteqdot \frac{a_i^2\ \sigma_i}{\sigma_p} \Delta\sigma_i$$

which using equations 7.2, 7.3 and 7.5 becomes:

$$e_{i,4} \doteqdot \frac{S_i^2\ q_i\ k_i^2}{\sigma_p} \tag{7.11}$$

Suppose now that X_i and X_j are dependent with coefficient of correlation $\rho_{ij.\mu p}$ remains unchanged while σ_p^2 increases by:

$$2\ a_i\ a_j\ \rho_{ij}\ \sigma_i\ \sigma_j$$

and assuming that this is small in relation to σ_p^2 it can be deduced that σp itself increases by approximately

$$\frac{a_i\ a_j\ \rho_{ij}\ \sigma_i\ \sigma_j}{\sigma_p}$$

Hence

$$g_{ij} = 0 \tag{7.12}$$

and

$$h_{ij} \doteqdot \frac{a_i\ a_j\ \sigma_i\ \sigma_j}{\sigma_p}$$

Using equations 7.3 and 7.5:

$$h_{ij} \doteqdot \frac{S_i\ S_j\ k_i\ k_j}{\sigma_p} \tag{7.13}$$

Situation No. 2

Suppose:

$$P = f(X_1, X_2 \ldots X_n) = b_{ij} X_i X_j + Z$$

where b_{ij} is a constant and Z is a function of all variables except X_i and X_j.

$$S_i = b_{ij} E_j (U_i - L_i) \qquad 7.14$$

$$S_j = b_{ij} E_i (U_j - L_j)$$

$$\mu_P = b_{ij} \mu_i \mu_j + \mu_Z$$

where μ_Z is the mean of Z

$$\frac{\partial \mu_P}{\partial \mu_i} = b_{ij} \mu_j$$

Hence

$$e_{i,1} = b_{ij} \mu_j \Delta\mu_i$$

and using equations 7.1, 7.4 and 7.14 this becomes

$$e_{i,1} = K_j p_i S_i \qquad 7.15$$

Similarly:

$$e_{j,1} = K_i p_j S_j$$

$$\frac{\partial \mu_P}{\partial \sigma_i} = 0$$

and therefore:

$$e_{i,2} = 0 \qquad 7.16$$

Also it is true that:

$$\sigma_P^2 = b_{ij}^2 \mu_i^2 \sigma_j^2 + b_{ij}^2 \mu_j^2 \sigma_i^2 + b_{ij}^2 \sigma_i^2 \sigma_j^2 + \sigma_Z^2$$

where σ_Z is the standard deviation of Z.

Hence:

$$2\sigma_P \frac{\partial \sigma_P}{\partial \mu_i} = 2 b_{ij}^2 \mu_i \sigma_j^2$$

i.e.
$$\frac{\partial\sigma_P}{\partial\mu_i} = \frac{b_{ij}^2\ \mu_i\ \sigma_j^2}{\sigma_P}$$

If $\Delta\mu_i$ is small:

$$e_{i,3} \doteq \frac{b_{ij}^2\ \mu_i\ \sigma_j^2}{\sigma_P}\ \Delta\mu_i$$

which using equations 7.1, 7.4 and 7.14 becomes

$$e_{i,3} \doteq \frac{S_i^2\ p_i\ \mu_i\ \sigma_j^2\ k_i\ K_j^2}{\mu_j^2\ \sigma_i\ \sigma_P} \qquad 7.17$$

Also:

$$2\sigma_P\ \frac{\partial\sigma_P}{\partial\sigma_i} = 2\ b_{ij}^2\ \mu_j^2\ \sigma_i + 2\ b_{ij}^2\ \sigma_i\ \sigma_j^2$$

i.e.
$$\frac{\partial\sigma_P}{\partial\sigma_i} = \frac{b_{ij}^2\ \mu_j^2\ \sigma_i}{\sigma_P} + \frac{b_{ij}^2\ \sigma_j^2\ \sigma_i}{\sigma_P}$$

which if $\Delta\sigma_i$ is small gives eventually (using equations 7.2, 7.3, 7.4 and 7.14).

$$e_{i,4} \doteq \frac{q_j\ k_i^2\ K_j^2\ S_i^2}{\sigma_P}\left[\ 1 + \frac{\sigma_j^2}{\mu_j^2}\right] \qquad 7.18$$

Using equations 4.3 - 4.6 it can also be shown that:

$$g_{ij} = b_{ij}\ \sigma_i\ \sigma_j$$

and that in certain circumstances (corresponding to the last 4 terms in equation 4.6 being small):

$$h_{ij} \doteq \frac{b_{ij}^2\ \mu_i\ \mu_j\ \sigma_i\ \sigma_j}{\sigma_P}$$

These equations reduce to:

$$g_{ij} = \left[\frac{S_i\ S_j\ k_i\ k_j\ K_i\ K_j}{\mu_i\ \mu_j}\right]^{\frac{1}{2}} \qquad 7.19$$

$$h_{ij} \doteq \frac{S_i\ S_j\ k_i\ k_j\ K_i\ K_j}{\sigma_P} \qquad 7.20$$

7.3 DISCUSSION CONCERNING THE OUTPUT FROM A SENSITIVITY ANALYSIS

The main results produced in section 7.2 are as follows:

(i) If $P = \sum_{i=1}^{n} a_i X_i$

where the a_is are constants then:

$$e_{i,1} = p_i S_i \qquad 7.7$$

$$e_{i,2} = 0 \qquad 7.8$$

$$e_{i,3} = 0 \qquad 7.10$$

$$e_{i,4} \doteq \frac{q_i\ S_i^2\ k_i^2}{\sigma_P} \qquad 7.11$$

$$g_{ij} = 0 \qquad 7.12$$

$$h_{ij} \doteq \frac{S_i\ S_j\ k_i\ k_j}{\sigma_P} \qquad 7.13$$

(ii) If $P = b_{ij} X_i X_j + Z$

where b_{ij} is a constant and Z is a function of all variables except X_i and X_j then:

$$e_{i,1} = K_j\ p_i\ S_i \qquad 7.15$$

$$e_{i,2} = 0 \qquad 7.16$$

$$e_{i,3} \doteq \frac{p_i\ S_i^2\ \mu_i\ \sigma_j^2\ k_i\ K_j^2}{\mu_j^2\ \sigma_i\ \ \sigma_P} \qquad 7.17$$

$$e_{i,4} \doteq \frac{q_j\ S_i^2\ k_i^2\ K_j^2}{\sigma_P}\left[1 + \frac{\sigma_j^2}{\mu_j^2}\right] \qquad 7.18$$

$$g_{ij} = \left[\frac{S_i\ S_j\ k_i\ k_j\ K_i\ K_j}{\mu_i\ \ \mu_j}\right]^{\frac{1}{2}} \qquad 7.19$$

$$h_{ij} \doteq \frac{S_i\ S_j\ k_i\ k_j\ K_i\ K_j}{\sigma_P} \qquad 7.20$$

Assume k_i is approximately constant for all i. (This corresponds to assuming that the standard deviation of a variable is approximately proportional to its range). The results for the first model:

$$P = \sum_{i=1}^{n} a_i \; X_i \qquad a_i \text{ constant}$$

show that:

(a) the effect of an error in the mean of variable i on the mean of the performance measure is equal to the product of S_i and the error (the error being expressed as a proportion of the variable's range).

(b) the effect of a certain percentage error in the standard deviation of variable i on the standard deviation of the performance measure is approximately proportional to the product of S_i^2 and the percentage error.

(c) the effect of total dependence between variables i and j on the standard deviation of the performance measure is approximately proportional to the product of S_i and S_j.

The results corresponding to the second model are rather more complicated than these, but it is worth noting that for both models:

(a) it is S_i which enters into calculations concerned with the effect of errors in the mean of variable i

(b) it is S_i^2 which enters into calculations concerned with the effect of errors in the standard deviation of variable i.

and (c) it is the product of S_i and S_j which enters into calculations concerned with the effect of a dependence between X_i and X_j.

Unfortunately, it is not easy to extend the results in section 7.2 to deal with a wide range of cash flow models. If the standard deviations of all variables are small then Taylor's theorem can be used to show that the results produced for the model:

$$P = \sum_{i=1}^{n} a_i \; X_i \qquad a_i\text{s constant}$$

are approximately true, but in general the results produced for this model do not provide a good indication as to the relative importance of the means and standard deviations of different variables. The analysis of case A in appendix H illustrates this point.

An attempt was made to estimate the effect of dependencies between pairs of variables i and j in case A by approximating the functional form:

$$P = f\ (X_1, X_2 \ldots X_n)$$

to $$P = \alpha + \beta X_i + \gamma X_j + \delta X_i X_j$$

where α, β, γ and δ are independent of X_i and X_j. However this was not successful as it only gave the effect of a dependence between the variables on the basis of an assumption that α, β, γ and δ were constants. (The effect of a dependence between variables i and j on the term

$$\delta X_i X_j$$

can be considerably greater than δ times its effect on $X_i X_j$ if δ itself has a large standard deviation. Also, as the same variables are often used in the calculation of two or more of α, β, γ and δ, the latter are, in many cases, not independent of each other).

7.4 THE POSSIBILITY OF A SENSITIVITY ANALYSIS BEING TOTALLY MISLEADING

Even when P is a relatively simple function of the X_is, the results from a sensitivity analysis can in theory be totally misleading. Suppose:

$$P = X_1 X_2 + X_3$$

and that:

$$\begin{array}{lll} L_1 = -11\ ; & E_1 = 1\ ; & U_1 = 13 \\ L_2 = -12\ ; & E_2 = 0\ ; & U_2 = 12 \\ L_3 = 0\ ; & E_3 = 12\ ; & U_3 = 24 \end{array}$$

with the distributions of the three variables being independent and triangular in shape.

Then

$$S_1 = 0$$
$$S_2 = 24$$
$$S_3 = 24$$

giving the impression that errors in the distributions of X_1 are totally unimportant when compared with errors in the distributions of X_2 and X_3 and that errors in the distributions of X_2 and X_3 are equally important. In fact it can be shown, from the analyses given in the previous section that:

$$e_{1,4} \doteq \frac{256 q_1}{\sigma_P}$$

$$e_{2,4} \doteqdot \frac{272q_2}{\sigma_p}$$

$$e_{3,4} \doteqdot \frac{16q_3}{\sigma_p}$$

i.e. a small percentage error in σ_1 has 16 times the effect on σ_p as the same percentage error on σ_3 and only marginally less effect than the same percentage error in σ_2.

Another example of a situation where the results of a sensitivity analysis are liable to be misleading is suggested by case D. Suppose that a certain capital investment project involves building a plant to manufacture a product for which there are two markets: market I and market II. Suppose that:

(a) market II will only be supplied if market I has been completely satisfied.

and (b) on the basis of best estimates market I cannot be completely satisfied.

A sensitivity analysis would then produce the result

$$S_i = 0$$

for all variables i relating to market II even though there might in fact be a high probability of market II being supplied.

7.5 THE USE OF A SENSITIVITY ANALYSIS TO SCREEN VARIABLES INITIALLY

In this section and the sections which follow the following definitions will be used:

Category A variables:	variables which need to be described by a probability distribution for the purposes of the investment decision being considered.
Category B variables:	variables which can satisfactorily be described by a single point estimate.

The analyses in sections 7.2, 7.3, 7.4 and appendix H show that:

(a) When the model:

$$P = \sum_{i=1}^{n} a_i X_i \qquad a_i\text{s constant}$$

is appropriate the sensitivity coefficients can be used to provide a good indication as to the relative importance of the different means, standard deviations and coefficients of correlation.

(b) With more complicated models the sensitivity coefficients may only provide rough indications as to the relative importance of the different means, standard deviations and coefficients of correlation.

(c) In some situations the sensitivity coefficients are liable to be totally misleading.

This section considers whether, in view of these results, a sensitivity analysis can be used to classify certain of the variables in a risk evaluation model as 'category B'. (Many authors recommend that this is precisely what a sensitivity analysis should be used for. Pouliquen (1970) for example in the Port of Mogadiscio study started with 27 variables and after carrying out a sensitivity analysis came to the conclusion that the variability of the performance measure was primarily explained by 7 of the variables. The other 20 variables were then classified as 'category B' and eliminated from further consideration).

Define for each variable, i:

$$R_i = \frac{|S_i|}{\text{Max}\left\{|S_k| : k = 1, 2.....n\right\}}$$

R_i will be termed the 'range coefficient' of variable i. It is a measure of how sensitive variable i is relative to the most sensitive variable and is in many ways easier to interpret than S_i. The values of $|S_i|$ and R_i (as produced by the computer program RISKANAL1) for the five case studies described in section 3.5 are shown in appendix G.

One rule which could be used when variables are screened initially is the following:

If $R_i \leqslant C$ assign variable i to category B;

If $R_i > C$ assign variable i to category A

for some value of C between 0 and 1.

To explore how this rule would work in practice four risk simulations were carried out for each of the five case studies described in section

3.5. In the first of these simulations each of the variables was described by a triangular distribution using the data in appendix C. The other three simulations tested the consequences, for C = 0.05, 0.1 and 0.25, of using the following rule:

If $R_i \leqslant C$ assume variable i is equal to its best estimate

If $R_i > C$ assume variable i has a triangular distribution as in the first simulation.

The results which were obtained are shown in tables 7.1, 7.2 and 7.3. (Note in the interpretation of tables 7.2 and 7.3 that C = 0 corresponds to the first simulation where all the variables were described by distributions).

Table 7.1 Number of Category A Variables for Different Values of the 'Cut-off Range Coefficient' C

Case Study	Total No. of Variables	Number of Category A Variables when C equals		
		0.05	0.1	0.25
A	9	7	5	4
B	12	10	6	4
C	7	7	7	6
D	17	8	7	6
E	4	4	4	2

Table 7.2 Effect of Value of C on Mean of NPV (Discount Rate = 10%; No. of Simulation Runs = 1000)

Case Study	Mean of NPV assuming C equals			
	0	0.05	0.1	0.25
A	-2590	-2470	-3020	-3030
B	1750	1860	2220	3060
C	74	74	74	78
D	8870	8870	8860	8880
E	2980	2980	2980	2630

Table 7.3 Effect of Value of C on S.D. of NPV (Discount Rate = 10%; No. of Simulation Runs = 1000)

Case Study	S.D. of NPV assuming C equals			
	0	0.05	0.1	0.25
A	10040	10030	9940	9940
B	5690	5670	5640	5670
C	93	93	93	94
D	535	533	532	524
E	1990	1990	1990	1920

It can be seen from table 7.3 that errors in the standard deviation of the performance measure resulting from the application of the rule described above are very small indeed even when C = 0.25 (In fact as the standard error of a standard deviation, σ, in table 7.3 is approximately:

$$\frac{\sigma}{\sqrt{2000}}$$

$$= \frac{\sigma}{44.72}$$

the errors in table 7.3 caused by increasing C from 0 to 0.25 are the same order of magnitude as potential sampling errors).

It can be seen from table 7.2 that the errors in the mean of the performance measure resulting from the application of the rules which were used are on occasion quite large. Further investigations revealed that this is almost entirely because some of the variables i for which:

$$R_i \leqslant C$$

are highly skewed and have the property that the difference between E_i and μ_i is quite large.

A sensible procedure aimed at reducing the errors in table 7.2 would be to assume that a category B variable is equal to an estimate of μ_i rather than that it is equal to E_i. The PERT formula

$$\mu_i = \frac{1}{6}\left[L_i + 4\,E_i + U_i\right]$$

which is discussed in section 5.1 could be used to provide the estimate of μ_i. (The logic of this procedure is reinforced by the observation that, for most simple cash flow models, the standard deviation of a variable has very little effect on the mean of the performance measure).

Section 7.4 produced examples to show that in theory the results of a sensitivity analysis can be totally misleading when the performance measure is not a linear function of the variables. No serious difficulties of this nature were however encountered in case studies A, B, C, D and E and the use of a sensitivity analysis to screen variables initially in the way which has been described in this section would seem on the basis of the case studies which have been examined to be justifiable. The analyst should of course be aware of potential pitfalls and in some cases it may be desirable for him to vary the values of two or more variables simultaneously in a sensitivity analysis to confirm that it is justifiable to assign a certain variable to category B (For example, in case D the fact that the 'sales vol. in market 2' variable can be assigned to category B should be confirmed by investigating the effect on the performance measure of this variable when production capacity is high).

7.6 THE BEST VALUE OF THE CUT-OFF RANGE COEFFICIENT

This section considers analytically arguments concerned with the way in which the cut-off range coefficient, C should be set in the case of the model:

$$P = \sum_{i=1}^{n} a_i X_i \qquad a_i\text{s constant} \qquad 7.21$$

Suppose that variable m is the most sensitive variable and that:

$$T_j = \frac{\text{Error in } \sigma_P \text{ caused by a small error equal to } q\sigma_m \text{ in } \sigma_m}{\text{Error in } \sigma_P \text{ caused by assuming that } \sigma_j = 0}$$

From the analyses in section 7.2 it can easily be deduced that:

$$T_j \doteq \frac{2q\, a_m^2\, \sigma_m^2}{a_j^2\, \sigma_j^2}$$

and if it is assumed that k_i is constant for all variables i it follows from this that:

$$T_j \doteq \frac{2q\, S_m^2}{S_j^2}$$

$$= \frac{2q}{R_j^2}$$

$T_j = 1$ when:

$$q \doteq \frac{R_j^2}{2}$$

and from this it can be deduced that the error in the performance measure which is caused by assuming that a variable, j, with range coefficient R_j can be replaced by its best estimate is approximately equal to the error in the performance measure caused by an error of:

$$\frac{R_j^2\, \sigma_m}{2}$$

in σ_m

Replacing a variable with range coefficient equal to 0.25 by a single point estimate does therefore have the same effect on the standard deviation of the performance measure as an error of approximately 3% in the standard deviation of the most sensitive variable. Section 5.4 shows that, even if management make 7 perfectly accurate assessments for the distribution of a variable, the error in the standard deviation which is calculated for the variable averages about 8% and on that basis a case could be made out for choosing a value of C equal to, or even greater than, 0.25.

There are however a number of reasons why it might in practice be considered desirable to choose a value for C less than 0.25:

(i) Non-linearities in the function:

$$P = f\ (X_1, X_2 \ldots\ldots X_n)$$

are liable to cause the error in σ_P resulting from σ_j being ignored to be greater than that assumed above (Appendix G shows that this happens in case A).

(ii) The mean of a variable j for which $R_j = 0.25$ does have a significant effect on σ_P. The decision to describe variable j by a distribution may eventually result in a more accurate value for μ_j than that provided by the PERT formula being found.

and (iii) If all the variables i for which

$$R_i \leqslant 0.25$$

are replaced by a single point estimate then there is a possibility that a correlation ρ_{mj} between the most sensitive variable, m, and a variable j for which $R_j = 0.25$ will be ignored. Define:

$$T_j' = \frac{\text{Error in } \sigma_p \text{ caused by an error equal to } q\sigma_m \text{ in } \sigma_m}{\text{Error in } \sigma_p \text{ caused by assuming } \rho_{mj} = 0}$$

On the assumption of the linear cash flow model in equation 7.21 it can easily be shown that:

$$T_j' \doteqdot \frac{q\, a_m\, \sigma_m}{\rho_{mj}\, a_j\, \sigma_j}$$

which assuming k_i is approximately constant for all i becomes:

$$T_j' \doteqdot \frac{q\, S_m}{\rho_{mj}\, S_j}$$

$$= \frac{q}{\rho_{mj}\, R_j}$$

and it can be seen that, if $\rho_{kj} = 0.5$, then assuming $\rho_{kj} = 0$ is equivalent to accepting a 12½% error in the standard deviation of the most sensitive variable when $R_j = 0.25$.

As a 'rule of thumb' C = 0.1 does, to the present author, seem better than C = 0.25 for the reasons just mentioned. An analysis similar to the one above indicates that C = 0.05 would for the linear model being considered be unnecessarily conservative.

One point which is worth noting however is that in some situations the number of variables which would be replaced by a point estimate with C = 0.1 might greatly exceed the number remaining. Direct comparisons between the errors in σ_p and μ_p caused by errors in σ_m and μ_m and the errors in σ_p and μ_p arising from other sources might then be misleading. As a guide, it can easily be shown that in the linear model in equation 7.21 the condition:

$$\frac{\text{Sum of Range Coefficients for Category B Variables}}{\text{Sum of Range Coefficients for Category A Variables}} \leqslant C$$

should be satisified if arguments similar to those given above are to be used.

It is possible (although unlikely in view of Taylor's theorem) that in a non-linear model the variables i for which:

$$R_i \leqslant C$$

could collectively have a far greater effect on μ_P and σ_P than the sum of their range coefficients would suggest. Whether this does in fact happen could be tested by calculating the value of the performance measure when:

(a) all the variables which are candidates for category B equal their pessimistic estimates and all other variables equal their best estimates.

and (b) all the variables which are candidates for category B equal their optimistic estimates and all other variables equal their best estimates.

and comparing the difference between the two values with:

$$\sum S_i$$

where the summation is over all the variables which are candidates for category B.

7.7 OUTLINE OF AN APPROACH TO RISK SIMULATION

Once a sensitivity analysis has been used to assign variables to either category A or category B, an analysis using risk simulation can be embarked upon. This section outlines an approach to carrying out the analysis which is designed specifically with the following two observations in mind:

(i) a major reason why risk simulation has not been widely accepted is the large number of probability assessments which management are typically required to make for a risk simulation (see Longbottom (1971) and Carter (1972)).

(ii) the cost of the computer time used to carry out a risk simulation once the cash flow model has been developed is, at the present time, trivial (see Bonini (1975)).

In essence, the approach is an attempt to reduce the total number of probability assessments which management are required to make by carrying out a series of 'test' risk simulations.

The proposed approach is illustrated diagrammatically in figure 7.1 As a first stage management identify any dependencies between the variables assigned to category A, stating simply whether the dependencies are positive or negative. A series of risk simulations are then carried out in order to:

(a) provide a 'crude estimate' of the probability distribution of the performance measure based on assumptions as to the distributions of the input variables.

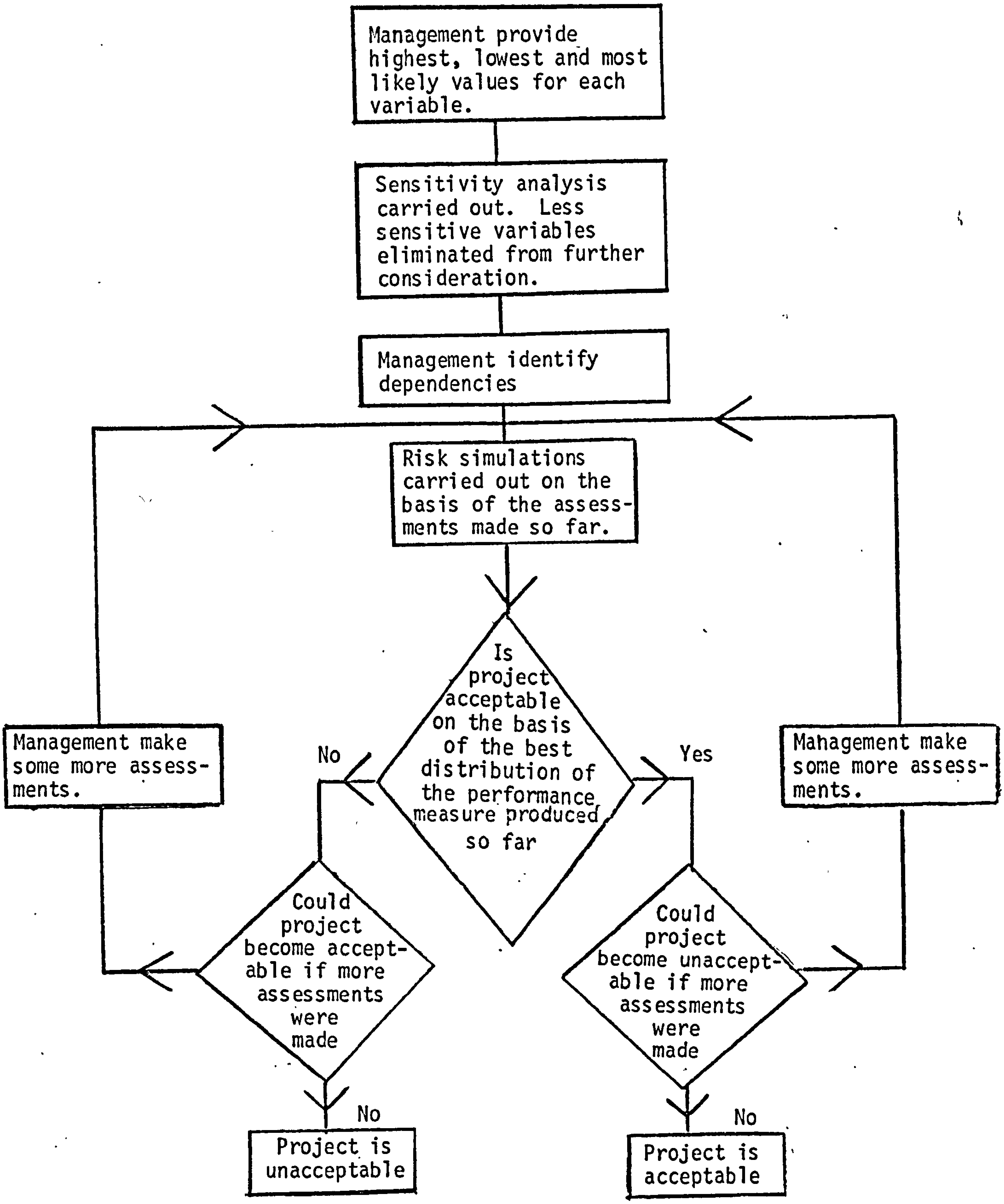

Figure 7.1 Proposed Procedure for Carrying Out Risk Simulations

(b) provide measures of the maximum potential effect on the distribution of the performance measure of each of the dependencies identified by management.

(c) provide, for each variable assigned to category A, measures of the maximum potential error in the distribution of the performance measure resulting from the 'true' distribution of the variable being different from that assumed in (a).

The distribution produced in (a) is then examined and a conclusion is reached as to whether the investment is acceptable on the basis of that distribution. If the investment is considered acceptable, an analysis is carried out to determine the extent to which the distribution of the performance measure would have to change before the investment became unacceptable. If the investment is considered unacceptable, an analysis is carried out to determine the extent to which the distribution of the performance measure would have to change before the investment became acceptable. In either case, the results produced in (b) and (c) above are then used to calculate, first, whether any further probability assessments are necessary and, second, if they are, which ones should be made, In the event of no further probability assessments being necessary, the analysis is finished. If further probability assessments are required, they are made, a further risk simulation is carried out and the results are reviewed again to determine whether further probability assessments are necessary.

7.8 ILLUSTRATION OF THE PROPOSED APPROACH

This section uses case A to provide an illustration of one of the ways in which the approach to risk simulation which was outlined in section 7.7 could be used in practice. The data used in the illustration is taken from Eilon and Fowkes (1973) and is tabulated in section 6.7. The performance measure is assumed to be NPV with a discount rate of 10% p.a.

Table 7.4 shows highest, lowest and most likely values for each of the nine variables in the case study and table 7.5 shows the results of a sensitivity analysis. Under the version of the proposed approach which is being described here, table 7.5 would be used to assign each of the variables to either category A or category B. Assuming a cut-off range coefficient equal to 0.1, the variables assigned to category A would be:

Selling Price
Operating Costs
Mkt. Share
Init. Mkt. Size
Life of Investment

and the variables assigned to category B would be:

Mkt. Growth
Init. Investment

Fixed Costs
Residual Value

Table 7.4 Initial Estimates made in Case A

Variable	Lowest Value	Most Likely Value	Highest Value
Init. Mkt. Size (thousands of tons).	100	250	340
Mkt. Growth (% p,a,)	0	3	6
Selling Price ($ per ton)	385	510	575
Mkt. Share (%)	3	12	17
Init. Investment ($M)	7.0	9.5	10.5
Life of Investment (yrs).	5	10	15
Residual Value (SM)	3.5	4.5	5.0
Operating Costs ($ per ton)	370	435	545
Fixed Costs ($'000s p.a.)	250	300	375

Table 7.5 Sensitivity Analysis: Case A

Variable	NPV at lowest Value ($'000s)	NPV at highest Value ($'000s)	Range of NPV ($'000s)	Range Coefficient
Selling Price	-19934	19303	39237	1.00
Operating Cost	-16836	19303	36139	0.92
Mkt. Share	-5736	12334	18070	0.46
Init. Mkt. Size	-3413	11456	14870	0.38
Life of Investment	9450	1163	8287	0.21
Mkt. Growth	4217	7804	3587	0.09
Init. Investment	4880	8380	3500	0.09
Fixed Costs	6187	5419	768	0.02
Residual Value	6072	5494	578	0.01

Management would then be asked if any dependencies existed between the category A variables. We shall assume that:

(i) Management consider market share to be negatively dependent on initial market size.

and (ii) Management consider selling price to be positively dependent on operating costs.

A total of 13 risk simulations would then be carried out simultaneously in order to provide figure 7.2 and tables 7.6, 7.7 and 7.8.

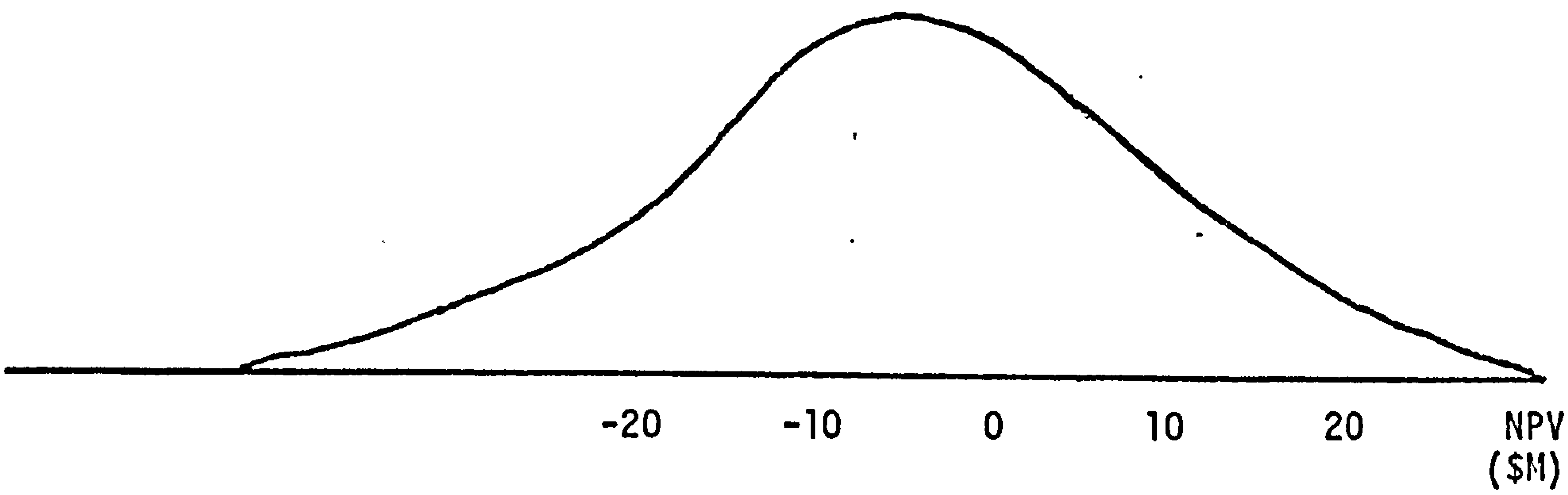

Figure 7.2 Distribution of NPV Assuming Triangular Distributions

Table 7.6 Table Shows Potential Effects of Dependencies ($'000s)

Independent Variable	Dependent Variable	Change in Mean of NPV if total dependence	Change in S.D. of NPV if total dependence
Init. Mkt Size	Mkt. Share	-530	-1360
Operating Costs	Selling Price	+330	-7340

Table 7.7 Effects on distribution of NPV of an increase in Mean of a variable equal to 5% of range ($'000s)

Variable	Effect of increase in mean equal to 5% of range on	
	Mean of NPV	S.D. of NPV
Selling Price	+1620	+170
Operating Costs	-1490	-120
Mkt. Share	+420	+600
Init. Mkt. Size	+340	+490
Life of Investment	+100	+340

Table 7.8 Effect on Distribution of NPV of a 30% Increase in S.D. of a Variable ($'000s)

Variable	Effect of 30% increase in S.D. on	
	Mean of NPV	S.D. of NPV
Selling Price	+10	+1620
Operating Costs	+30	+1390
Mkt. Share	+50	+350
Init. Mkt. Size	+10	+250
Life of Investment	-20	+10

Figure 7.2 shows the probability distribution of the performance measure based on the following assumptions.

(a) the category A variables have the independent probability distributions shown in figure 7.3

and (b) the category B variables are equal to estimates of their means.

The distribution has (in $'000s) a mean of -2920 and a standard deviation of of 10100 with Prob. (NPV > 0) being 0.36. Table 7.6 shows the effect on figure 7.2 of the dependencies in (i) and (ii) above being 'total' (for a definition of 'total dependence' see section 6.3). Table 7.7 shows the effect of increasing the mean of each category A variable by 5% of its range in the way indicated in figure 7.4. Table 7.8 shows the effect of increasing the standard deviation of each category A variable by 30% in the way indicated in figure 7.5.

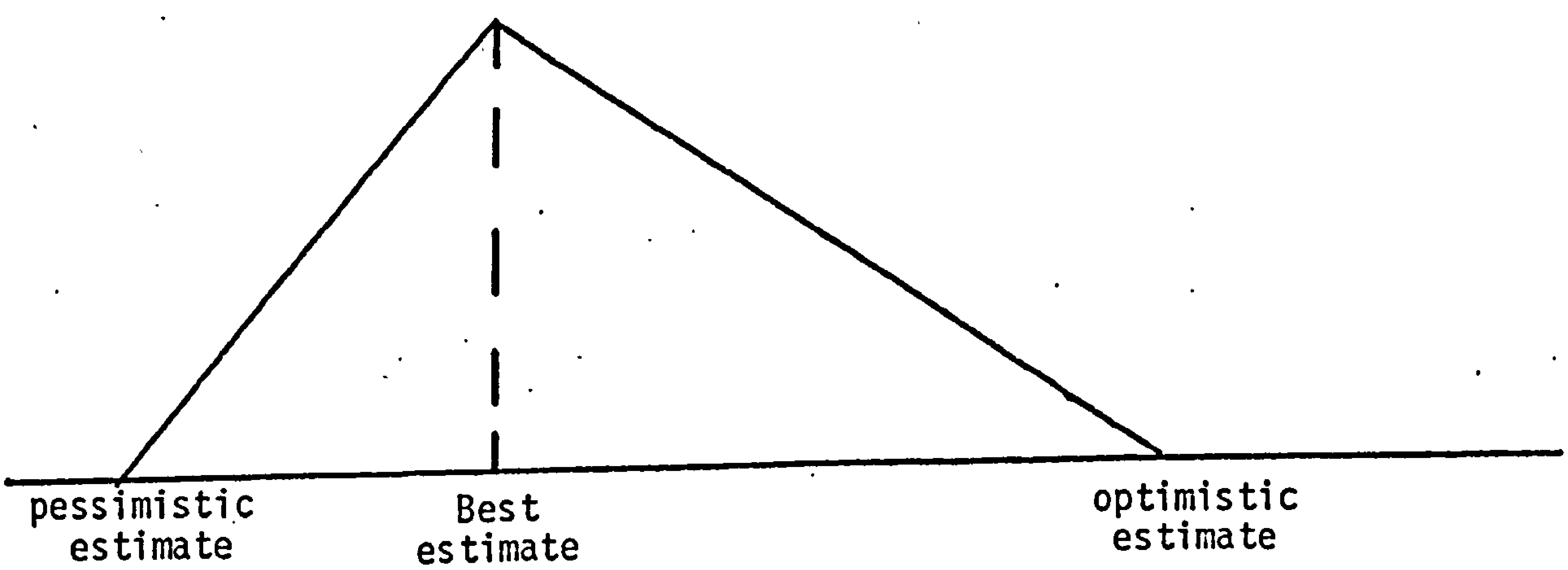

Figure 7.3 Triangular Distribution Initially Assumed for Each Category A Variable

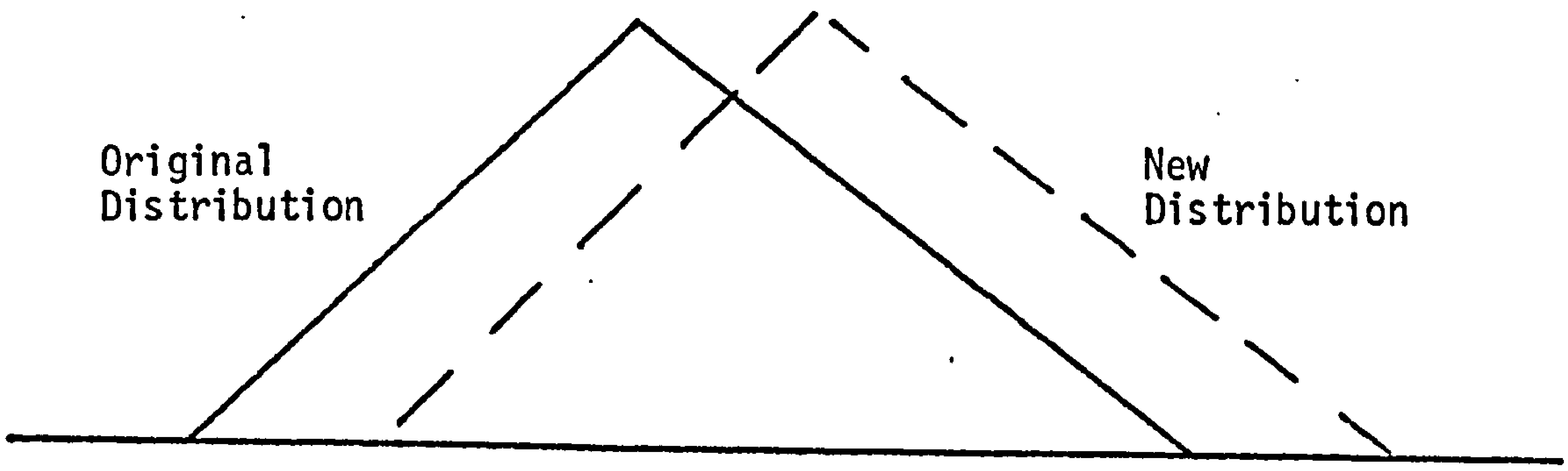

Figure 7.4 Mean of Variable is Increased by 5% of Variable's Range; S.D. Remains Unchanged

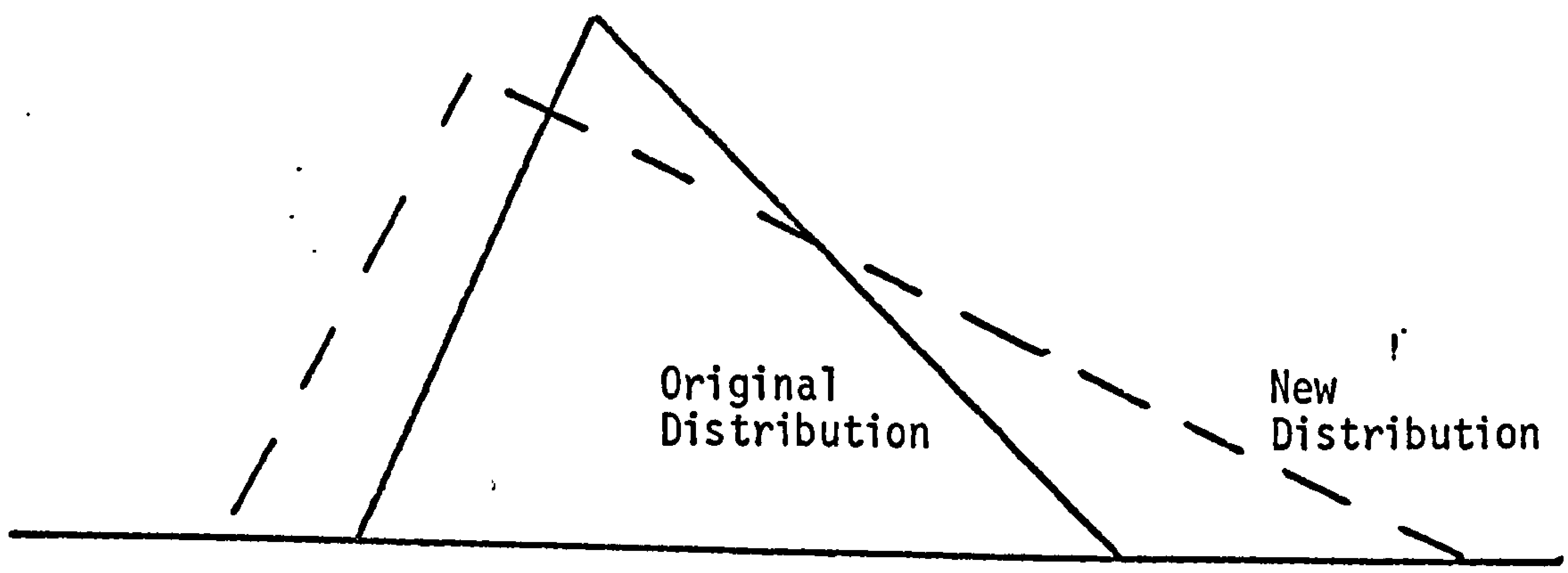

Figure 7.5 S.D. of Variable in Increased by 30%; Mean Remains Unchanged

Once figure 7.2 and tables 7.3, 7.4 and 7.5 had been produced management would be asked whether the investment was acceptable on the basis of the distribution in figure 7.2. Assuming that it is not acceptable, a further question of the form:

> By how much would the distribution in figure 7.2 have to improve before the investment became acceptable?

would be asked. The answer to this question might take a number of difference forms. We assume that, management indicate that for investments of this type they require:

Prob (NPV positive) > 0.9

As the distribution of NPV is, in this case, approximately normal, this criterion is equivalent to:

$$\mu_{NPV} - 1.28\ \sigma_{NPV} > 0$$

where μ_{NPV} and σ_{NPV} are the mean and standard deviation of NPV.

On the basis of independent triangular distributions:

$$\mu_{NPV} - 1.28\ \sigma_{NPV} = -\ 15850$$

(measured in $'000s).

It would then be necessary for the analyst to determine whether, if further assessments were made, there could be an increased of 15850 in $\mu_{NPV} - 1.28\ \sigma_{NPV}$.Appendix J provides a discussion of the ways in which the analyst might approach this problem and shows how, if certain assumptions are made, table 7.9 can be obtained from tables 7.6, 7.7 and 7.8.

Table 7.9 Potential Effects of Different Factors on the Distribution of NPV which is Produced Assuming Triangular Distributions for the Variables

Factor	Maximum positive effect on μ_{NPV} -1.28 σ_{NPV}	Maximum negative effect on μ_{NPV} -1.28 σ_{NPV}
Errors in Selling Price Distn.	6400	4900
Errors in Op. Costs Distn.	5300	4100
Errors in Mkt. Share Distn.	1200	800
Errors in Init. Mkt. Distn.	800	600
Errors in Life of Invt. Distn.	0	0
Init. Mkt. Size Dependent on Mkt. Share	1200	0
Operating Costs Dependent on Selling Price.	9700	0

Table 7.9 shows that an increase of 15850 in μ_{NPV} -1.28 σ_{NPV} is not altogether out of the question and that the most important factors are:

(i) the selling price distribution

(ii) the operating costs distribution

(iii) the dependence between selling price and operating costs.

Further assessments would then be made for each of these factors. We assume that the eight assessments indicated in table 6.6 are made for selling price and operating costs (accuracy parameter = 0.01) and that the two assessments:

If operating costs = 422.5, the median value of price is 500

If operating costs = 492.5, the median value of price is 523.5

are made to provide information (in the way described in chapter 6) about the dependence in (iii).

A single further risk simulation would then be carried out incorporating these assessments. It would with 1000 simulation runs give the results:

$\mu_{NPV} = 640$ (Standard error = 240)

$\sigma_{NPV} = 7540$ (Standard error = 170)

$\mu_{NPV} = -1.28\ \sigma_{NPV} = -9010$

It would then be necessary for the analyst to determine whether, if further assessments were made, there could be an increase of 9010 in μ_{NPV} $-1.28\ \sigma_{NPV}$. Appendix J shows how, if certain assumptions are made table 7.10 can be produced from tables 7.6, 7.7 and 7.8.

Table 7.10 Potential effects of different factors on the distributions of NPV which is produced after further assessments have been made for Selling Price Distribution, Op. Costs Distribution and the Dependency between Selling Price and Op. Costs.

Factor	Maximum positive effect on μ_{NPV} $-1.28\ \sigma_{NPV}$	Maximum negative effect on μ_{NPV} $-1.28\ \sigma_{NPV}$
Errors in Selling Price Distn.	1900	1900
Errors in Op. Costs Distn.	1600	1600
Errors in Mkt. Share Distn.	1200	800
Errors in Init. Mkt. Distn.	800	600
Errors in life of Invt. Distn.	0	0
Init. Mkt. Size Dependent on Mkt. Share	1200	0
Op. Costs Dependent on Selling Price	1500	1500

On the basis of table 7.10 it is inconceivable that further assessments could lead to an increase of 9010 in:

$$\mu_{NPV} -1.28\ \sigma_{NPV}$$

The investment would therefore be rejected without further analysis.

7.9 DISCUSSION OF PROPOSED APPROACH

The analytic procedures used in section 7.8 are it should be emphasised merely intended to illustrate a general approach to risk simulation. It is not suggested that the same procedures would be applicable in all situations. For example, if the distribution of NPV in case A had not been approximately normal or if management's response to the initial distribution of the performance measure had indicated that a more complicated decision criterion than:

$$\text{Prob. (NPV positive)} > 0.9$$

were applicable, it would have been necessary to use tables 7.7 and 7.8 in a different way to arrive at tables 7.9 and 7.10.

The main assumptions upon which the proposed approach is based can be listed as follows:

(i) all variables and all dependencies are type I.

(ii) μ_p depends linearly on μ_i, ρ_{ij} but not on σ_i for all i and j; σ_p depends linearly on μ_i, σ_i and ρ_{ij} for all i and j. (The notation used here is the same as that in section 7.2).

and (iii) managerial attitudes to risk can, for the set of probability distributions of the performance measure which are considered during the analysis be expressed entirely in terms of μ_p and σ_p.

Evidence which suggests that the first assumption is, in many situations, not unreasonable was presented in chapter 4. Some empirical evidence in support of assumption (ii) is contained in appendix I. (In view of the formulae given in chapter 4 for the means and standard deviations of sums and products of variables a more reasonable assumption that (ii) might be:

μ_p depends linearly on μ_i for all i

μ_p does not depend on σ_i for all i

σ_p^2 depends linearly on μ_i^2 for all i

σ_p^2 depends linearly on σ_i^2 for all i

μ_p depends linearly on ρ_{ij} for all i and j

σ_p^2 depends linearly on ρ_{ij} for all i and j

However, for most of the errors which are considered it can easily be shown that there is very little difference between this assumption and the one in (ii) above). Whether the third assumption is reasonable depends to a large extent on whether the general shape of the distribution of the performance measure remains the same for the range of values of μ_i, σ_i and ρ_{ij} which are considered during the analysis and some indication as to whether this is so will in general be provided as the analysis proceeds.

The proposed procedure is based on the assumption that it is worth using extra computing time in order to reduce the number of assessments which management are required to make. In this connection it is interesting to note that only approximately 200 seconds of computer time were required to carry out all the 13 initial simulations for case A. (Although 1000 simulation runs were used in the first of the simulations to produce figure 7.2, it was found that 500 runs were sufficient in the case of the other 12 simulations which provided tables 7.6, 7.7 and 7.8. This is because, in those 12 simulations, only changes to the mean and standard deviation of the performance measure were of interest and standard variance reduction techniques could be used).

One disadvantage of the proposed approach is that there is no wholly satisfactory way of interpreting tables such as table 7.9 and 7.10. One 'safe' procedure (and the procedure which is used in section 7.8) is to assume that the maximum effect of all the factors is the sum of the individual maximum effects. It is of course possible that, at some stage during the analysis, the conclusion could be reached that, no matter how many assessments are made, it is impossible to reach a conclusion on the investment with the accuracy parameter being used by management.

One aspect of the approach which has been proposed should be particularly noted. If, as in section 7.8, the criterion which management specify is:

$$\text{Prob. (NPV positive)} \geqslant K \tag{7.22}$$

for some value K, then calculating directly the effect of different factors on:

Prob. (NPV positive)

is liable to be misleading. This is because Prob. (NPV positive) is not linearly dependent on μ_{NPV} and σ_{NPV} and cannot, therefore, be expected to be linearly dependent on the μ_is, σ_is and ρ_{ij}s. (In fact if $\mu_{NPV} < 0$ then Prob. (NPV positive) increases as σ_{NPV} increases whereas if $\mu_{NPV} > 0$ it decreases as σ_{NPV} increases). The criterion in equation 7.22 should always be replaced by one involving μ_{NPV} and σ_{NPV}. (If this had not been done in the previous example a dependence between selling price and operating costs would have appeared harmful!)

One improvement to the procedures in section 7.8 might well be to use a distribution which is different from the triangular distribution in figure 7.3 to represent the variables initially. The triangular distribution has the advantage that it is easy - and inexpensive in terms of computer time - to use. Another distribution (e.g. a member of the beta family) might have the advantage that its mean and its standard deviation do in many situations correspond more closely with those of the 'true' distribution. Possibly a sensible idea in view of the results in chapter 4 is to choose a triangular distribution with a mean and standard deviation given by the PERT formulae.

Finally, one advantage of the proposed approach which has not as yet been mentioned is the fact that the analyst, when he does ask management to make a probability assessment, is able to explain the importance of that probability assessment. To quote Spetzler and Stael von Holstein (1975):

> 'Choose only uncertain quantities that are important to the decision, as determined by a sensitivity analysis. Be prepared to explain to the subject why the quantity is important to the decision. This demonstrates the relevance of the encoding process and is essential in gaining the subject's full co-operation'.

7.10 SUMMARY AND CONCLUSIONS

This chapter started by considering the interpretation which can be put on the output from a sensitivity analysis. It reached the following conclusions:

(i) In the situation where all variables are either inflows or outflows of money the sensitivity analysis does provide a good indication of the relative importance of errors in the means and standard deviations of, and the coefficients of correlation between, the variables. If errors in the mean are measured as a percentage of the range and errors in the standard deviation are measured as a percentage of the standard deviation, then (a) a comparison of the magnitudes of the range coefficients provides a good indication of the relative importance of errors in the means of different variables, (b) a comparison of the squares of the range coefficients provides a good indication of the relative importance of errors in the standard deviations of different variables and (c) a comparison of products of pairs of range coefficients provides a good indication of the relative importance of different dependencies.

(ii) The results in (i) cannot easily be extended to deal with situations where the cash flow model is non-linear.

(iii) In case studies A, B, C, D and E a sensitivity analysis would be useful as a rough first indication of the relative importance of different variables. It is suggested that as 'a rule of thumb' a variables with a range coefficient less than 0.1 should be replaced by a 'point estimate' in risk simulations and that the ' point estimate' chosen should be an estimate of the mean of the variable rather than its most likely value.

(iv) It is possible - even in relatively simple cash flow models - for the results from a sensitivity analysis to be totally misleading and the analyst should be aware of potential pitfalls.

The chapter then proceeded to suggest a multistage approach to carrying out risk simulations which is such that assessments are only made by management at any given stage if they have been indicated as being necessary at a previous stage. Although the approach uses slightly more computer time than single stage approaches it has the advantage that it avoids alienating managers by asking them to make a large number of unnecessary assessments. Case Study A was used to illustrate the approach. It was shown that, once highest, lowest and most likely values had been assessed for each variable it was possible to reach a conclusion on the acceptability of the investment with only 10 further probability assessments being made by management.

CHAPTER 8

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

8.1 INTRODUCTION

There are four parts to the research described in this thesis:

Part 1: An analysis of the nature of the distributions which are input to and output from risk evaluation models.

Part 2: A comparison of different methods of assessing subjective probability distributions in terms of the accuracy with which they are capable of providing the inputs to risk evaluation models.

Part 3: An analysis of the problems created by dependencies between the variables in a risk evaluation model and of how the problems can be dealt with.

Part 4: An investigation to determine how the total number of probability assessments which management we asked to make for a risk evaluation model can be kept as low as possible.

This chapter summarises the main conclusions which have been reached in each part and suggests areas where further research could fruitfully be carried out.

8.2 CONCLUSIONS FROM FIRST PART OF THE RESEARCH

In the first part of this research (see Chapter 4) five case studies were analysed in depth with a view to determining:

(i) the extent to which the distributions of NPV and IRR output from a risk evaluation model are approximately normal.

and (ii) the extent to which it can be said that the only important parameters of the distributions of the variables which are input to a risk evaluation model are their means and standard deviations.

(This part of the research is motivated by the work of Hillier and Wagle. Hillier (1969) presents a number of arguments which suggest that the distributions of NPV and IRR are often in practice approximately normal; Wagle (1967) shows analytically that, for certain types of cash flow models, it is possible to determine the mean and standard deviation of NPV from the means and standard deviations of the input variables and coefficients of correlation between the input variables).

As far as (i) above is concerned it was found that the distribution of NPV could be termed 'approximately normal' in three out of the five case studies for a wide range of input distributions and that the distribution of IRR could be so termed in one of the five case studies. The following reasons why NPV and IRR might not be approximately normal in a given situation were identified:

(a) the investor might have options (e.g. abandonment or expansion options) open to him at stages during the project's life.

(b) the distribution of NPV might be heavily influenced by non-linearities in the cash flow model. (Non-linearities can be caused by the presence of variables such as 'growth rate' and 'life of project' or by conditions within the model itself e.g. the condition: supply the export market only if there is sufficient capacity after the home market has been satisfied).

and (c) there might be an inadequate number of uncertain variables (or the uncertainty in one variable might dominate all other considerations).

In addition, it should be noted that IRR will not be approximately normal if there is a significant probability that all cash flows have the same sign.

On the basis of the results it seems likely that, if a cash flow model is sufficiently 'well-behaved' for the Hillier-Wagle analytic approach to be applied, then the distribution of NPV will be approximately normal providing that there are a sufficient number of uncertain variables in the model.

As far as (ii) above is concerned, the results which have been produced suggest that, in four out of five of the case studies, for a wide of shapes of input distributions, it is only the means and standard deviations of those distributions which are of importance. There are of course a number of reasons why parameters other than the mean and standard deviation might, for a particular variable X, be important in a given situation. For example:

(a) the decision on whether to abandon the investment might depend on whether X is less than some pre-determined level.

(b) the uncertainty in the performance measure might depend almost entirely on the uncertainty in X.

and (c) there might be very few uncertain variables apart from X in the cash flow model.

Nevertheless, the results which have been obtained do suggest that in a wide variety of situations the only important parameters of the distributions of variables are their means and standard deviations. It is particularly notable that, in one of the case studies, the results indicated that this was so even though the distributions of both NPV and IRR were definitely non-normal.

8.3 CONCLUSIONS FROM SECOND PART OF THE RESEARCH

The subjective probability distributions which are used in risk evaluation studies are, at best, only approximate representations of managerial judgement because they are based on a small number of individual assessments. The second part of the research in this thesis (see Chapter 5) explored this aspect of risk evaluation in some detail. It first postulated a number of different 'true' distributions. (A 'true' distribution is defined in a given situation as the distribution which the assessor would produce if he were capable of making an infinitely large number of infinitely accurate assessments). It then investigated the accuracy with which the distributions would be assessed using a number of different assessment procedures. It should be emphasised that the conclusions in this part of the research are based on the assumption that the 'true' distribution is the same for all probability assessment procedures. In view of the experimental evidence which is available on the assessment of subjective probability distributions (see section 2.3) this assumption is questionable and further research aimed at developing a detailed model of the process by which probability assessments are made could be usefully carried. This model would incorporate the biases which are present when different procedures are used.

In view of the results from the first part of the research, the accuracy of an assessed distribution was measured in terms of the accuracy of the estimates which would be calculated for the distribution's mean and standard deviation. Two separate sets of analyses were carried out. The first assumed the assessor was capable of making a finite number of infinitely accurate probability assessments; the second assumed that the assessor made a finite number of assessments but that his ability to discriminate between different probabilities and different values of the variable was limited.

55 'true' distributions (all unimodal beta distributions) and six different assessment procedures were investigated. The results which were obtained when it was assumed that management were capable of making infinitely accurate assessments showed that, in the case of the distributions which were considered:

(i) For the same number of assessments, fixed interval methods provide better estimates of both the mean and the standard deviation than variable interval methods.

(ii) For the same number of assessments, procedure 5 provides better estimates of both the mean and the standard deviation than procedure 6. (See section 5.2 for a description of the assessment procedures).

(iii) The accuracy of an estimate for the mean of a distribution tends to depend more heavily on the distribution's skewness than the accuracy of an estimate for its standard deviation.

The assumption that the assessor could only make assessments to the nearest 0.1 or the nearest 0.2 significantly reduced the accuracy of the estimates made in a number of cases, but the general conclusions in (i), (ii), and (iii) above still held.

If the assumption is made that the 55 'true' distributions which were investigated are representative of a far wider class of distributions, the results can be used to provide rough estimates of the maximum errors in assessed means and standard deviations for all distributions in the class.

8.4 CONCLUSIONS FROM THIRD PART OF THE RESEARCH

Methods for dealing with dependencies between the variables in a risk evaluation study have received relatively little attention in the literature. After demonstrating that dependencies between variables can in certain situations be extremely important, the third part of the research in this thesis (see Chapter 6) reviewed those methods which have been suggested in the literature for assessing dependence. It found that all of the methods suffer from one or more of the following disadvantages:

(i) management are required to make an unreasonably large number of individual probability assessments.

(ii) management make no quantitative judgements at all as to the extent of the dependence.

(iii) the conditional distributions used for sampling cannot in general be made consistent with pre-determined unconditional distributions.

(iv) a coefficient of correlation rather than a complete sampling scheme is produced. (This is only a disadvantage in simulation approaches to risk evaluation; it could be a positive advantage in analytic approaches).

It was pointed out one way of judging the importance of a particular dependence in risk simulation is to carry out two separate simulations, the first assuming no dependence and the second assuming 'total dependence'. The following definition of 'total dependence' was suggested:

> X and Y are totally positively dependent if when X takes a value equal to its k-th fractile Y also takes a value equal to its k-th fractile. X and Y are totally negatively dependent if when X takes a value equal to its k-th fractile. Y takes a value equal to its (1-k) -th fractile.

(This is an improvement over defining 'total dependence' to mean the situation where the coefficient of correlation equals +1 or -1 as it allows distributions which have been assessed as different in shape to be totally dependent on each other).

A new method for assessing partial dependencies which suffers from none of the disadvantages in (i) - (iv) above was suggested and demonstrated using data in Eilon and Fowkes (1973). The method involves first asking management to assess independent distributions for all the variables and then asking management to make further assessments of the form:

if X = Q, my median estimate for Y is P

where X is the independent variable and Y is the dependent variable. All of the assessments are then fitted to a model which is obtained by applying monotonic transformations to the variables in a bivariate normal distribution. The method can, it is worth noting, be extended, using properties of the multivariate normal distribution, to deal with situations where the value of a variable is dependent on the values of two or more other variables.

What might be termed 'the growth rate problem' was also discussed in the course of the third part of the research. When a variable is expected to have a different value in each year of the life of a project there are a number of alternatives open to the analyst:

(a) assume an 'initial value + constant growth rate' model and obtain probability distributions for the initial value and the growth rate.

(b) identify growth parameters which are different from those in (a) and obtain probability distributions for them.

(c) assume that the variable has different growth rates at different stages during the project's life.

(d) use a family of growth curves.

(e) use a probabilistic growth model.

As far as (e) is concerned it was pointed out that the model which is proposed for the assessment of the dependence between two variables can also be formulated as a model of probabilistic growth.

8.5 CONCLUSIONS FROM FOURTH PART OF THE RESEARCH

It is unreasonable in risk evaluation to expect management to provide detailed probabilistic descriptions of every single variable and every single dependence. The analyst must make some attempt to distinguish important probability assessments from unimportant ones so that management attention can be directed towards the former. The fourth part of the research in this thesis (see Chapter 7) started by considering the usefulness of the output from a sensitivity analysis. It reached the following conclusions:

(i) In the situation where all variables are either inflows or outflows of money the sensitivity analysis does provide a good indication of the relative importance of errors in the means and standard deviations of, and the coefficients of correlation between the variables. If errors in the mean are measured as a percentage of the range and errors in the standard deviation are measured as a percentage of the standard deviation, then (a) a comparison of the magnitudes of the range coefficients provides a good indication of the relative importance of errors in the means of different variables. (b) a comparison of the squares of the range coefficients provides a good indication of the relative importance of errors in the standard deviations of different variables and (c) a comparison of products of pairs of range coefficients provides a good indication of the relative importance of different dependencies. (See section 7.5 for a definition of the term ' range coefficient).

(ii) The results in (i) cannot easily be extended to deal with situations where the cash flow model is non-linear.

(iii) A sensitivity analysis would, in the five case studies which have been considered, be useful as a rough first indication of the relative importance of different variables. It is suggested that as 'a rule of thumb' a variable with a range coefficient less than 0.1 should be replaced by a 'point estimate' in risk simulations and that the 'point estimate' chosen should be an estimate of the mean of the variable rather than its most likely value.

(iv) It is possible - even in relatively simple cash flow models - for the results from a sensitivity analysis to be totally misleading and the analyst should be aware of potential pitfalls.

The fourth part of the research then proceeded to suggest a multistage approach to carrying out risk simulations which is such that assessments are only made by management at any given stage if they have been indicated as being necessary at a previous stage. During the first stage distributions are hypothesised for each variable on the basis of highest, lowest and most likely values, possible dependencies are identified and simulations are carried out in order to:

(a) provide a 'crude estimate' of the probability distribution of the performance measure based on the hypothesised distributions.

(b) provide measures of the maximum potential effect on the distribution of the performance measure of each of the dependencies identified by management.

(c) provide, for each variable assigned to category A, measures of the maximum potential error in the distribution of the performance measure resulting from the 'true' distribution of the variable being different from that hypothesised.

The distribution produced in (a) is then examined and a conclusion is reached as to whether the investment is acceptable on the basis of that distribution. If the investment is considered acceptable, an analysis is carried out to determine the extent to which the distribution of the performance measure would have to change before the investment became unacceptable. If the investment is considered unacceptable, an analysis is carried out to determine the extent to which the distribution of the performance measure would have to change before the investment became acceptable. In either case, the results produced in (b) and (c) above are then used to calculate, first, whether any further probability assessments are necessary and, second, if they are, which ones should be made. In the event of no further probability assessments being necessary, the analysis is finished. If further probability assessments are required, they are made, a further risk simulation is carried out and the results are reviewed again to determine whether further probability assessments are necessary. (This approach is illustrated graphically on page 127).

Although the proposed approach uses slightly more computer time than single stage approaches, it has the advantage that it avoids alienating management by asking them to make unnecessary assessments. The Hertz model was used to illustrate the approach. It was shown that, once highest, lowest and most likely values had been assessed for each variable it was possible to reach a conclusion on the acceptability of the investment with only 10 further probability assessments being made by management.

8.6 SUGGESTIONS FOR FURTHER RESEARCH

It is appropriate to conclude this thesis with a few suggestions as to areas where further research might usefully be carried out.

A great deal of further research is clearly necessary to deal with what has been termed 'the growth rate problem' (i.e. the problem which is present when the value of a variable in one time period is expected to depend on its value in one or more previous time periods. Section 6.9 of this thesis provides a number of interesting ideas as far as this is concerned, but further research is now necessary in order to develop satisfactory assessment procedures and satisfactory sampling schemes. It would be particularly interesting to identify a set of parameters describing the growth of a variable over time and then to determine which of them have most effect on the distributions which are output from risk evaluation studies.

Some consideration should also be given to simpler ways of approaching the whole risk evaluation problem. It is possible that a heuristic procedure based on the output from a sensitivity analysis could be developed to provide a reasonable 'risk index' for a project. The procedure would probably be fairly complicated in that it would require variables to be categorised in some way (e.g variables representing fixed costs would almost certainly have to be considered differently from those representing variable costs within the heuristic), but it might be quite attractive to finance specialists within industry.

Finally, as computer time becomes cheaper, it seems likely that more approaches to risk evaluation along the lines of the one suggested in Chapter 7 will be developed. Indeed it is not totally unreasonable to suggest that, in the forseeable future, a computer program will be developed which will output directly messages of the form:

> 'The most crucial probability assessments necessary for a decision on this investment are........'

once management have supplied certain basic information.

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APPENDIX A

THE CASH FLOW MODELS ASSUMED FOR THE CASE STUDIES

Figures A.1 to A.5 show the FORTRAN subroutines which were used to calculate the net cash flows from the values of the input variables in the five case studies examined in this thesis. The following notation is used in all subroutines:

NYEAR	:	life of project
CINIT	:	net cash flow in year 0
C(I)	:	net cash flow in year I
V(I)	:	value of the I-th input variable.

(The input variables for each case study are listed in section 3.4. Thus, in case A,V(1) is 'initial market size', V(2) is 'market growth rate' etc).

Other notation used is as follows:

Case A:	MKT(I)	:	Market size in year I
	W	:	Proportion of the final year of its life for which the project lasts.
Case B:	PRICE(I)	:	Price in year I
	VC(I)	:	Total variable costs in year I
	PROD(I)	:	Production in year I
	FC(I)	:	Fixed cost in year I
	INFLOW(I)	:	Inflow in year I
	W	:	Proportion of the final year of its life for which project lasts.
	OUTFLOW(I)	:	Outflow on capital equipment in year I
	INITOUTFLOW	:	Initial expenditure on capital equipment
	BALOUTST(I)	:	Balance of capital outstanding in year I assuming depreciation at 20% p.a.
	TAXALLOW(I)	:	Tax allowance in year I
	INITTAXALLOW	:	Initial tax allowance
Case C:	MKTSIZE(I)	:	Total size of market in year I
	SALES(I)	:	Co. sales in year I
Case D:	SALES 1	:	Sales in market 1 per year
	SALES 2	:	Sales in market 2 per year
	NSY	:	Start year
	NFY	:	Year in which project ends
	PRETAXINFLOW	:	Pre-tax inflow per year

	GRANT(I)	:	Grant in year I
	TAXALLOW(I)	:	Tax allowance in year I
	CAPEXP(I)	:	Capital expenditure in year I
	PTI(I)	:	Pre-tax inflow in year I
	ANNCAPEXP	:	Capital expenditure per year initially
	BALOUTST(I)	:	Balance of capital not allowed against tax in year I.
Case E:	NYHA	:	No. of year of hostile act
	NYFGC	:	No. of year of introduction of fourth generation computers.
	SHARE(I)	:	Market share (external market) in year I
	INITMKTINT	:	Initial internal market
	INITMKTEXT	:	Initial external market
	MKTINT(I)	:	Internal market in year I
	MKTEXT(I)	:	External market in year I
	TOTMKT(I)	:	Total market in year I
	INITTOTMKT	:	Initial total market
	INITCAPEXP	:	Initial capital expenditure
	CAPEXP(I)	:	Capital expenditure in year I
	TGE(I)	:	Stock of third generation equipment in year I.
	FGE(I)	:	Stock of fourth generation equipment in year I.
	BALOUTST(I)	:	Balance of capital not allowed against tax in year I.
	TAXALLOW(I)	:	Tax allowance in year I

Figure A.1: Cash Flow Model used for Case A

```
      SUBROUTINE MODEL(V)
      REAL MKT(50)
      DIMENSION V(20),C(50)
      COMMON /CF/CINIT,C,NYEAR
      NYEAR=V(6)
      CINIT=-V(5)
      MKT(1)=V(1)
      C(1)=V(1)*V(4)*(V(3)-V(8))/100.0-V(9)
      DO 1 IYEAR=2,NYEAR
      MKT(IYEAR)=MKT(IYEAR-1)*(1+V(2)/100.0)
      C(IYEAR)=MKT(IYEAR)*V(4)*(V(3)-V(8))/100.0-V(9)
1     CONTINUE
      W=V(6)-NYEAR
      IF(W.GT.0.001) GO TO 2
      C(NYEAR)=C(NYEAR)+V(7)
      RETURN
2     NYEAR=NYEAR+1
      MKT(NYEAR)=MKT(NYEAR-1)*(1+V(2)/100.0)
      C(NYEAR)=V(7)+W*(MKT(NYEAR)*V(4)*(V(3)-V(8))/100.0-V(9))
      RETURN
      END
```

Figure A.2: Cash Flow Model used for Case B

```
      SUBROUTINE MODEL(V)
      DIMENSION INFLOW(50),OUTFLOW(50),TAXALLOW(50),BALOUTST(50)
      DIMENSION V(20),C(50),PRICE(50),VC(50),PROD(3),FC(3)
      REAL INITOUTFLOW,OUTFLOW,INFLOW,INITTAXALLOW
      COMMON /CF/CINIT,C,NYEAR
      DO 5 IYEAR=1,50
 5    C(IYEAR)=0
      PRICE(1)=V(1)
      PRICE(2)=PRICE(1)*(1+V(2)/100.0)
      PRICE(3)=PRICE(2)*(1+V(2)/100.0)
      VC(1)=V(3)+V(4)+V(5)+V(6)
      VC(2)=VC(1)*(1+V(8)/100.0)
      VC(3)=VC(2)*(1+V(8)/100.0)
      PROD(1)=V(10)
      PROD(2)=2.1538*V(10)
      PROD(3)=2.8462*V(10)
      FC(1)=V(7)
      FC(2)=2.0*V(7)
      FC(3)=2.4*V(7)
      DO 1 IYEAR=1,3
 1    INFLOW(IYEAR)=(PRICE(IYEAR)-VC(IYEAR))*PROD(IYEAR)-FC(IYEAR)
      NYEAR=V(11)
      DO 2 IYEAR=4,NYEAR
      PRICE(IYEAR)=PRICE(IYEAR-1)*(1+V(2)/100.0)
      VC(IYEAR)=VC(IYEAR-1)*(1+V(8)/100.0)
 2    INFLOW(IYEAR)=(PRICE(IYEAR)-VC(IYEAR))*PROD(3)-FC(3)
      W=V(11)-NYEAR
      IF(W.GT.0.001) GO TO 20
      INFLOW(NYEAR)=INFLOW(NYEAR)+2.0707*V(12)
      GO TO 30
 20   NYEAR=NYEAR+1
      PRICE(NYEAR)=PRICE(NYEAR-1)*(1+V(2)/100.0)
      VC(NYEAR)=VC(NYEAR-1)*(1+V(8)/100.0)
      INFLOW(NYEAR)=((PRICE(NYEAR)-VC(NYEAR))*PROD(3)-FC(3))*W
     1+2.0707*V(12)
 30   CONTINUE
      INITOUTFLOW=V(9)+V(12)
      OUTFLOW(1)=2.5087*V(9)+1.1522*V(12)
      OUTFLOW(2)=V(9)+.6384*V(12)
      OUTFLOW(3)=0.2301*V(12)
      INITTAXALLOW=0.2*V(9)
      BALOUTST(1)=0.8*V(9)
      TAXALLOW(1)=0.2*BALOUTST(1)+0.2*2.5087*V(9)
      BALOUTST(2)=0.8*BALOUTST(1)+0.8*2.5087*V(9)
      TAXALLOW(2)=0.2*BALOUTST(2)+0.2*V(9)
      BALOUTST(3)=0.8*BALOUTST(2)+0.8*V(9)
      TAXALLOW(3)=0.2*BALOUTST(3)
      DO 3 IYEAR=4,NYEAR-1
      OUTFLOW(IYEAR)=0
      BALOUTST(IYEAR)=0.8*BALOUTST(IYEAR-1)
 3    TAXALLOW(IYEAR)=0.2*BALOUTST(IYEAR)
      TAXALLOW(NYEAR)=BALOUTST(NYEAR-1)*0.8
      OUTFLOW(NYEAR)=0
      CINIT=-INITOUTFLOW+INITTAXALLOW*0.545
      DO 4 IYEAR=1,NYEAR
 4    C(IYEAR)=INFLOW(IYEAR)*0.455-OUTFLOW(IYEAR)+TAXALLOW(IYEAR)*0.545
      RETURN
      END
```

Figure A.3: Cash Flow Model used for Case C

```
      SUBROUTINE MODEL(V)
      DIMENSION V(20),C(50)
      DIMENSION MKTSIZE(5),SALES(5)
      REAL MKTSIZE
      COMMON /CF/CINIT,C,NYEAR
      NYEAR=5
      CINIT=0
      MKTSIZE(1)=V(1)
      SALES(1)=MKTSIZE(1)*(1+V(4)/100)*V(3)/100
      C(1)=SALES(1)*(1-V(5)/100.0-V(6)/100.0)-V(7)-100.0
      IF (C(1).LE.0) GO TO 2
      DO 1 IYEAR=2,5
      MKTSIZE(IYEAR)=MKTSIZE(IYEAR-1)*(1+V(2)/100.0)
      SALES(IYEAR)=MKTSIZE(IYEAR)*(1+V(4)/100.0)*V(3)/100.0
    1 C(IYEAR)=SALES(IYEAR)*(1-V(5)/100.0-V(6)/100.0)-V(7)-100.0
      RETURN
    2 DO 3 IYEAR=2,5
    3 C(IYEAR)=0
      RETURN
      END
```

Figure A.4: Cash Flow Model used for Case D

```
      SUBROUTINE MODEL(V)
      DIMENSION V(20),C(50)
      DIMENSION PTI(50),CAPEXP(50),GRANT(50),BALOUTST(50),TAXALLOW(50)
      COMMON /CF/CINIT,C,NYEAR
      IF(V(4).LT.V(17)) GO TO 1
      SALES1=V(17)
      SALES2=0
      GO TO 2
1     SALES1=V(4)
      IF(V(8).LT.V(17)-V(4)) GO TO 3
      SALES2=V(17)-V(4)
      GO TO 2
3     SALES2=V(8)
2     CONTINUE
      NSY=V(10)
      NFY=V(10)+V(11)+1
      PRETAXINFLOW=SALES1*(V(3)-V(2))-V(1)+SALES2*(V(7)-V(6))-V(5)
     1-(SALES1+SALES2)+V(15)-V(14)
      NYEAR=NFY+1
      DO 100 IYEAR=1,NYEAR
      GRANT(IYEAR)=0
      TAXALLOW(IYEAR)=0
      CAPEXP(IYEAR)=0
100   PTI(IYEAR)=0
      DO 4 IYEAR=NSY,NFY
4     PTI(IYEAR)=PRETAXINFLOW
      PTI(NSY)=(NSY+1-V(10))*PTI(NSY)-V(12)
      PTI(NFY)=(V(11)+V(10)-NFY+1)*PTI(NFY)+V(13)*V(9)/100.0
      ANNCAPEXP=V(9)/V(10)
      CINIT=-ANNCAPEXP
      IYEAR=1
12    IF(NSY.GT.IYEAR) GO TO 10
      CAPEXP(IYEAR)=ANNCAPEXP*(V(10)-NSY)
      GO TO 11
10    CAPEXP(IYEAR)=ANNCAPEXP
      IYEAR=IYEAR+1
      GO TO 12
11    CONTINUE
      GRANT(1)=0.45*ANNCAPEXP
      DO 20 IYEAR=2,NSY+1
20    GRANT(IYEAR)=0.45*CAPEXP(IYEAR-1)
      BALOUTST(1)=ANNCAPEXP
      DO 30 IYEAR=1,NFY
      TAXALLOW(IYEAR)=(BALOUTST(IYEAR)-GRANT(IYEAR))*0.2
      IY=IYEAR+1
30    BALOUTST(IY)=BALOUTST(IYEAR)-GRANT(IYEAR)-TAXALLOW(IYEAR)
     1+CAPEXP(IYEAR)
      IY=NFY+1
      TAXALLOW(IY)=BALOUTST(IY)-GRANT(IY)
      C(1)=PTI(1)+GRANT(1)-CAPEXP(1)+TAXALLOW(1)*0.45
      DO 40 IYEAR=2,NFY+1
40    C(IYEAR)=PTI(IYEAR)+GRANT(IYEAR)-CAPEXP(IYEAR)+TAXALLOW(IYEAR)*0.4
     15-0.45*PTI(IYEAR-1)
      C(NSY)=C(NSY)-V(16)
      C(NFY)=C(NFY)+V(16)
      RETURN
      END
```

Figure A.5: Cash Flow Model used for Case E

```
      SUBROUTINE MODEL(V)
      DIMENSION V(20),C(50)
      COMMON /CF/CINIT,C,NYEAR
      REAL INITMKTINT,INITMKTEXT,INITTOTMKT,INITCAPEXP,INITEQUIP
      DIMENSION SHARE(6),MKTINT(6),MKTEXT(6),TOTMKT(6),CAPEXP(6),TGE(6),
     1FGE(6),BALOUTST(7),TAXALLOW(6)
      REAL MKTINT,MKTEXT
      NYEAR=6
      INITMKTINT=20000
      INITMKTEXT=280000
      SHARE(1)=0.05
      SHARE(2)=0.05
      SHARE(3)=0.04
      SHARE(4)=0.04
      SHARE(5)=0.03
      SHARE(6)=0.02
      NYHA=V(3)
      NYFGC=V(4)
      MKTINT(1)=INITMKTINT*(1+V(1)/100.0)
      MKTEXT(1)=INITMKTEXT*(1+V(2)/100.0)*1.3
      IF(NYHA.EQ.1) GO TO 1
      DO 2 IYEAR=2,NYHA
      MKTEXT(IYEAR)=MKTEXT(IYEAR-1)*(1+V(2)/100.0)*1.3
      MKTINT(IYEAR)=MKTINT(IYEAR-1)*(1+V(1)/100.0)
2     TOTMKT(IYEAR)=MKTINT(IYEAR)+MKTEXT(IYEAR)*SHARE(IYEAR)
1     N1=NYHA+1
      MKTEXT(N1)=MKTEXT(NYHA)*(1+V(2)/100.0)*(1.3**(V(3)-NYHA))
      MKTINT(N1)=MKTINT(NYHA)*(1+V(1)/100.0)
      TOTMKT(N1)=MKTINT(N1)+MKTEXT(N1)*SHARE(N1)
      DO 3 IYEAR=NYHA+2,NYEAR
      MKTEXT(IYEAR)=MKTEXT(IYEAR-1)*(1+V(2)/100.0)
      MKTINT(IYEAR)=MKTINT(IYEAR-1)*(1+V(1)/100.0)
3     TOTMKT(IYEAR)=MKTINT(IYEAR)+MKTEXT(IYEAR)*SHARE(IYEAR)
      TOTMKT(1)=MKTINT(1)+MKTEXT(1)*SHARE(1)
      INITTOTMKT=INITMKTINT
      INITCAPEXP=INITTOTMKT
      CAPEXP(1)=AMAX1(0,TOTMKT(1)-INITTOTMKT)
      INITEQUIP=INITCAPEXP
      TGE(1)=INITEQUIP+CAPEXP(1)
      FGE(1)=0
      DO 4 IYEAR=2,NYFGC
      CAPEXP(IYEAR)=AMAX1(0,TOTMKT(IYEAR)-TGE(IYEAR-1)-FGE(IYEAR-1))
      TGE(IYEAR)=TGE(IYEAR-1)+CAPEXP(IYEAR)
```

(Continued on next page)

```
4     FGE(IYEAR)=0
      IF(NYFGC.EQ.6) GO TO 9
      N2=NYFGC+1
      CAPEXP(N2)=AMAX1(0,TOTMKT(IYEAR)-TGE(IYEAR-1)-FGE(IYEAR-1))
      TGE(N2)=TGE(NYFGC)+CAPEXP(IYEAR)*(V(4)-NYFGC)
      FGE(N2)=CAPEXP(IYEAR)*(1-V(4)+NYFGC)
      IF(N2.EQ.6) GO TO 9
      DO 5 IYEAR=NYFGC+2,NYEAR
      CAPEXP(IYEAR)=AMAX1(0,TOTMKT(IYEAR)-TGE(IYEAR-1)-FGE(IYEAR-1))
      TGE(IYEAR)=TGE(IYEAR-1)
5     FGE(IYEAR)=FGE(IYEAR-1)+CAPEXP(IYEAR)
9     CONTINUE
      CINIT=-INITCAPEXP+INITTOTMKT*0.1
      BALOUTST(1)=INITCAPEXP
      DO 6 IYEAR=1,NYEAR
      TAXALLOW(IYEAR)=0.2*BALOUTST(IYEAR)
      IY=IYEAR+1
6     BALOUTST(IY)=BALOUTST(IYEAR)-TAXALLOW(IYEAR)+CAPEXP(IYEAR)
      TAXALLOW(NYEAR)=TAXALLOW(NYEAR)+BALOUTST(NYEAR+1)
      DO 7 IYEAR=1,NYFGC
7     C(IYEAR)=TOTMKT(IYEAR)*0.12+TAXALLOW(IYEAR)*0.5-CAPEXP(IYEAR)
      IF(NYFGC.EQ.6) GO TO 10
      DO 8 IYEAR=NYFGC+1,NYEAR
8     C(IYEAR)=0.12*AMIN1(TOTMKT(IYEAR),FGE(IYEAR))
     1+AMAX1((TOTMKT(IYEAR)-FGE(IYEAR))*0.06,0)
     2+TAXALLOW(IYEAR)*0.5-CAPEXP(IYEAR)
10    CONTINUE
      C(NYEAR)=C(NYEAR)+BALOUTST(NYEAR)
      RETURN
      END
```

APPENDIX B

DESCRIPTION OF THE COMPUTER PROGRAMS WHICH HAVE BEEN DEVELOPED

A complete listing of all computer programs developed in the course of this research would be too voluminous to include in this thesis. To give an indication as to how the programs were written, this appendix describes very briefly the main subroutines in each one.

RISKANAL 1

This program carries out a sensitivity analysis. The master segment reads data concerned with:

(i) title for the project

(ii) no. of different discount rates to be tested (up to five are allowable).

(iii) the discount rates (a negative discount rate indicates that IRR is the performance measure).

(iv) no. of different input variables (up to 20 allowed)

(v) variable names

(vi) highest, most likely and lowest values for each variable.

The subroutines are as follows:

PRESVAL: Calculates present value of a stream of cash flows.

IRR: Calculates internal rate of return from a stream of cash flows.

SENSITIVITY: Calculates sensitivity coefficients and range coefficients for each variable and outputs results.

SORT: Arranges variables in the order of decreasing range coefficients prior to output.

MODEL: Calculates net cash flows from the values of the input variables for the particular investment being considered (Examples of this subroutine are in appendix A).

RISKANAL 2

This program carries out a risk simulation. The master segment reads data concerned with:

(i) title for the project

(ii) number X (0 < X < 67101323) necessary to start the generation of a sequence of random numbers (see Downham and Roberts (1967)).

(iii) No. of input variables, no. of sets of distributions to be considered, no. of runs.

(iv) discount rate.

(v) variable names, distributions for the variables

(vi) dependencies between variables.

The main subroutines are as follows:

SIM:	Controls simulation by calling other subroutines which sample from distributions etc.
UNIFORM:	Samples from uniform distribution
TRIANGSAMPLE:	Samples from a triangular distribution
NORMAL:	Samples from normal distribution
HISTOSAMPLE:	Samples from histograms
QUADSAMPLE:	Samples from a piecewise quadratic distribution
PRESVAL:	Calculates present value of a stream of cash flows.
IRR:	Calculates internal rate of return from a stream of cash flows.
RANDOM:	Samples random number R: $0 \leq R \leq 1$
MODEL:	Calculates net cash flows from values of input variables for particular investment being considered.
GRAPHPAR:	Calculates on the basis of a small number of simulations parameters for the graphical output (i.e. it calculates the first class interval, the width of each class interval, the no. of class intervals etc).
GRAPHPLOT:	Plots graphical output.
CALCOL:	Calculates values of the performance measure on one simulation run and assigns it to appropriate class interval for output.

(A number of different versions of RISKANAL 2 were used to provide the results in this thesis).

PROBSIM

This program simulates the assessment of a number of different beta distributions using a number of different assessment procedures. The master segment reads data concerned with:

(i) Accuracy parameters to be tested

and (ii) No. of assessments to be assumed

The main subroutines are as follows:

CONTROL:	Carries out most of the calculations, calls other subroutines as necessary and outputs results.

MNSDSTEP:	Calculates the mean and standard deviation of a step distribution.
MNSDPIECEWISE:	Calculates the mean and standard deviation of a piecewise quadratic distribution.
BETACUMPROB:	Calculates, for a beta distribution, the cumulative probability corresponding to a value of the variable.
BETAINVCUM:	Calculates, for a beta distribution, the value of the variable corresponding to a cumulative probability.
BETAORD 1:	Calculates the ordinate of the beta distributions for a particular value of the variable.
BETAORD 2:	Calculates two values of the beta distribution with particular ordinates.
PIECEQUADFIT:	Fits a piecewise quadratic distribution to a set of cumulative probabilities and values.
DISCVALUE:	Determines how values assessed for a variable would be altered by the existence of an accuracy parameter.
DISCDIST:	Determines how probabilities assesssed would be altered by the existence of an accuracy parameter.

APPENDIX C

VALUES ASSUMED FOR THE VARIABLES IN THE CASE STUDIES

Tables C.1 - C.5 show the highest, most likely and lowest values which were assumed for the variables in the five case studies in order to produce the results in chapters 4 and 7.

Table C.1 Highest, most likely and lowest values for variables in case A

Variable	Highest Value	Most Likely Value	Lowest Value
Initial Market Size ('000s tons)	340	250	100
Market Growth (% p.a.)	6	3	0
Selling Price ($ per ton)	575	510	385
Market Share (%)	17	12	3
Initial Investment ($M)	10.5	9.5	7.0
Life of Investment (yrs)	15	10	5
Residual Value ($M)	5.0	4.5	3.5
Operating Costs ($ per ton)	545	435	370
Fixed Costs ($'000s p.a.)	375	300	250

Table C.2 Highest, most likely and lowest values for variables in case B

Variable	Highest Value	Most Likely Value	Lowest Value
Price of product ($ per unit)	140	128	115
Price Growth rate (% p.a.)	4.6	2.7	0
Variable Op. cost now ($ per unit)	55	51	48
Extra variable op. costs 1 ($ per unit)	10.0	6.25	3.75
Extra variable op. costs 2 ($ per unit)	37.5	25.0	22.5
Extra variable op. costs 3 ($ per unit)	2.5	0.38	0
Extra fixed costs ($'000s p.a.)	320	170	120
Cost growth rate (% p.a.)	8.3	2.6	-0.3
Capital Costs in year 0 ($'000s)	1165	965	565
Production in year 1 (units)	27000	15000	5000
Life of Project (yrs)	40	30	15
Extra Working Capital in year 0 ($'000s)	173	123	43

Table C.3 Highest, most likely and lowest values for variables in case C.

Variable	Highest Value	Most Likely Value	Lowest Value
Initial Market Size (Lire M.)	12000	10000	8000
Market Growth (% p.a.)	9	3	-3
Market Share (%)	7	5	3
Price adjustment factor (%)	5	1	-5
Cost of Goods Sold (% of sales)	58	56	54
Variable Selling Expenses (% of sales)	19	17	15
Fixed Costs (Lire M. p.a.).	35	28	21

Table C.4 Highest, most likely and lowest values for variables in case D

Variable	Highest Value	Most Likely Value	Lowest Value
Fixed Costs (Home Mkt) £'000s p.a.	110	100	90
Variable Costs (Home Mkt) £ per ton	1.6	1.3	1.1
Price (Home Mkt) £ per ton	23.0	22.5	22.0
Potential Sales (Home Mkt) '000s tons	210	200	150
Fixed Costs (Export Mkt) £'000s p.a.	55	50	45
Var. Costs (Export Mkt) £ per ton	2.9	2.4	2.0
Price (Export Mkt) £ per ton	21.5	21.0	20.5
Potential Sales (Export Mkt) '000s tons	100	50	30
Capital Costs (£'000s)	2650	2550	2350
Start Year	2.25	1.75	1.25
Life of plant (yrs)	15.0	12.0	10.0
Start Costs (£'000s)	200	180	160
Salvage Value (£'000s)	6.0	5.0	4.0
Fixed Prodn. Costs (£'000s p.a.)	1050	1000	900
Var. Costs Prodn. (£ per ton)	2.25	2.20	2.15
Working Capital (£'000s)	550	500	450
Prodn. Capacity ('000s tons)	205	200	190

Table C.5 Highest, most likely, and lowest values for variables in case E.

Variable	Highest Value	Most Likely Value	Lowest Value
Rate of Growth of external mkt. (% p.a.)	10	5	3
Rate of Growth of internal mkt. (% p.a.)	10	5	3
Year of Hostile Act	4	2	1
Year of Introduction of fourth generation equipment	6	3	1

APPENDIX D

DISTRIBUTIONS INITIALLY OBTAINED FOR NPV AND IRR USING THE FIVE CASE STUDIES

The distributions in this appendix are those which were obtained for NPV and IRR in simulations numbers 1, 2 and 3 using case studies A, B, C, D and E. The reader is reminded of the following abbreviations:

Simulation No. 1:	Simulation where all variables have triangular distributions.
Simulation No. 2:	Simulation where all variables have uniform distributions with the same means and standard deviations as the corresponding triangular distributions in simulation no. 1.
Simulation No. 3:	Simulation where all variables have ◺ shaped distributions with the same means and standard deviations as the corresponding triangular distributions in simulation no. 1.
Case A:	Hertz Model
Case B:	Kryzanowski et al Model
Case C:	Interchemical Model
Case D:	ICI Model
Case E:	Economos Model

When interpreting the distributions it should be noted that in situations where either:

(i) all cash flows were positive

or (ii) the IRR was finite but greater than 100%.

the IRR was taken as 100%. When all cash flows were negative the IRR was taken as -100%.

FIGURE D.1

PERTZ MODEL

SIMULATION NO 1 2000 RUNS

DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT

MEAN= -2718.83 S.D.= 9922.34

RANGE			PROB	
LESS THAN		-32000.0	0.0030	IXX
-32000.0	TO	-30000.0	0.0025	IX
-30000.0	TO	-28000.0	0.0005	I
-28000.0	TO	-26000.0	0.0030	IXX
-26000.0	TO	-24000.0	0.0100	IXXXXX
-24000.0	TO	-22000.0	0.0070	IXXXX
-22000.0	TO	-20000.0	0.0120	IXXXXXX
-20000.0	TO	-18000.0	0.0175	IXXXXXXXXX
-18000.0	TO	-16000.0	0.0280	IXXXXXXXXXXXXXX
-16000.0	TO	-14000.0	0.0230	IXXXXXXXXXXXX
-14000.0	TO	-12000.0	0.0415	IXXXXXXXXXXXXXXXXXXXXX
-12000.0	TO	-10000.0	0.0590	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-10000.0	TO	-8000.0	0.0780	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-8000.0	TO	-6000.0	0.0870	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-6000.0	TO	-4000.0	0.0895	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-4000.0	TO	-2000.0	0.0890	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-2000.0	TO	0.0	0.0825	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0	TO	2000.0	0.0720	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
2000.0	TO	4000.0	0.0670	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4000.0	TO	6000.0	0.0560	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
6000.0	TO	8000.0	0.0465	IXXXXXXXXXXXXXXXXXXXXXXX
8000.0	TO	10000.0	0.0305	IXXXXXXXXXXXXXXX
10000.0	TO	12000.0	0.0250	IXXXXXXXXXXXXX
12000.0	TO	14000.0	0.0200	IXXXXXXXXXX
14000.0	TO	16000.0	0.0125	IXXXXXX
16000.0	TO	18000.0	0.0110	IXXXXXX
18000.0	TO	20000.0	0.0100	IXXXXX
20000.0	TO	22000.0	0.0050	IXXX
22000.0	TO	24000.0	0.0025	IX
24000.0	TO	26000.0	0.0015	IX
GREATER THAN		26000.0	0.0075	IXXXX

FIGURE D.2

```
                                        HERTZ  MODEL
                            SIMULATION NO 2                    2000 RUNS
                            DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                                        MEAN=  -2713.67      S.D.=   9883.69

        RANGE                PROB

   LESS THAN   -32000.0    0.0010   IX
-32000.0  TO   -30000.0    0.0005   I
-30000.0  TO   -28000.0    0.0025   IX
-28000.0  TO   -26000.0    0.0025   IX
-26000.0  TO   -24000.0    0.0030   IXX
-24000.0  TO   -22000.0    0.0075   IXXXX
-22000.0  TO   -20000.0    0.0160   IXXXXXXXX
-20000.0  TO   -18000.0    0.0155   IXXXXXXXX
-18000.0  TO   -16000.0    0.0300   IXXXXXXXXXXXXXXX
-16000.0  TO   -14000.0    0.0315   IXXXXXXXXXXXXXXXX
-14000.0  TO   -12000.0    0.0460   IXXXXXXXXXXXXXXXXXXXXXXX
-12000.0  TO   -10000.0    0.0640   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-10000.0  TO    -8000.0    0.0665   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -8000.0  TO    -6000.0    0.0935   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -6000.0  TO    -4000.0    0.0875   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -4000.0  TO    -2000.0    0.0885   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -2000.0  TO        0.0    0.0840   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
     0.0  TO     2000.0    0.0735   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0  TO     4000.0    0.0605   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0  TO     6000.0    0.0460   IXXXXXXXXXXXXXXXXXXXXXXX
  6000.0  TO     8000.0    0.0440   IXXXXXXXXXXXXXXXXXXXXXX
  8000.0  TO    10000.0    0.0350   IXXXXXXXXXXXXXXXXXX
 10000.0  TO    12000.0    0.0210   IXXXXXXXXXXX
 12000.0  TO    14000.0    0.0220   IXXXXXXXXXXX
 14000.0  TO    16000.0    0.0105   IXXXXX
 16000.0  TO    18000.0    0.0110   IXXXXXX
 18000.0  TO    20000.0    0.0090   IXXXXX
 20000.0  TO    22000.0    0.0060   IXXX
 22000.0  TO    24000.0    0.0055   IXXX
 24000.0  TO    26000.0    0.0030   IXX
GREATER THAN    28000.0    0.0070   IXXXX
```

FIGURE D.3

HERTZ MODEL

SIMULATION NO 3 2000 RUNS

DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT

MEAN= -2697.11 S.D.= 9869.41

RANGE			PROB	
LESS THAN		-32000.0	0.0015	IX
-32000.0	TO	-30000.0	0.0025	IX
-30000.0	TO	-28000.0	0.0015	IX
-28000.0	TO	-26000.0	0.0010	IX
-26000.0	TO	-24000.0	0.0055	IXXX
-24000.0	TO	-22000.0	0.0050	IXXX
-22000.0	TO	-20000.0	0.0125	IXXXXXX
-20000.0	TO	-18000.0	0.0185	IXXXXXXXXX
-18000.0	TO	-16000.0	0.0255	IXXXXXXXXXXXXXX
-16000.0	TO	-14000.0	0.0330	IXXXXXXXXXXXXXXXXXX
-14000.0	TO	-12000.0	0.0435	IXXXXXXXXXXXXXXXXXXXXXXXX
-12000.0	TO	-10000.0	0.0570	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-10000.0	TO	-8000.0	0.0725	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-8000.0	TO	-6000.0	0.0885	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-6000.0	TO	-4000.0	0.1010	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-4000.0	TO	-2000.0	0.0980	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-2000.0	TO	0.0	0.0865	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0	TO	2000.0	0.0695	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
2000.0	TO	4000.0	0.0545	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4000.0	TO	6000.0	0.0670	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
6000.0	TO	8000.0	0.0370	IXXXXXXXXXXXXXXXXXXXXX
8000.0	TO	10000.0	0.0260	IXXXXXXXXXXXXXX
10000.0	TO	12000.0	0.0160	IXXXXXXXXX
12000.0	TO	14000.0	0.0135	IXXXXXXXXX
14000.0	TO	16000.0	0.0145	IXXXXXXXXX
16000.0	TO	18000.0	0.0100	IXXXXX
18000.0	TO	20000.0	0.0075	IXXXX
20000.0	TO	22000.0	0.0055	IXXX
22000.0	TO	24000.0	0.0055	IXXX
24000.0	TO	26000.0	0.0035	IXX
GREATER THAN		26000.0	0.0075	IXXXX

FIGURE D:4

```
                                   HERTZ  MODEL
                      SIMULATION NO 1                    2000 RUNS
                              DISTRIBUTION OF IRR
                              MEAN=      1.79      S.D.=      22.55

      RANGE              PROB

 LESS THAN       -65.0  0.0065   IXXX
 -65.0   TO      -60.0  0.0015   IX
 -60.0   TO      -55.0  0.0055   IXXX
 -55.0   TO      -50.0  0.0065   IXXX
 -50.0   TO      -45.0  0.0110   IXXXXXX
 -45.0   TO      -40.0  0.0145   IXXXXXXX
 -40.0   TO      -35.0  0.0170   IXXXXXXXXX
 -35.0   TO      -30.0  0.0185   IXXXXXXXXX
 -30.0   TO      -25.0  0.0300   IXXXXXXXXXXXXXXX
 -25.0   TO      -20.0  0.0405   IXXXXXXXXXXXXXXXXXXXX
 -20.0   TO      -15.0  0.0470   IXXXXXXXXXXXXXXXXXXXXXXXX
 -15.0   TO      -10.0  0.0660   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -10.0   TO       -5.0  0.0850   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -5.0   TO        0.0  0.0880   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   0.0   TO        5.0  0.1010   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   5.0   TO       10.0  0.0945   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  10.0   TO       15.0  0.0880   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  15.0   TO       20.0  0.0760   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  20.0   TO       25.0  0.0645   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  25.0   TO       30.0  0.0450   IXXXXXXXXXXXXXXXXXXXXXXX
  30.0   TO       35.0  0.0370   IXXXXXXXXXXXXXXXXXXX
  35.0   TO       40.0  0.0225   IXXXXXXXXXXX
  40.0   TO       45.0  0.0110   IXXXXXX
  45.0   TO       50.0  0.0090   IXXXXX
  50.0   TO       55.0  0.0040   IXX
  55.0   TO       60.0  0.0030   IXX
  60.0   TO       65.0  0.0020   IX
 GREATER THAN     70.0  0.0020   IX
```

FIGURE D.5

```
                                    HERTZ  MODEL
                      SIMULATION NO 2                              2000 RUNS
                              DISTRIBUTION OF IRR
                              MEAN=       1.85      S.D.=      22.09

      RANGE              PROB

 LESS THAN       -65.0   0.0020   IX
 -65.0   TO      -60.0   0.0020   IX

 -60.0   TO      -55.0   0.0065   IXXX
 -55.0   TO      -50.0   0.0050   IXXX

 -50.0   TO      -45.0   0.0080   IXXXX
 -45.0   TO      -40.0   0.0100   IXXXXX
 -40.0   TO      -35.0   0.0170   IXXXXXXXXX

 -35.0   TO      -30.0   0.0290   IXXXXXXXXXXXXXXX
 -30.0   TO      -25.0   0.0370   IXXXXXXXXXXXXXXXXXXX

 -25.0   TO      -20.0   0.0415   IXXXXXXXXXXXXXXXXXXXXX
 -20.0   TO      -15.0   0.0515   IXXXXXXXXXXXXXXXXXXXXXXXXXX
 -15.0   TO      -10.0   0.0640   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -10.0   TO       -5.0   0.0855   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -5.0   TO        0.0   0.0910   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   0.0   TO        5.0   0.0970   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   5.0   TO       10.0   0.0960   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  10.0   TO       15.0   0.0905   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  15.0   TO       20.0   0.0695   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  20.0   TO       25.0   0.0565   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  25.0   TO       30.0   0.0495   IXXXXXXXXXXXXXXXXXXXXXXXXX
  30.0   TO       35.0   0.0340   IXXXXXXXXXXXXXXXXX

  35.0   TO       40.0   0.0210   IXXXXXXXXXXX
  40.0   TO       45.0   0.0120   IXXXXXX

  45.0   TO       50.0   0.0105   IXXXXX
  50.0   TO       55.0   0.0050   IXXX
  55.0   TO       60.0   0.0020   IX

  60.0   TO       65.0   0.0040   IXX
GREATER THAN      70.0   0.0025   IX
```

FIGURE D.6

```
                              HERTZ  MODEL
               SIMULATION NO 3                         2000 RUNS
                        DISTRIBUTION OF IRP
                        MEAN=      1.87      S.D.=      22.23

     RANGE              PROB

  LESS THAN     -65.0  0.0060   IXXX
  -65.0   TO    -60.0  0.0020   IX
  -60.0   TO    -55.0  0.0025   IX
  -55.0   TO    -50.0  0.0055   IXXX
  -50.0   TO    -45.0  0.0050   IXXX
  -45.0   TO    -40.0  0.0120   IXXXXXX
  -40.0   TO    -35.0  0.0190   IXXXXXXXXXX
  -35.0   TO    -30.0  0.0260   IXXXXXXXXXXXXX
  -30.0   TO    -25.0  0.0315   IXXXXXXXXXXXXXXXX
  -25.0   TO    -20.0  0.0375   IXXXXXXXXXXXXXXXXXXX
  -20.0   TO    -15.0  0.0500   IXXXXXXXXXXXXXXXXXXXXXXXXX
  -15.0   TO    -10.0  0.0610   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -10.0   TO     -5.0  0.0915   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   -5.0   TO      0.0  0.0940   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    0.0   TO      5.0  0.1110   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    5.0   TO     10.0  0.0990   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   10.0   TO     15.0  0.0835   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   15.0   TO     20.0  0.0795   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   20.0   TO     25.0  0.0595   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   25.0   TO     30.0  0.0350   IXXXXXXXXXXXXXXXXXX
   30.0   TO     35.0  0.0270   IXXXXXXXXXXXXXX
   35.0   TO     40.0  0.0260   IXXXXXXXXXXXXX
   40.0   TO     45.0  0.0155   IXXXXXXXX
   45.0   TO     50.0  0.0065   IXXX
   50.0   TO     55.0  0.0045   IXX
   55.0   TO     60.0  0.0030   IXX
   60.0   TO     65.0  0.0015   IX
 GREATER THAN    70.0  0.0050   IXXX
```

FIGURE D.7

KRYZANOWSKI ET AL MODEL

SIMULATION NO 1 2000 RUNS
DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
MEAN= 1680.40 S.D.= 5669.11

RANGE			PROB	
LESS THAN		-15000.0	0.0060	IXXX
-15000.0	TO	-14000.0	0.0045	IXX
-14000.0	TO	-13000.0	0.0025	IX
-13000.0	TO	-12000.0	0.0030	IXX
-12000.0	TO	-11000.0	0.0010	IX
-11000.0	TO	-10000.0	0.0060	IXXX
-10000.0	TO	-9000.0	0.0080	IXXXX
-9000.0	TO	-8000.0	0.0105	IXXXXX
-8000.0	TO	-7000.0	0.0130	IXXXXXXX
-7000.0	TO	-6000.0	0.0195	IXXXXXXXXXX
-6000.0	TO	-5000.0	0.0220	IXXXXXXXXXXX
-5000.0	TO	-4000.0	0.0320	IXXXXXXXXXXXXXXXX
-4000.0	TO	-3000.0	0.0470	IXXXXXXXXXXXXXXXXXXXXXXXX
-3000.0	TO	-2000.0	0.0600	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-2000.0	TO	-1000.0	0.0655	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-1000.0	TO	0.0	0.0755	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0	TO	1000.0	0.0770	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1000.0	TO	2000.0	0.0780	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
2000.0	TO	3000.0	0.0755	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
3000.0	TO	4000.0	0.0785	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4000.0	TO	5000.0	0.0610	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5000.0	TO	6000.0	0.0555	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
6000.0	TO	7000.0	0.0370	IXXXXXXXXXXXXXXXXXXX
7000.0	TO	8000.0	0.0400	IXXXXXXXXXXXXXXXXXXXX
8000.0	TO	9000.0	0.0265	IXXXXXXXXXXXXX
9000.0	TO	10000.0	0.0255	IXXXXXXXXXXXXX
10000.0	TO	11000.0	0.0175	IXXXXXXXXX
11000.0	TO	12000.0	0.0140	IXXXXXXX
12000.0	TO	13000.0	0.0095	IXXXXX
13000.0	TO	14000.0	0.0090	IXXXXX
14000.0	TO	15000.0	0.0065	IXXX
15000.0	TO	16000.0	0.0045	IXX
16000.0	TO	17000.0	0.0035	IXX
17000.0	TO	18000.0	0.0020	IX
GREATER THAN		19000.0	0.0030	IXX

FIGURE D.8

```
                                KRYZANOWSKI ET AL  MODEL
                     SIMULATION NO 2                      2000 RUNS
                     DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                               MEAN=    1710.56      S.D.=    5452.69

       RANGE                PROB

   LESS THAN   -15000.0   0.0020   IX
-15000.0  TO   -14000.0   0.0010   IX
-14000.0  TO   -13000.0   0.0015   IX
-13000.0  TO   -12000.0   0.0015   IX
-12000.0  TO   -11000.0   0.0015   IX
-11000.0  TO   -10000.0   0.0055   IXXX
-10000.0  TO    -9000.0   0.0070   IXXXX
 -9000.0  TO    -8000.0   0.0085   IXXXX
 -8000.0  TO    -7000.0   0.0115   IXXXXXX
 -7000.0  TO    -6000.0   0.0200   IXXXXXXXXXX
 -6000.0  TO    -5000.0   0.0310   IXXXXXXXXXXXXXXXX
 -5000.0  TO    -4000.0   0.0430   IXXXXXXXXXXXXXXXXXXXXXX
 -4000.0  TO    -3000.0   0.0505   IXXXXXXXXXXXXXXXXXXXXXXXXX
 -3000.0  TO    -2000.0   0.0655   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -2000.0  TO    -1000.0   0.0710   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -1000.0  TO        0.0   0.0620   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
     0.0  TO     1000.0   0.0870   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1000.0  TO     2000.0   0.0810   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0  TO     3000.0   0.0685   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3000.0  TO     4000.0   0.0740   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0  TO     5000.0   0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5000.0  TO     6000.0   0.0515   IXXXXXXXXXXXXXXXXXXXXXXXXXX
  6000.0  TO     7000.0   0.0350   IXXXXXXXXXXXXXXXXXX
  7000.0  TO     8000.0   0.0390   IXXXXXXXXXXXXXXXXXXXX
  8000.0  TO     9000.0   0.0295   IXXXXXXXXXXXXXXX
  9000.0  TO    10000.0   0.0245   IXXXXXXXXXXXX
 10000.0  TO    11000.0   0.0175   IXXXXXXXXX
 11000.0  TO    12000.0   0.0130   IXXXXXXX
 12000.0  TO    13000.0   0.0090   IXXXXX
 13000.0  TO    14000.0   0.0100   IXXXXX
 14000.0  TO    15000.0   0.0080   IXXXX
 15000.0  TO    16000.0   0.0045   IXX
 16000.0  TO    17000.0   0.0050   IXXX
 17000.0  TO    18000.0   0.0015   IX
GREATER THAN    19000.0   0.0030   IXX
```

FIGURE D.9

```
                                         KRYZANOWSKI ET AL  MODEL
                         SIMULATION NO 3                          2000 RUNS
                         DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                                    MEAN=    1670.77      S.D.=    5853.12

        RANGE                PROB

   LESS THAN   -15000.0   0.0105    IXXXXX
 -15000.0  TO  -14000.0   0.0015    IX
 -14000.0  TO  -13000.0   0.0025    IX
 -13000.0  TO  -12000.0   0.0030    IXX
 -12000.0  TO  -11000.0   0.0045    IXX
 -11000.0  TO  -10000.0   0.0065    IXXX
 -10000.0  TO   -9000.0   0.0075    IXXXX
  -9000.0  TO   -8000.0   0.0120    IXXXXXX
  -8000.0  TO   -7000.0   0.0095    IXXXXX
  -7000.0  TO   -6000.0   0.0150    IXXXXXXX
  -6000.0  TO   -5000.0   0.0285    IXXXXXXXXXXXXXX
  -5000.0  TO   -4000.0   0.0320    IXXXXXXXXXXXXXXXX
  -4000.0  TO   -3000.0   0.0470    IXXXXXXXXXXXXXXXXXXXXXXX
  -3000.0  TO   -2000.0   0.0515    IXXXXXXXXXXXXXXXXXXXXXXXXXX
  -2000.0  TO   -1000.0   0.0595    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -1000.0  TO       0.0   0.0720    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      0.0  TO    1000.0   0.0830    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   1000.0  TO    2000.0   0.0805    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   2000.0  TO    3000.0   0.0790    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   3000.0  TO    4000.0   0.0800    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   4000.0  TO    5000.0   0.0615    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   5000.0  TO    6000.0   0.0580    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6000.0  TO    7000.0   0.0465    IXXXXXXXXXXXXXXXXXXXXXXX
   7000.0  TO    8000.0   0.0320    IXXXXXXXXXXXXXXXX
   8000.0  TO    9000.0   0.0260    IXXXXXXXXXXXXX
   9000.0  TO   10000.0   0.0175    IXXXXXXXXX
  10000.0  TO   11000.0   0.0170    IXXXXXXXXX
  11000.0  TO   12000.0   0.0155    IXXXXXXXX
  12000.0  TO   13000.0   0.0105    IXXXXX
  13000.0  TO   14000.0   0.0075    IXXXX
  14000.0  TO   15000.0   0.0065    IXXX
  15000.0  TO   16000.0   0.0055    IXXX
  16000.0  TO   17000.0   0.0030    IXX
  17000.0  TO   18000.0   0.0025    IX
  GREATER THAN  19000.0   0.0050    IXXX
```

FIGURE D.10

```
                              KRYZANOWSKI ET AL  MODEL
          SIMULATION NO 1                       2000 RUNS
                               DISTRIBUTION OF IPR
                        MEAN=    -17.11      S.D.=      58.29

      RANGE                PROB

  LESS THAN      -190.0  0.0000   I
 -190.0   TO     -180.0  0.0000   I
 -180.0   TO     -170.0  0.0000   I
 -170.0   TO     -160.0  0.0000   I
 -160.0   TO     -150.0  0.0000   I
 -150.0   TO     -140.0  0.0000   I
 -140.0   TO     -130.0  0.0000   I
 -130.0   TO     -120.0  0.0000   I
 -120.0   TO     -110.0  0.0000   I
 -110.0   TO     -100.0  0.0000   I
 -100.0   TO      -90.0  0.3200   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -90.0   TO      -80.0  0.0075   IXXXX
  -80.0   TO      -70.0  0.0005   I
  -70.0   TO      -60.0  0.0010   IX
  -60.0   TO      -50.0  0.0005   I
  -50.0   TO      -40.0  0.0000   I
  -40.0   TO      -30.0  0.0000   I
  -30.0   TO      -20.0  0.0000   I
  -20.0   TO      -10.0  0.0000   I
  -10.0   TO        0.0  0.0030   IXX
    0.0   TO       10.0  0.0435   IXXXXXXXXXXXXXXXXXXXXXX
   10.0   TO       20.0  0.2025   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   20.0   TO       30.0  0.2645   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   30.0   TO       40.0  0.1240   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   40.0   TO       50.0  0.0310   IXXXXXXXXXXXXXXXX
   50.0   TO       60.0  0.0020   IX
   60.0   TO       70.0  0.0000   I
   70.0   TO       80.0  0.0000   I
   80.0   TO       90.0  0.0000   I
   90.0   TO      100.0  0.0000   I
  100.0   TO      110.0  0.0000   I
  110.0   TO      120.0  0.0000   I
  120.0   TO      130.0  0.0000   I
  130.0   TO      140.0  0.0000   I
  140.0   TO      150.0  0.0000   I
 GREATER THAN     160.0  0.0000   I
```

FIGURE D.11 SIMULATION NO 2

KRYZANOWSKI ET AL MODEL 2000 RUNS

DISTRIBUTION OF IRR

MEAN= -19.01 S.D.= 59.08

RANGE			PROB	
LESS THAN		-190.0	0.0000	I
-190.0	TO	-180.0	0.0000	I
-180.0	TO	-170.0	0.0000	I
-170.0	TO	-160.0	0.0000	I
-160.0	TO	-150.0	0.0000	I
-150.0	TO	-140.0	0.0000	I
-140.0	TO	-130.0	0.0000	I
-130.0	TO	-120.0	0.0000	I
-120.0	TO	-110.0	0.0000	I
-110.0	TO	-100.0	0.0000	I
-100.0	TO	-90.0	0.3335	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-90.0	TO	-80.0	0.0045	IXX
-80.0	TO	-70.0	0.0020	IX
-70.0	TO	-60.0	0.0000	I
-60.0	TO	-50.0	0.0000	I
-50.0	TO	-40.0	0.0005	I
-40.0	TO	-30.0	0.0000	I
-30.0	TO	-20.0	0.0000	I
-20.0	TO	-10.0	0.0000	I
-10.0	TO	0.0	0.0020	IX
0.0	TO	10.0	0.0355	IXXXXXXXXXXXXXXXXXX
10.0	TO	20.0	0.2065	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
20.0	TO	30.0	0.2405	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
30.0	TO	40.0	0.1300	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
40.0	TO	50.0	0.0300	IXXXXXXXXXXXXXXX
50.0	TO	60.0	0.0020	IX
60.0	TO	70.0	0.0000	I
70.0	TO	80.0	0.0000	I
80.0	TO	90.0	0.0000	I
90.0	TO	100.0	0.0000	I
100.0	TO	110.0	0.0000	I
110.0	TO	120.0	0.0000	I
120.0	TO	130.0	0.0000	I
130.0	TO	140.0	0.0000	I
140.0	TO	150.0	0.0000	I
GREATER THAN		150.0	0.0006	I

FIGURE D.12

```
                                   KRYZANOWSKI ET AL  MODEL
                   SIMULATION NO 3                     2000 RUNS
                                DISTRIBUTION OF IRR
                             MEAN=    -16.31      S.D.=      57.98

     RANGE                PROB .

  LESS THAN      -190.0  0.0000   I
 -190.0   TO     -180.0  0.0000   I
 -180.0   TO     -170.0  0.0000   I
 -170.0   TO     -160.0  0.0000   I
 -160.0   TO     -150.0  0.0000   I
 -150.0   TO     -140.0  0.0000   I
 -140.0   TO     -130.0  0.0000   I
 -130.0   TO     -120.0  0.0000   I
 -120.0   TO     -110.0  0.0000   I
 -110.0   TO     -100.0  0.0000   I
 -100.0   TO      -90.0  0.3155   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -90.0   TO      -80.0  0.0030   IXX
  -80.0   TO      -70.0  0.0030   IXX
  -70.0   TO      -60.0  0.0005   I
  -60.0   TO      -50.0  0.0005   I
  -50.0   TO      -40.0  0.0000   I
  -40.0   TO      -30.0  0.0000   I
  -30.0   TO      -20.0  0.0000   I
  -20.0   TO      -10.0  0.0000   I
  -10.0   TO        0.0  0.0005   I
    0.0   TO       10.0  0.0400   IXXXXXXXXXXXXXXXXXXXX
   10.0   TO       20.0  0.2250   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   20.0   TO       30.0  0.2530   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   30.0   TO       40.0  0.1170   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   40.0   TO       50.0  0.0345   IXXXXXXXXXXXXXXXXX
   50.0   TO       60.0  0.0025   IX
   60.0   TO       70.0  0.0000   I
   70.0   TO       80.0  0.0000   I
   80.0   TO       90.0  0.0000   I
   90.0   TO      100.0  0.0000   I
  100.0   TO      110.0  0.0000   I
  110.0   TO      120.0  0.0000   I
  120.0   TO      130.0  0.0000   I
  130.0   TO      140.0  0.0000   I
  140.0   TO      150.0  0.0000   I
GREATER THAN.     160.0  0.0000   I
```

FIGURE D.13

```
                                        INTERCHEM MODEL
                         SIMULATION NO 1                          2000 RUNS
                         DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                                  MEAN=      71.23      S.D.=      92.05

      RANGE                  PROB

   LESS THAN     -200.0   0.0000    I
 -200.0   TO     -180.0   0.0000    I
 -180.0   TO     -160.0   0.0000    I
 -160.0   TO     -140.0   0.0000    I
 -140.0   TO     -120.0   0.0000    I
 -120.0   TO     -100.0   0.0000    I
 -100.0   TO      -80.0   0.0000    I
  -80.0   TO      -60.0   0.0000    I
  -60.0   TO      -40.0   0.0175    IXXXXXX
  -40.0   TO      -20.0   0.1150    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -20.0   TO        0.0   0.2740    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    0.0   TO       20.0   0.0160    IXXXXXXXX
   20.0   TO       40.0   0.0380    IXXXXXXXXXXXXXXXXXX
   40.0   TO       60.0   0.0545    IXXXXXXXXXXXXXXXXXXXXXXXXXX
   60.0   TO       80.0   0.0705    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   80.0   TO      100.0   0.0680    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  100.0   TO      120.0   0.0505    IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  120.0   TO      140.0   0.0515    IXXXXXXXXXXXXXXXXXXXXXXXXX
  140.0   TO      160.0   0.0545    IXXXXXXXXXXXXXXXXXXXXXXXXXX
  160.0   TO      180.0   0.0470    IXXXXXXXXXXXXXXXXXXXXXXX
  180.0   TO      200.0   0.0430    IXXXXXXXXXXXXXXXXXXXXX
  200.0   TO      220.0   0.0230    IXXXXXXXXXXXX
  220.0   TO      240.0   0.0210    IXXXXXXXXXXX
  240.0   TO      260.0   0.0175    IXXXXXXXXX
  260.0   TO      280.0   0.0095    IXXXXX
  280.0   TO      300.0   0.0065    IXXX
  300.0   TO      320.0   0.0055    IXXX
  320.0   TO      340.0   0.0050    IXXX
   GREATER THAN   360.0   0.0030    IXXXX
```

FIGURE D.14

```
                         INTERCHEM MODEL
         SIMULATION NO 2                      2000 RUNS
         DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                    MEAN=      71.55      S.D.=      91.84

     RANGE                PROB

 LESS THAN     -200.0  0.0000  I
-200.0  TO     -180.0  0.0000  I
-180.0  TO     -160.0  0.0000  I
-160.0  TO     -140.0  0.0000  I
-140.0  TO     -120.0  0.0000  I
-120.0  TO     -100.0  0.0000  I
-100.0  TO      -80.0  0.0000  I
 -80.0  TO      -60.0  0.0000  I
 -60.0  TO      -40.0  0.0045  IXX
 -40.0  TO      -20.0  0.1275  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -20.0  TO        0.0  0.2725  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   0.0  TO       20.0  0.0150  IXXXXXXXX
  20.0  TO       40.0  0.0335  IXXXXXXXXXXXXXXXXX
  40.0  TO       60.0  0.0540  IXXXXXXXXXXXXXXXXXXXXXXXXXX
  60.0  TO       80.0  0.0555  IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  80.0  TO      100.0  0.0715  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 100.0  TO      120.0  0.0610  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 120.0  TO      140.0  0.0460  IXXXXXXXXXXXXXXXXXXXXXXXX
 140.0  TO      160.0  0.0510  IXXXXXXXXXXXXXXXXXXXXXXXXXX
 160.0  TO      180.0  0.0490  IXXXXXXXXXXXXXXXXXXXXXXXXX
 180.0  TO      200.0  0.0430  IXXXXXXXXXXXXXXXXXXXXXX
 200.0  TO      220.0  0.0380  IXXXXXXXXXXXXXXXXXXX
 220.0  TO      240.0  0.0195  IXXXXXXXXXX
 240.0  TO      260.0  0.0185  IXXXXXXXXX
 260.0  TO      280.0  0.0085  IXXXX
 280.0  TO      300.0  0.0090  IXXXXX
 300.0  TO      320.0  0.0060  IXXX
 320.0  TO      340.0  0.0045  IXX
GREATER THAN    360.0  0.0030  IXX
```

FIGURE D.15

```
                              INTERCHEM MODEL
              SIMULATION NO 3                         2000 RUNS
              DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                         MEAN=      69.93      S.D.=      96.13

      RANGE                  PROB

  LESS THAN       -200.0   0.0000   I
 -200.0   TO      -180.0   0.0000   I
 -180.0   TO      -160.0   0.0000   I
 -160.0   TO      -140.0   0.0000   I
 -140.0   TO      -120.0   0.0000   I
 -120.0   TO      -100.0   0.0000   I
 -100.0   TO       -80.0   0.0000   I
  -80.0   TO       -60.0   0.0000   I
  -60.0   TO       -40.0   0.0005   I
  -40.0   TO       -20.0   0.1045   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -20.0   TO         0.0   0.3275   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    0.0   TO        20.0   0.0180   IXXXXXXXXX
   20.0   TO        40.0   0.0425   IXXXXXXXXXXXXXXXXXXXXX
   40.0   TO        60.0   0.0510   IXXXXXXXXXXXXXXXXXXXXXXXXXX
   60.0   TO        80.0   0.0535   IXXXXXXXXXXXXXXXXXXXXXXXXXXX
   80.0   TO       100.0   0.0615   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  100.0   TO       120.0   0.0570   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  120.0   TO       140.0   0.0530   IXXXXXXXXXXXXXXXXXXXXXXXXXXX
  140.0   TO       160.0   0.0445   IXXXXXXXXXXXXXXXXXXXXXX
  160.0   TO       180.0   0.0405   IXXXXXXXXXXXXXXXXXXXX
  180.0   TO       200.0   0.0415   IXXXXXXXXXXXXXXXXXXXXX
  200.0   TO       220.0   0.0230   IXXXXXXXXXXXX
  220.0   TO       240.0   0.0190   IXXXXXXXXXX
  240.0   TO       260.0   0.0160   IXXXXXXXX
  260.0   TO       280.0   0.0130   IXXXXXXX
  280.0   TO       300.0   0.0080   IXXXX
  300.0   TO       320.0   0.0065   IXXX
  320.0   TO       340.0   0.0045   IXX
 GREATER THAN      360.0   0.0145   IXXXXXXX
```

FIGURE D.16

INTERCHEM MODEL

SIMULATION NO 1 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 19.70 S.D.= 98.06

RANGE			PROB
LESS THAN		-260.0	0.0000
-260.0	TO	-240.0	0.0000
-240.0	TO	-220.0	0.0000
-220.0	TO	-200.0	0.0000
-200.0	TO	-180.0	0.0000
-180.0	TO	-160.0	0.0000
-160.0	TO	-140.0	0.0000
-140.0	TO	-120.0	0.0000
-120.0	TO	-100.0	0.0000
-100.0	TO	-80.0	0.4015
-80.0	TO	-60.0	0.0000
-60.0	TO	-40.0	0.0000
-40.0	TO	-20.0	0.0000
-20.0	TO	0.0	0.0000
0.0	TO	20.0	0.0000
20.0	TO	40.0	0.0000
40.0	TO	60.0	0.0000
60.0	TO	80.0	0.0000
80.0	TO	100.0	0.0000
100.0	TO	120.0	0.5985
120.0	TO	140.0	0.0000
140.0	TO	160.0	0.0000
160.0	TO	180.0	0.0000
180.0	TO	200.0	0.0000
200.0	TO	220.0	0.0000
220.0	TO	240.0	0.0000
240.0	TO	260.0	0.0000
260.0	TO	280.0	0.0000
280.0	TO	300.0	0.0000
GREATER THAN		320.0	0.0000

FIGURE D.17

INTERCHEM MODEL

SIMULATION NO 2 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 17.90 S.D.= 98.41

RANGE			PROB	
LESS THAN		-260.0	0.0000	I
-260.0	TO	-240.0	0.0000	I
-240.0	TO	-220.0	0.0000	I
-220.0	TO	-200.0	0.0000	I
-200.0	TO	-180.0	0.0000	I
-180.0	TO	-160.0	0.0000	I
-160.0	TO	-140.0	0.0000	I
-140.0	TO	-120.0	0.0000	I
-120.0	TO	-100.0	0.0000	I
-100.0	TO	-80.0	0.4145	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-80.0	TO	-60.0	0.0000	I
-60.0	TO	-40.0	0.0000	I
-40.0	TO	-20.0	0.0000	I
-20.0	TO	0.0	0.0000	I
0.0	TO	20.0	0.0000	I
20.0	TO	40.0	0.0000	I
40.0	TO	60.0	0.0000	I
60.0	TO	80.0	0.0000	I
80.0	TO	100.0	0.0000	I
100.0	TO	120.0	0.5395	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
120.0	TO	140.0	0.0000	I
140.0	TO	160.0	0.0000	I
160.0	TO	180.0	0.0000	I
180.0	TO	200.0	0.0000	I
200.0	TO	220.0	0.0000	I
220.0	TO	240.0	0.0000	I
240.0	TO	260.0	0.0000	I
260.0	TO	280.0	0.0000	I
280.0	TO	300.0	0.0000	I
GREATER THAN		320.0	0.0000	I

FIGURE D.18

INTERCHEM MODEL

SIMULATION NO 3 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 13.50 S.D.= 99.11

RANGE			PROB	
LESS THAN		-260.0	0.0000	I
-260.0	TO	-240.0	0.0000	I
-240.0	TO	-220.0	0.0000	I
-220.0	TO	-200.0	0.0000	I
-200.0	TO	-180.0	0.0000	I
-180.0	TO	-160.0	0.0000	I
-160.0	TO	-140.0	0.0000	I
-140.0	TO	-120.0	0.0000	I
-120.0	TO	-100.0	0.0000	I
-100.0	TO	-80.0	0.4325	I
-80.0	TO	-60.0	0.0000	I
-60.0	TO	-40.0	0.0000	I
-40.0	TO	-20.0	0.0000	I
-20.0	TO	0.0	0.0000	I
0.0	TO	20.0	0.0000	I
20.0	TO	40.0	0.0000	I
40.0	TO	60.0	0.0000	I
60.0	TO	80.0	0.0000	I
80.0	TO	100.0	0.0000	I
100.0	TO	120.0	0.5675	I
120.0	TO	140.0	0.0000	I
140.0	TO	160.0	0.0000	I
160.0	TO	180.0	0.0000	I
180.0	TO	200.0	0.0000	I
200.0	TO	220.0	0.0000	I
220.0	TO	240.0	0.0000	I
240.0	TO	260.0	0.0000	I
260.0	TO	280.0	0.0000	I
280.0	TO	300.0	0.0000	I
GREATER THAN		320.0	0.0000	I

FIGURE D.19

JCI MODEL

SIMULATION NO 1 2000 RUNS
DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
MEAN= 8834.70 S.D.= 534.57

RANGE			PROB	
LESS	THAN	7200.0	0.0000	I
7200.0	TO	7300.0	0.0005	I
7300.0	TO	7400.0	0.0015	IX
7400.0	TO	7500.0	0.0030	IXX
7500.0	TO	7600.0	0.0040	IXX
7600.0	TO	7700.0	0.0070	IXXXX
7700.0	TO	7800.0	0.0115	IXXXXXX
7800.0	TO	7900.0	0.0090	IXXXXX
7900.0	TO	8000.0	0.0170	IXXXXXXXXX
8000.0	TO	8100.0	0.0250	IXXXXXXXXXXXXX
8100.0	TO	8200.0	0.0365	IXXXXXXXXXXXXXXXXXX
8200.0	TO	8300.0	0.0380	IXXXXXXXXXXXXXXXXXXX
8300.0	TO	8400.0	0.0540	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
8400.0	TO	8500.0	0.0620	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8500.0	TO	8600.0	0.0725	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8600.0	TO	8700.0	0.0720	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8700.0	TO	8800.0	0.0740	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8800.0	TO	8900.0	0.0705	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8900.0	TO	9000.0	0.0685	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9000.0	TO	9100.0	0.0660	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9100.0	TO	9200.0	0.0630	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9200.0	TO	9300.0	0.0550	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9300.0	TO	9400.0	0.0355	IXXXXXXXXXXXXXXXXXX
9400.0	TO	9500.0	0.0375	IXXXXXXXXXXXXXXXXXXX
9500.0	TO	9600.0	0.0290	IXXXXXXXXXXXXXXX
9600.0	TO	9700.0	0.0235	IXXXXXXXXXXXX
9700.0	TO	9800.0	0.0225	IXXXXXXXXXXX
9800.0	TO	9900.0	0.0135	IXXXXXXX
9900.0	TO	10000.0	0.0080	IXXXX
10000.0	TO	10100.0	0.0075	IXXXX
10100.0	TO	10200.0	0.0045	IXX
10200.0	TO	10300.0	0.0020	IX
10300.0	TO	10400.0	0.0015	IX
GREATER	THAN	10500.0	0.0015	IX

FIGURE D.20

```
                                   ICI MODEL
                 SIMULATION NO 2                        2000 RUNS
                 DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                           MEAN=    8828.69       S.D.=     531.13

        RANGE              PROB

   LESS THAN     7200.0   0.0000   I
  7200.0   TO    7300.0   0.0000   I
  7300.0   TO    7400.0   0.0005   I
  7400.0   TO    7500.0   0.0015   IX
  7500.0   TO    7600.0   0.0030   IXX
  7600.0   TO    7700.0   0.0075   IXXXX
  7700.0   TO    7800.0   0.0100   IXXXXX
  7800.0   TO    7900.0   0.0150   IXXXXXXXX
  7900.0   TO    8000.0   0.0145   IXXXXXXX
  8000.0   TO    8100.0   0.0365   IXXXXXXXXXXXXXXX
  8100.0   TO    8200.0   0.0380   IXXXXXXXXXXXXXXXXXXX
  8200.0   TO    8300.0   0.0450   IXXXXXXXXXXXXXXXXXXXXXXX
  8300.0   TO    8400.0   0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8400.0   TO    8500.0   0.0615   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8500.0   TO    8600.0   0.0730   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8600.0   TO    8700.0   0.0645   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8700.0   TO    8800.0   0.0715   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8800.0   TO    8900.0   0.0650   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8900.0   TO    9000.0   0.0680   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9000.0   TO    9100.0   0.0675   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9100.0   TO    9200.0   0.0570   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9200.0   TO    9300.0   0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9300.0   TO    9400.0   0.0445   IXXXXXXXXXXXXXXXXXXXXXX
  9400.0   TO    9500.0   0.0355   IXXXXXXXXXXXXXXXXXX
  9500.0   TO    9600.0   0.0320   IXXXXXXXXXXXXXXXX
  9600.0   TO    9700.0   0.0305   IXXXXXXXXXXXXXXX
  9700.0   TO    9800.0   0.0185   IXXXXXXXXX
  9800.0   TO    9900.0   0.0120   IXXXXXX
  9900.0   TO   10000.0   0.0100   IXXXXX
 10000.0   TO   10100.0   0.0065   IXXX
 10100.0   TO   10200.0   0.0035   IXX
 10200.0   TO   10300.0   0.0000   I
 10300.0   TO   10400.0   0.0020   IX
 GREATER THAN   10500.0   0.0005   I
```

FIGURE D.21

ICI MODEL

SIMULATION NO 3 2000 RUNS

DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT

MEAN= 8830.05 S.D.= 520.75

RANGE			PROB	
LESS THAN		7200.0	0.0000	I
7200.0	TO	7300.0	0.0000	I
7300.0	TO	7400.0	0.0000	I
7400.0	TO	7500.0	0.0000	I
7500.0	TO	7600.0	0.0020	IX
7600.0	TO	7700.0	0.0035	IXX
7700.0	TO	7800.0	0.0080	IXXXX
7800.0	TO	7900.0	0.0145	IXXXXXXX
7900.0	TO	8000.0	0.0110	IXXXXXX
8000.0	TO	8100.0	0.0295	IXXXXXXXXXXXXXXX
8100.0	TO	8200.0	0.0345	IXXXXXXXXXXXXXXXXX
8200.0	TO	8300.0	0.0505	IXXXXXXXXXXXXXXXXXXXXXXXXX
8300.0	TO	8400.0	0.0570	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8400.0	TO	8500.0	0.0695	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8500.0	TO	8600.0	0.0805	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8600.0	TO	8700.0	0.0770	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8700.0	TO	8800.0	0.0685	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8800.0	TO	8900.0	0.0750	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8900.0	TO	9000.0	0.0640	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9000.0	TO	9100.0	0.0615	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9100.0	TO	9200.0	0.0545	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
9200.0	TO	9300.0	0.0525	IXXXXXXXXXXXXXXXXXXXXXXXXXX
9300.0	TO	9400.0	0.0380	IXXXXXXXXXXXXXXXXXXX
9400.0	TO	9500.0	0.0385	IXXXXXXXXXXXXXXXXXXX
9500.0	TO	9600.0	0.0310	IXXXXXXXXXXXXXXXX
9600.0	TO	9700.0	0.0255	IXXXXXXXXXXXXX
9700.0	TO	9800.0	0.0135	IXXXXXXX
9800.0	TO	9900.0	0.0115	IXXXXXX
9900.0	TO	10000.0	0.0085	IXXXX
10000.0	TO	10100.0	0.0080	IXXXX
10100.0	TO	10200.0	0.0040	IXX
10200.0	TO	10300.0	0.0050	IXXX
10300.0	TO	10400.0	0.0010	IX
GREATER THAN		10500.0	0.0020	IX

FIGURE D.22

```
                                   ICI MODEL
                   SIMULATION NO 1                               2000 RUNS
                                   DISTRIBUTION OF IRR
                              MEAN=      74.67      S.D.=        6.33

      RANGE                PROB

  LESS THAN        55.0  0.0000   I
   55.0   TO       56.0  0.0000   I
   56.0   TO       57.0  0.0000   I
   57.0   TO       58.0  0.0000   I
   58.0   TO       59.0  0.0000   I
   59.0   TO       60.0  0.0000   I
   60.0   TO       61.0  0.0000   I
   61.0   TO       62.0  0.0005   I
   62.0   TO       63.0  0.0015   IX
   63.0   TO       64.0  0.0035   IXXXX
   64.0   TO       65.0  0.0055   IXXX
   65.0   TO       66.0  0.0260   IXXXXXXXXXXXX
   66.0   TO       67.0  0.0370   IXXXXXXXXXXXXXXXXXX
   67.0   TO       68.0  0.0455   IXXXXXXXXXXXXXXXXXXXXXXX
   68.0   TO       69.0  0.0690   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   69.0   TO       70.0  0.0655   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   70.0   TO       71.0  0.0730   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   71.0   TO       72.0  0.0755   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   72.0   TO       73.0  0.0630   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   73.0   TO       74.0  0.0605   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   74.0   TO       75.0  0.0450   IXXXXXXXXXXXXXXXXXXXXXXX
   75.0   TO       76.0  0.0475   IXXXXXXXXXXXXXXXXXXXXXXXX
   76.0   TO       77.0  0.0515   IXXXXXXXXXXXXXXXXXXXXXXXXXX
   77.0   TO       78.0  0.0435   IXXXXXXXXXXXXXXXXXXXXXXXX
   78.0   TO       79.0  0.0425   IXXXXXXXXXXXXXXXXXXXXX
   79.0   TO       80.0  0.0355   IXXXXXXXXXXXXXXXXXX
   80.0   TO       81.0  0.0300   IXXXXXXXXXXXXXXX
   81.0   TO       82.0  0.0430   IXXXXXXXXXXXXXXXXXXXXXX
   82.0   TO       83.0  0.0205   IXXXXXXXXXXXXXXX
   83.0   TO       84.0  0.0250   IXXXXXXXXXXXXX
   84.0   TO       85.0  0.0145   IXXXXXXX
   85.0   TO       86.0  0.0115   IXXXXXX-
   86.0   TO       87.0  0.0155   IXXXXXXXX
   87.0   TO       88.0  0.0035   IXXXX
   88.0   TO       89.0  0.0100   IXXXXX
   89.0   TO       90.0  0.0035   IXX
   90.0   TO       91.0  0.0000   I
   91.0   TO       92.0  0.0000   I
   92.0   TO       93.0  0.0000   I
  GREATER THAN     94.0  0.01[illegible]   IXXXXX
```

FIGURE D.23

ICI MODEL

SIMULATION NO 2 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 74.64 S.D.= 6.18

RANGE			PROB	
LESS THAN		55.0	0.0000	I
55.0	TO	56.0	0.0000	I
56.0	TO	57.0	0.0000	I
57.0	TO	58.0	0.0000	I
58.0	TO	59.0	0.0000	I
59.0	TO	60.0	0.0000	I
60.0	TO	61.0	0.0000	I
61.0	TO	62.0	0.0015	IX
62.0	TO	63.0	0.0030	IXX
63.0	TO	64.0	0.0070	IXXXX
64.0	TO	65.0	0.0160	IXXXXXXX
65.0	TO	66.0	0.0235	IXXXXXXXXXXX
66.0	TO	67.0	0.0350	IXXXXXXXXXXXXXXXXX
67.0	TO	68.0	0.0515	IXXXXXXXXXXXXXXXXXXXXXXXXX
68.0	TO	69.0	0.0660	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
69.0	TO	70.0	0.0575	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
70.0	TO	71.0	0.0665	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
71.0	TO	72.0	0.0810	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.0	TO	73.0	0.0555	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.0	TO	74.0	0.0520	IXXXXXXXXXXXXXXXXXXXXXXXXX
74.0	TO	75.0	0.0445	IXXXXXXXXXXXXXXXXXXXXX
75.0	TO	76.0	0.0425	IXXXXXXXXXXXXXXXXXXXX
76.0	TO	77.0	0.0410	IXXXXXXXXXXXXXXXXXXXX
77.0	TO	78.0	0.0415	IXXXXXXXXXXXXXXXXXXXX
78.0	TO	79.0	0.04[illegible]5	IXXXXXXXXXXXXXXXXXXXXXXX
79.0	TO	80.0	0.0435	IXXXXXXXXXXXXXXXXXXXXX
80.0	TO	81.0	0.0315	IXXXXXXXXXXXXXXX
81.0	TO	82.0	0.0400	IXXXXXXXXXXXXXXXXXXXXXXXX
82.0	TO	83.0	0.0340	IXXXXXXXXXXXXXXXX
83.0	TO	84.0	0.0315	IXXXXXXXXXXXXXXX
84.0	TO	85.0	0.02[illegible]5	IXXXXXXXXX
85.0	TO	86.0	0.0145	IXXXXXXX
86.0	TO	87.0	0.0140	IXXXXXXX
87.0	TO	88.0	0.01[illegible]5	IXXXXX
88.0	TO	89.0	0.00[illegible]5	IXXXX
89.0	TO	90.0	0.00[illegible]5	IX
90.0	TO	91.0	0.0000	I
91.0	TO	92.0	0.0000	I
92.0	TO	93.0	0.0000	I
GREATER THAN		94.0	[illegible]	IXX

FIGURE D.24

```
                                      ICI MODEL
                          SIMULATION NO 3                                 2000 RUNS
                                    DISTRIBUTION OF IRR
                                  MEAN=      74.73       S.D.=        5.62

      RANGE              PROB

  LESS THAN      55.0  0.0000  I
   55.0   TO     56.0  0.0000  I
   56.0   TO     57.0  0.0000  I
   57.0   TO     58.0  0.0000  I
   58.0   TO     59.0  0.0000  I
   59.0   TO     60.0  0.0000  I
   60.0   TO     61.0  0.0000  I
   61.0   TO     62.0  0.0005  I
   62.0   TO     63.0  0.0020  IX
   63.0   TO     64.0  0.0050  IXXX
   64.0   TO     65.0  0.0115  IXXXXXX
   65.0   TO     66.0  0.0225  IXXXXXXXXXXX
   66.0   TO     67.0  0.0305  IXXXXXXXXXXXXXXX
   67.0   TO     68.0  0.0430  IXXXXXXXXXXXXXXXXXXXXXX
   68.0   TO     69.0  0.0660  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   69.0   TO     70.0  0.0580  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   70.0   TO     71.0  0.0595  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   71.0   TO     72.0  0.0650  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   72.0   TO     73.0  0.0620  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   73.0   TO     74.0  0.0570  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   74.0   TO     75.0  0.0480  IXXXXXXXXXXXXXXXXXXXXXXXX
   75.0   TO     76.0  0.0500  IXXXXXXXXXXXXXXXXXXXXXXXXX
   76.0   TO     77.0  0.0580  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   77.0   TO     78.0  0.0510  IXXXXXXXXXXXXXXXXXXXXXXXXXX
   78.0   TO     79.0  0.0600  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   79.0   TO     80.0  0.0460  IXXXXXXXXXXXXXXXXXXXXXXX
   80.0   TO     81.0  0.0520  IXXXXXXXXXXXXXXXXXXXXXXXXXX
   81.0   TO     82.0  0.0405  IXXXXXXXXXXXXXXXXXXXX
   82.0   TO     83.0  0.0370  IXXXXXXXXXXXXXXXXXXX
   83.0   TO     84.0  0.0315  IXXXXXXXXXXXXXXXX
   84.0   TO     85.0  0.0095  IXXXXX
   85.0   TO     86.0  0.0160  IXXXXXXXX
   86.0   TO     87.0  0.0090  IXXXXX
   87.0   TO     88.0  0.0045  IXX
   88.0   TO     89.0  0.0025  IX
   89.0   TO     90.0  0.0010  IX
   90.0   TO     91.0  0.0000  I
   91.0   TO     92.0  0.0000  I
   92.0   TO     93.0  0.0000  I
 GREATER THAN    94.0  0.0010  IX
```

FIGURE D.25

```
                                  ECONOMOS MODEL
                     SIMULATION NO 1                          2000 RUNS
                     DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                                 MEAN=   3055.14      S.D.=   2024.65

        RANGE                 PROB

   LESS THAN       -3000.0   0.0000   I
 -3000.0   TO      -2500.0   0.0000   I
 -2500.0   TO      -2000.0   0.0000   I
 -2000.0   TO      -1500.0   0.0000   I
 -1500.0   TO      -1000.0   0.0005   I
 -1000.0   TO       -500.0   0.0025   IX
  -500.0   TO          0.0   0.0080   IXXXX
     0.0   TO        500.0   0.0940   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   500.0   TO       1000.0   0.0840   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1000.0   TO       1500.0   0.0860   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1500.0   TO       2000.0   0.0620   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0   TO       2500.0   0.0970   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2500.0   TO       3000.0   0.1005   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3000.0   TO       3500.0   0.0790   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3500.0   TO       4000.0   0.0715   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0   TO       4500.0   0.0670   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4500.0   TO       5000.0   0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5000.0   TO       5500.0   0.0530   IXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5500.0   TO       6000.0   0.0510   IXXXXXXXXXXXXXXXXXXXXXXXXXX
  6000.0   TO       6500.0   0.0280   IXXXXXXXXXXXXXX
  6500.0   TO       7000.0   0.0230   IXXXXXXXXXXXX
  7000.0   TO       7500.0   0.0165   IXXXXXXXX
  7500.0   TO       8000.0   0.0100   IXXXXX
  8000.0   TO       8500.0   0.0060   IXXX
  8500.0   TO       9000.0   0.0015   IX
 GREATER THAN       9500.0   0.0025   IX
```

FIGURE D.26

```
                                    ECONOMOS MODEL
                   SIMULATION NO 2                          2000 RUNS
                   DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                               MEAN=    3058.44      S.D.=    1993.77

      RANGE                 PROB.

   LESS THAN     -3000.0   0.0000   I
 -3000.0  TO     -2500.0   0.0000   I
 -2500.0  TO     -2000.0   0.0000   I
 -2000.0  TO     -1500.0   0.0000   I
 -1500.0  TO     -1000.0   0.0000   I
 -1000.0  TO      -500.0   0.0055   IXXX
  -500.0  TO         0.0   0.0155   IXXXXXXXX
     0.0  TO       500.0   0.0825   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   500.0  TO      1000.0   0.0855   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1000.0  TO      1500.0   0.0900   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1500.0  TO      2000.0   0.0690   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0  TO      2500.0   0.0840   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2500.0  TO      3000.0   0.0910   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3000.0  TO      3500.0   0.0560   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3500.0  TO      4000.0   0.0780   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0  TO      4500.0   0.0745   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4500.0  TO      5000.0   0.0695   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5000.0  TO      5500.0   0.0570   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5500.0  TO      6000.0   0.0580   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  6000.0  TO      6500.0   0.0465   IXXXXXXXXXXXXXXXXXXXXXXX
  6500.0  TO      7000.0   0.0165   IXXXXXXXX
  7000.0  TO      7500.0   0.0070   IXXXX
  7500.0  TO      8000.0   0.0065   IXXX
  8000.0  TO      8500.0   0.0035   IXX
  8500.0  TO      9000.0   0.0020   IX
GREATER THAN      9500.0   0.0000   I
```

FIGURE D.27

```
                              ECONOMOS MODEL
              SIMULATION NO 3                       2000 RUNS
              DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                       MEAN=   2977.15      S.D.=   2110.28

      RANGE              PROB

   LESS THAN   -3000.0  0.0000  I
 -3000.0  TO   -2500.0  0.0000  I
 -2500.0  TO   -2000.0  0.0000  I
 -2000.0  TO   -1500.0  0.0000  I
 -1500.0  TO   -1000.0  0.0000  I
 -1000.0  TO    -500.0  0.0010  IX
  -500.0  TO       0.0  0.0030  IXXXX
     0.0  TO     500.0  0.1100  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   500.0  TO    1000.0  0.0965  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1000.0  TO    1500.0  0.0855  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1500.0  TO    2000.0  0.0580  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0  TO    2500.0  0.0975  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2500.0  TO    3000.0  0.0915  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3000.0  TO    3500.0  0.0665  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3500.0  TO    4000.0  0.0610  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0  TO    4500.0  0.0670  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4500.0  TO    5000.0  0.0525  IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5000.0  TO    5500.0  0.0430  IXXXXXXXXXXXXXXXXXXXXXXXX
  5500.0  TO    6000.0  0.0515  IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  6000.0  TO    6500.0  0.0290  IXXXXXXXXXXXXXXXX
  6500.0  TO    7000.0  0.0255  IXXXXXXXXXXXXXX
  7000.0  TO    7500.0  0.0195  IXXXXXXXXXXX
  7500.0  TO    8000.0  0.0115  IXXXXXX
  8000.0  TO    8500.0  0.0065  IXXX
  8500.0  TO    9000.0  0.0025  IX
 GREATER THAN   9500.0  0.0030  IXX
```

SIMULATION NO 1 2000 RUNS

FIGURE D.28

DISTRIBUTION OF IRR
MEAN= 12.26 S.D.= 1.46

RANGE			PROB	
LESS THAN		7.8	0.0000	I
7.8	TO	8.0	0.0000	I
8.0	TO	8.2	0.0000	I
8.2	TO	8.4	0.0000	I
8.4	TO	8.6	0.0000	I
8.6	TO	8.8	0.0000	I
8.8	TO	9.0	0.0005	I
9.0	TO	9.2	0.0010	IX
9.2	TO	9.4	0.0010	IX
9.4	TO	9.6	0.0005	I
9.6	TO	9.8	0.0050	IXXX
9.8	TO	10.0	0.0030	IXX
10.0	TO	10.2	0.0270	IXXXXXXXXXXX
10.2	TO	10.4	0.0875	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
10.4	TO	10.6	0.0315	IXXXXXXXXXXXXXXXX
10.6	TO	10.8	0.0445	IXXXXXXXXXXXXXXXXXXXXXX
10.8	TO	11.0	0.0415	IXXXXXXXXXXXXXXXXXXXXX
11.0	TO	11.2	0.0765	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.2	TO	11.4	0.0245	IXXXXXXXXXXXX
11.4	TO	11.6	0.0140	IXXXXXXX
11.6	TO	11.8	0.0570	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.8	TO	12.0	0.0535	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.0	TO	12.2	0.0545	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.2	TO	12.4	0.0405	IXXXXXXXXXXXXXXXXXXXX
12.4	TO	12.6	0.0900	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.6	TO	12.8	0.0345	IXXXXXXXXXXXXXXXXX
12.8	TO	13.0	0.0230	IXXXXXXXXXXXX
13.0	TO	13.2	0.0125	IXXXXXX
13.2	TO	13.4	0.0375	IXXXXXXXXXXXXXXXXXXX
13.4	TO	13.6	0.0350	IXXXXXXXXXXXXXXXXXX
13.6	TO	13.8	0.0245	IXXXXXXXXXXXX
13.8	TO	14.0	0.0550	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
14.0	TO	14.2	0.0280	IXXXXXXXXXXXXXX
14.2	TO	14.4	0.0150	IXXXXXXXX
14.4	TO	14.6	0.0140	IXXXXXXX
14.6	TO	14.8	0.0305	IXXXXXXXXXXXXXXX
14.8	TO	15.0	0.00[illegible]0	IXXXX
15.0	TO	15.2	0.00[illegible]0	IXXXXX
15.2	TO	15.4	0.0105	IXXXXXXXXXX
15.4	TO	15.6	0.0060	IXXX
15.6	TO	15.8	0.0020	IX
[illegible]	TO	[illegible]	[illegible]	IX

FIGURE D.29

SIMULATION NO 2 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 12.28 S.D.= 1.45

RANGE			PROB	
LESS THAN		7.8	0.0000	I
7.8	TO	8.0	0.0000	I
8.0	TO	8.2	0.0000	I
8.2	TO	8.4	0.0000	I
8.4	TO	8.6	0.0000	I
8.6	TO	8.8	0.0000	I
8.8	TO	9.0	0.0000	I
9.0	TO	9.2	0.0005	I
9.2	TO	9.4	0.0015	IX
9.4	TO	9.6	0.0035	IXX
9.6	TO	9.8	0.0090	IXXXXX
9.8	TO	10.0	0.0065	IXXX
10.0	TO	10.2	0.0255	IXXXXXXXXXXXXX
10.2	TO	10.4	0.0715	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
10.4	TO	10.6	0.0340	IXXXXXXXXXXXXXXXXX
10.6	TO	10.8	0.0430	IXXXXXXXXXXXXXXXXXXXXXX
10.8	TO	11.0	0.0395	IXXXXXXXXXXXXXXXXXXXX
11.0	TO	11.2	0.0800	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.2	TO	11.4	0.0305	IXXXXXXXXXXXXXXX
11.4	TO	11.6	0.0195	IXXXXXXXXXX
11.6	TO	11.8	0.0535	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.8	TO	12.0	0.0360	IXXXXXXXXXXXXXXXXXX
12.0	TO	12.2	0.0435	IXXXXXXXXXXXXXXXXXXXXXX
12.2	TO	12.4	0.0375	IXXXXXXXXXXXXXXXXXXX
12.4	TO	12.6	0.0715	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.6	TO	12.8	0.0405	IXXXXXXXXXXXXXXXXXXXX
12.8	TO	13.0	0.0185	IXXXXXXXXX
13.0	TO	13.2	0.0140	IXXXXXXX
13.2	TO	13.4	0.0460	IXXXXXXXXXXXXXXXXXXXXXXX
13.4	TO	13.6	0.0370	IXXXXXXXXXXXXXXXXXXX
13.6	TO	13.8	0.0310	IXXXXXXXXXXXXXXXX
13.8	TO	14.0	0.0720	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
14.0	TO	14.2	0.0405	IXXXXXXXXXXXXXXXXXXXX
14.2	TO	14.4	0.0360	IXXXXXXXXXXXXXXXXXX
14.4	TO	14.6	0.0190	IXXXXXXXXXX
14.6	TO	14.8	0.0145	IXXXXXXX
14.8	TO	15.0	0.0035	IXX
15.0	TO	15.2	0.0045	IXX
15.2	TO	15.4	0.0105	IXXXXX
15.4	TO	15.6	0.0040	IXX
15.6	TO	15.8	0.0015	IX
[illegible]	TO	[illegible]	[illegible]	IX

SIMULATION NO 3 2000 RUNS

FIGURE D.30

DISTRIBUTION OF IRR
MEAN= 12.20 S.D.= 1.52

RANGE			PROB	
LESS THAN		7.8	0.0000	I
7.8	TO	8.0	0.0000	I
8.0	TO	8.2	0.0000	I
8.2	TO	8.4	0.0000	I
8.4	TO	8.6	0.0000	I
8.6	TO	8.8	0.0000	I
8.8	TO	9.0	0.0000	I
9.0	TO	9.2	0.0000	I
9.2	TO	9.4	0.0000	I
9.4	TO	9.6	0.0010	IX
9.6	TO	9.8	0.0050	IXXX
9.8	TO	10.0	0.0030	IXX
10.0	TO	10.2	0.0245	IXXXXXXXXXXXX
10.2	TO	10.4	0.1005	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
10.4	TO	10.6	0.0430	IXXXXXXXXXXXXXXXXXXXXXX
10.6	TO	10.8	0.0465	IXXXXXXXXXXXXXXXXXXXXXXX
10.8	TO	11.0	0.0360	IXXXXXXXXXXXXXXXXXX
11.0	TO	11.2	0.0780	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.2	TO	11.4	0.0225	IXXXXXXXXXXX
11.4	TO	11.6	0.0170	IXXXXXXXXX
11.6	TO	11.8	0.0605	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.8	TO	12.0	0.0540	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.0	TO	12.2	0.0545	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.2	TO	12.4	0.0255	IXXXXXXXXXXXXX
12.4	TO	12.6	0.0820	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.6	TO	12.8	0.0275	IXXXXXXXXXXXXXX
12.8	TO	13.0	0.0200	IXXXXXXXXXX
13.0	TO	13.2	0.0120	IXXXXXX
13.2	TO	13.4	0.0365	IXXXXXXXXXXXXXXXXXX
13.4	TO	13.6	0.0310	IXXXXXXXXXXXXXXXX
13.6	TO	13.8	0.0265	IXXXXXXXXXXXXX
13.8	TO	14.0	0.0460	IXXXXXXXXXXXXXXXXXXXXXXX
14.0	TO	14.2	0.0245	IXXXXXXXXXXXX
14.2	TO	14.4	0.0150	IXXXXXXXX
14.4	TO	14.6	0.0140	IXXXXXXX
14.6	TO	14.8	0.0360	IXXXXXXXXXXXXXXXXXX
14.8	TO	15.0	0.0080	IXXXX
15.0	TO	15.2	0.0110	IXXXXXX
15.2	TO	15.4	0.0225	IXXXXXXXXXXX
15.4	TO	15.6	0.0065	IXXX
15.6	TO	15.8	0.0030	IXX
15.8	TO	16.0	0.0010	IX

APPENDIX E

FURTHER DISTRIBUTIONS OBTAINED FOR NPV AND IRR USING THE FIVE CASE STUDIES

Figures E.1 - E.3 show the distributions obtained for IRR in case study D when the variable 'start year' was held fixed at its best estimate. Chi-square statistics were calculated to test the normality of the distributions in the same way as in section 4.3. Their values were as follows:

Simulation No. 1 (See figure E.1: all variables except 'start year' have triangular distributions). Chi-square statistic = 35 (Degrees of freedom = 29).

Simulation No. 2 (See figure E.2: all variables except 'start year' have uniform distributions). Chi-square statistic = 67 (Degrees of freedom = 29).

Simulation No. 3 (See figure E.3: all variables except 'start year' have ◺ shaped distributions). Chi-square statistic = 57 (Degrees of freedom = 29).

The results of other tests of normality are shown in table E.1.

Table E.1 Comparison of tails of distribution of IRR in case D with normal distribution when 'start year' is constant.

	Percentage of the Distribution which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Simulation No. 1	2.10	16.75	16.15	2.20
Simulation No. 2	1.80	17.35	16.30	2.50
Simulation No. 3	2.25	16.95	15.95	1.90
Normal Dist.	2.28	15.87	15.87	2.28

These results support the statement (see section 4.4) that the non-normality of the distribution of IRR in case D is due to the uncertainty in the non-linear variable 'start year'.

Figures E.4 - E.8 show the distributions obtained for NPV in case studies A - E when V-shaped distributions were assumed for the variables (see section 4.8). The same tests for normality as in the main part of the research in chapter 4 were made. The results are shown in tables E.2 and E.3

Table E.2 Values of chi-square statistic as a test for normality when V-shaped input distributions were assumed (Degrees of freedom = 29).

	Case Study				
	A	B	C	D	E
Chi-square Statistic	181	116	2424	67	868

Table E.3 Comparison of tails of distributions of NPV with tails of normal distribution when V-shaped input distributions were assumed

	Percentage of the Distribution which is			
	Greater than 2 S.D.s below mean	Greater than 1 S.D. below mean	Greater than 1 S.D. above mean	Greater than 2 S.D.s above mean
Case A	1.25	17.25	15.05	4.00
Case B	1.15	16.05	15.55	4.30
Case C	0.00	12.50	20.20	3.20
Case D	1.45	17.40	17.45	2.20
Case E	0.00	22.60	21.60	0.00
Normal Dist.	2.28	15.87	15.87	2.28

The distributions obtained for NPV in case D assuming V-shaped distributions passes the test of approximate normality which is given in section 4.4. The distributions obtained for NPV in cases A and B both fail the test because

Prob. (Value of variable is 2 S.D.s above mean)

is too high. No doubt the explanation for these results lies partly in the fact that case D has more uncertain variables than either case A or case B.

The shape of the distribution obtained for NPV in case E (see figure E.8) has been markedly influenced by the shapes of the input distributions.

FIGURE E.1

EXTRA RUN CARRIED OUT USING
CASE STUDY D WITH 'START YEAR'
HELD FIXED AT ITS BEST ESTIMATE.

ICI MODEL

SIMULATION NO 1 — 2000 RUNS

DISTRIBUTION OF IRR

MEAN= 73.51 S.D.= 2.44

RANGE			PROB	
LESS THAN		66.0	0.0000	I
66.0	TO	66.5	0.0010	IX
66.5	TO	67.0	0.0030	IXX
67.0	TO	67.5	0.0010	IX
67.5	TO	68.0	0.0060	IXXX
68.0	TO	68.5	0.0055	IXXX
68.5	TO	69.0	0.0135	IXXXXXXXX
69.0	TO	69.5	0.0145	IXXXXXXX
69.5	TO	70.0	0.0220	IXXXXXXXXXX
70.0	TO	70.5	0.0310	IXXXXXXXXXXXXXX
70.5	TO	71.0	0.0470	IXXXXXXXXXXXXXXXXXXXX
71.0	TO	71.5	0.0745	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
71.5	TO	72.0	0.0510	IXXXXXXXXXXXXXXXXXXXXXX
72.0	TO	72.5	0.0720	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.5	TO	73.0	0.0660	IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.0	TO	73.5	0.0745	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.5	TO	74.0	0.0905	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.0	TO	74.5	0.0760	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.5	TO	75.0	0.0645	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
75.0	TO	75.5	0.0595	IXXXXXXXXXXXXXXXXXXXXXXXXX
75.5	TO	76.0	0.0575	IXXXXXXXXXXXXXXXXXXXXXXXX
76.0	TO	76.5	0.0575	IXXXXXXXXXXXXXXXXXXXXXXXX
76.5	TO	77.0	0.0320	IXXXXXXXXXXXXXX
77.0	TO	77.5	0.0250	IXXXXXXXXXXX
77.5	TO	78.0	0.0125	IXXXXXX
78.0	TO	78.5	0.0125	IXXXXXX
78.5	TO	79.0	0.0115	IXXXXXX
79.0	TO	79.5	0.0035	IXX
79.5	TO	80.0	0.0030	IXX
80.0	TO	80.5	0.0020	IX-
GREATER THAN		81.0	0.0020	IX

FIGURE E.2
EXTRA RUN CARRIED OUT USING
CASE STUDY D WITH 'START YEAR'
HELD FIXED AT ITS BEST ESTIMATE.

ICI MODEL
SIMULATION NO 2 2000 RUNS
DISTRIBUTION OF IRR
MEAN= 73.48 S.D.= 2.40

RANGE			PROB	
LESS THAN		66.0	0.0005	I
66.0	TO	66.5	0.0005	I
66.5	TO	67.0	0.0015	IX
67.0	TO	67.5	0.0010	IX
67.5	TO	68.0	0.0030	IXX
68.0	TO	68.5	0.0060	IXXXX
68.5	TO	69.0	0.0165	IXXXXXXXX
69.0	TO	69.5	0.0170	IXXXXXXXXX
69.5	TO	70.0	0.0225	IXXXXXXXXXXX
70.0	TO	70.5	0.0355	IXXXXXXXXXXXXXXXXXX
70.5	TO	71.0	0.0515	IXXXXXXXXXXXXXXXXXXXXXXXXXX
71.0	TO	71.5	0.0715	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
71.5	TO	72.0	0.0605	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.0	TO	72.5	0.0650	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.5	TO	73.0	0.0615	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.0	TO	73.5	0.0740	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.5	TO	74.0	0.1030	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.0	TO	74.5	0.0665	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.5	TO	75.0	0.0735	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
75.0	TO	75.5	0.0555	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
75.5	TO	76.0	0.0465	IXXXXXXXXXXXXXXXXXXXXXXX
76.0	TO	76.5	0.0505	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
76.5	TO	77.0	0.0310	IXXXXXXXXXXXXXXXX
77.0	TO	77.5	0.0230	IXXXXXXXXXXXXXX
77.5	TO	78.0	0.0150	IXXXXXXXX
78.0	TO	78.5	0.0065	IXXXXX
78.5	TO	79.0	0.0125	IXXXXXX
79.0	TO	79.5	0.0035	IXX
79.5	TO	80.0	0.0025	IX
80.0	TO	80.5	0.0000	I
GREATER THAN		81.0	0.0015	IX

FIGURE E.3
EXTRA RUN CARRIED OUT USING
CASE STUDY D WITH 'START YEAR'
HELD FIXED AT ITS BEST ESTIMATE.

ICI MODEL
SIMULATION NO 3 2000 RUNS
DISTRIBUTION OF IRR
MEAN= 73.45 S.D.= 2.38

RANGE			PROB	
LESS THAN		66.0	0.0005	I
66.0	TO	66.5	0.0000	I
66.5	TO	67.0	0.0010	IX
67.0	TO	67.5	0.0025	IX
67.5	TO	68.0	0.0040	IXX
68.0	TO	68.5	0.0045	IXX
68.5	TO	69.0	0.0165	IXXXXXXX
69.0	TO	69.5	0.0145	IXXXXXX
69.5	TO	70.0	0.0200	IXXXXXXXXXXXXX
70.0	TO	70.5	0.0205	IXXXXXXXXXXXXXX
70.5	TO	71.0	0.0545	IXXXXXXXXXXXXXXXXXXXXXXXXX
71.0	TO	71.5	0.0610	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
71.5	TO	72.0	0.0735	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.0	TO	72.5	0.0660	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
72.5	TO	73.0	0.0630	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.0	TO	73.5	0.0725	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
73.5	TO	74.0	0.1060	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.0	TO	74.5	0.0725	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
74.5	TO	75.0	0.0700	IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
75.0	TO	75.5	0.0600	IXXXXXXXXXXXXXXXXXXXXXXXXXXX
75.5	TO	76.0	0.0555	IXXXXXXXXXXXXXXXXXXXXXXXXX
76.0	TO	76.5	0.0520	IXXXXXXXXXXXXXXXXXXXXXXXX
76.5	TO	77.0	0.0260	IXXXXXXXXXXXX
77.0	TO	77.5	0.0195	IXXXXXXXXX
77.5	TO	78.0	0.0165	IXXXXXXX
78.0	TO	78.5	0.0135	IXXXXXX
78.5	TO	79.0	0.0080	IXXXX
79.0	TO	79.5	0.0030	IXX
79.5	TO	80.0	0.0000	I
80.0	TO	80.5	0.0040	IXX
GREATER THAN		81.0	0.0020	IX

FIGURE E.4

DISTRIBUTION OF NPV
ASSUMING V-SHAPED DISTRIBUTIONS
IN CASE A.

```
                                      HERTZ  MODEL
                          SIMULATION NO 4                        2000 RUNS
                          DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                                    MEAN=  -2692.11      S.D.=   9894.99

         RANGE               PROB

   LESS THAN  -32000.0     0.0000   I
 -32000.0  TO  -30000.0    0.0000   I
 -30000.0  TO  -28000.0    0.0010   IX
 -28000.0  TO  -26000.0    0.0035   IXX
 -26000.0  TO  -24000.0    0.0020   IX
 -24000.0  TO  -22000.0    0.0080   IXXXX
 -22000.0  TO  -20000.0    0.0105   IXXXXX
 -20000.0  TO  -18000.0    0.0280   IXXXXXXXXXXXXXX
 -18000.0  TO  -16000.0    0.0295   IXXXXXXXXXXXXXXX
 -16000.0  TO  -14000.0    0.0495   IXXXXXXXXXXXXXXXXXXXXXXXXX
 -14000.0  TO  -12000.0    0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -12000.0  TO  -10000.0    0.0405   IXXXXXXXXXXXXXXXXXXXX
 -10000.0  TO   -8000.0    0.0445   IXXXXXXXXXXXXXXXXXXXXXX
  -8000.0  TO   -6000.0    0.0765   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -6000.0  TO   -4000.0    0.1085   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -4000.0  TO   -2000.0    0.1065   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -2000.0  TO       0.0    0.0915   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      0.0  TO    2000.0    0.0720   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   2000.0  TO    4000.0    0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   4000.0  TO    6000.0    0.0420   IXXXXXXXXXXXXXXXXXXXXX
   6000.0  TO    8000.0    0.0370   IXXXXXXXXXXXXXXXXXXX
   8000.0  TO   10000.0    0.0365   IXXXXXXXXXXXXXXXXXX
  10000.0  TO   12000.0    0.0275   IXXXXXXXXXXXXXX
  12000.0  TO   14000.0    0.0175   IXXXXXXXXX
  14000.0  TO   16000.0    0.0120   IXXXXXX
  16000.0  TO   18000.0    0.0115   IXXXXXX
  18000.0  TO   20000.0    0.0095   IXXXXX
  20000.0  TO   22000.0    0.0065   IXXX
  22000.0  TO   24000.0    0.0055   IXXX
  24000.0  TO   26000.0    0.0055   IXXX
  GREATER THAN  26000.0    0.0060   IXXX
```

FIGURE E.5

DISTRIBUTION OF NPV
ASSUMING V-SHAPED
DISTRIBUTIONS IN CASE B

```
                              KRYZANOWSKI ET AL  MODEL
                   SIMULATION NO 4                2000 RUNS
                   DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                              MEAN=   1718.80      S.D.=   5378.43

       RANGE              PROB

   LESS THAN  -14000.0   0.0010   IX
-14000.0  TO  -13000.0   0.0005   I
-13000.0  TO  -12000.0   0.0010   IX
-12000.0  TO  -11000.0   0.0040   IXX
-11000.0  TO  -10000.0   0.0035   IXX
-10000.0  TO   -9000.0   0.0015   IX
 -9000.0  TO   -8000.0   0.0070   IXXXX
 -8000.0  TO   -7000.0   0.0130   IXXXXXXX
 -7000.0  TO   -6000.0   0.0225   IXXXXXXXXXXX
 -6000.0  TO   -5000.0   0.0355   IXXXXXXXXXXXXXXXXXX
 -5000.0  TO   -4000.0   0.0475   IXXXXXXXXXXXXXXXXXXXXXXXX
 -4000.0  TO   -3000.0   0.0590   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -3000.0  TO   -2000.0   0.0600   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -2000.0  TO   -1000.0   0.0635   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 -1000.0  TO       0.0   0.0800   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
     0.0  TO    1000.0   0.0865   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  1000.0  TO    2000.0   0.0705   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2000.0  TO    3000.0   0.0665   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3000.0  TO    4000.0   0.0695   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4000.0  TO    5000.0   0.0535   IXXXXXXXXXXXXXXXXXXXXXXXXXXX
  5000.0  TO    6000.0   0.0540   IXXXXXXXXXXXXXXXXXXXXXXXXXXX
  6000.0  TO    7000.0   0.0405   IXXXXXXXXXXXXXXXXXXXX
  7000.0  TO    8000.0   0.0350   IXXXXXXXXXXXXXXXXXX
  8000.0  TO    9000.0   0.0265   IXXXXXXXXXXXXX
  9000.0  TO   10000.0   0.0250   IXXXXXXXXXXXXX
 10000.0  TO   11000.0   0.0145   IXXXXXXX
 11000.0  TO   12000.0   0.0095   IXXXXX
 12000.0  TO   13000.0   0.0140   IXXXXXXX
 13000.0  TO   14000.0   0.0110   IXXXXXX
 14000.0  TO   15000.0   0.0090   IXXXXX
 15000.0  TO   16000.0   0.0075   IXXXX
 16000.0  TO   17000.0   0.0050   IXXX
 GREATER THAN  18000.0   0.0025   IX
```

FIGURE E.6

DISTRIBUTION OF NPV
ASSUMING V-SHAPED
DISTRIBUTIONS IN
CASE C.

```
                              INTERCHEM MODEL
               SIMULATION NO 4                          2000 RUNS
               DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                         MEAN=      71.14     S.D.=      92.07

      RANGE              PROB

 LESS THAN     -200.0   0.0000   I
 -200.0   TO   -180.0   0.0000   I
 -180.0   TO   -160.0   0.0000   I
 -160.0   TO   -140.0   0.0000   I
 -140.0   TO   -120.0   0.0000   I
 -120.0   TO   -100.0   0.0000   I
 -100.0   TO    -80.0   0.0000   I
  -80.0   TO    -60.0   0.0000   I
  -60.0   TO    -40.0   0.0005   I
  -40.0   TO    -20.0   0.1335   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  -20.0   TO      0.0   0.2855   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    0.0   TO     20.0   0.0215   IXXXXXXXXXXX
   20.0   TO     40.0   0.0355   IXXXXXXXXXXXXXXXXXX
   40.0   TO     60.0   0.0380   IXXXXXXXXXXXXXXXXXXX
   60.0   TO     80.0   0.0585   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   80.0   TO    100.0   0.0555   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  100.0   TO    120.0   0.0665   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  120.0   TO    140.0   0.0505   IXXXXXXXXXXXXXXXXXXXXXXXXX
  140.0   TO    160.0   0.0445   IXXXXXXXXXXXXXXXXXXXXXX
  160.0   TO    180.0   0.0490   IXXXXXXXXXXXXXXXXXXXXXXXXX
  180.0   TO    200.0   0.0520   IXXXXXXXXXXXXXXXXXXXXXXXXXX
  200.0   TO    220.0   0.0325   IXXXXXXXXXXXXXXXX
  220.0   TO    240.0   0.0270   IXXXXXXXXXXXXXX
  240.0   TO    260.0   0.0225   IXXXXXXXXXXX
  260.0   TO    280.0   0.0115   IXXXXXX
  280.0   TO    300.0   0.0080   IXXXX
  300.0   TO    320.0   0.0045   IXX
  320.0   TO    340.0   0.0025   IX
GREATER THAN    360.0   0.0005   I
```

FIGURE E.7

DISTRIBUTION OF NPV
ASSUMING V-SHAPED
DISTRIBUTIONS IN CASE D

```
                              ICI MODEL
              SIMULATION NO 4                          2000 RUNS
              DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                          MEAN=   8827.03      S.D.=    529.73

        RANGE              PROB

  LESS THAN     7200.0   0.0000   I
  7200.0  TO    7300.0   0.0000   I
  7300.0  TO    7400.0   0.0000   I
  7400.0  TO    7500.0   0.0010   IX
  7500.0  TO    7600.0   0.0015   IX
  7600.0  TO    7700.0   0.0030   IXX
  7700.0  TO    7800.0   0.0115   IXXXXXX
  7800.0  TO    7900.0   0.0150   IXXXXXXXX
  7900.0  TO    8000.0   0.0205   IXXXXXXXXXXX
  8000.0  TO    8100.0   0.0295   IXXXXXXXXXXXXXXX
  8100.0  TO    8200.0   0.0365   IXXXXXXXXXXXXXXXXXXX
  8200.0  TO    8300.0   0.0560   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8300.0  TO    8400.0   0.0515   IXXXXXXXXXXXXXXXXXXXXXXXXXX
  8400.0  TO    8500.0   0.0770   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8500.0  TO    8600.0   0.0630   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8600.0  TO    8700.0   0.0670   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8700.0  TO    8800.0   0.0645   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8800.0  TO    8900.0   0.0600   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  8900.0  TO    9000.0   0.0595   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9000.0  TO    9100.0   0.0715   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9100.0  TO    9200.0   0.0590   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  9200.0  TO    9300.0   0.0520   IXXXXXXXXXXXXXXXXXXXXXXXXXX
  9300.0  TO    9400.0   0.0450   IXXXXXXXXXXXXXXXXXXXXXXX
  9400.0  TO    9500.0   0.0375   IXXXXXXXXXXXXXXXXXXX
  9500.0  TO    9600.0   0.0335   IXXXXXXXXXXXXXXXXX
  9600.0  TO    9700.0   0.0310   IXXXXXXXXXXXXXXXX
  9700.0  TO    9800.0   0.0190   IXXXXXXXXXX
  9800.0  TO    9900.0   0.0140   IXXXXXXX
  9900.0  TO   10000.0   0.0110   IXXXXXX
 10000.0  TO   10100.0   0.0060   IXXX
 10100.0  TO   10200.0   0.0015   IX
 10200.0  TO   10300.0   0.0010   IX
 10300.0  TO   10400.0   0.0005   I
 GREATER THAN  10500.0   0.0005   I
```

FIGURE E.8

DISTRIBUTION OF NPV
ASSUMING V-SHAPED
DISTRIBUTIONS IN CASE E

```
                          ECONOMOS MODEL
        SIMULATION NO 4                          2000 RUNS
        DISTRIBUTION OF NPV WITH DISCOUNT RATE = 10.00 PERCENT
                   MEAN=   3026.78      S.D.=   2036.65

     RANGE              PROB

  LESS THAN   -3000.0  0.0000   I
-3000.0  TO   -2500.0  0.0000   I
-2500.0  TO   -2000.0  0.0000   I
-2000.0  TO   -1500.0  0.0000   I
-1500.0  TO   -1000.0  0.0000   I
-1000.0  TO    -500.0  0.0045   IXX
 -500.0  TO       0.0  0.0095   IXXXXX
    0.0  TO     500.0  0.1145   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  500.0  TO    1000.0  0.0985   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1000.0  TO    1500.0  0.0730   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1500.0  TO    2000.0  0.0870   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 2000.0  TO    2500.0  0.0910   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 2500.0  TO    3000.0  0.0485   IXXXXXXXXXXXXXXXXXXXXXXXX
 3000.0  TO    3500.0  0.0260   IXXXXXXXXXXXXX
 3500.0  TO    4000.0  0.0550   IXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 4000.0  TO    4500.0  0.0945   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 4500.0  TO    5000.0  0.0745   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 5000.0  TO    5500.0  0.0590   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 5500.0  TO    6000.0  0.0905   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 6000.0  TO    6500.0  0.0630   IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 6500.0  TO    7000.0  0.0110   IXXXXXX
 7000.0  TO    7500.0  0.0000   I
 7500.0  TO    8000.0  0.0000   I
 8000.0  TO    8500.0  0.0000   I
 8500.0  TO    9000.0  0.0000   I
GREATER THAN   9500.0  0.0000   I
```

APPENDIX F

FULL RESULTS CONCERNING THE ACCURACY OF DIFFERENT METHODS FOR ASSESSING SUBJECTIVE PROBABILITY DISTRIBUTIONS

The computer output at the end of this appendix shows the full results upon which tables 5.1 - 5.8 are based. In order to understand this output note that:

(i) N1 and N2 refer to the parameters of the beta distributions which were considered. In the notation of section 5.3, N1=a and N2=b).

(ii) The errors which are shown are actual errors in situations where the mean is being estimated and proportional errors in situations where the standard deviation is being estimated.

Tables F.1 - F.18 provide the following summary information:

μ_1 : Mean error in estimates

σ_1 : Standard deviation of errors in the estimates

ρ : Coefficient of correlation between the absolute sizes of the errors in the estimates and the skewness of the distributions (See section 5.5 for a definition of skewness).

a & b: Parameters in the best fit relationship. Absolute size of error = a + b x skewness.

σ_2 : Standard error of estimate of the error which is obtained using the best fit relationship.

As there are 55 observations values of ρ greater than 0.27 are significant at the 0.05 level.

Table F.1 Miscellaneous extra information concerning the estimation of the mean using 3 perfectly accurate assessments (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.1	0.1	.84	0.0	1.0	0.1
Procedure 2	0.5	0.4	.62	0.2	2.3	0.3
Procedure 3	2.4	1.9	.99	0.0	19.0	0.2
Procedure 4	0.7	0.5	.98	0.1	4.5	0.1
Procedure 5	0.9	0.9	.95	-0.2	8.8	0.3
Procedure 6	4.5	3.8	.99	-0.1	37.2	0.6

Table F.2 Miscellaneous extra information concerning the estimation of the S.D. using 3 perfectly accurate assessments (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	23.9	4.6	.18	22.8	8.2	4.6
Procedure 2	14.8	7.3	.05	14.3	3.9	7.3
Procedure 3	46.6	7.7	.32	43.6	24.4	7.3
Procedure 4	20.5	6.6	.35	17.7	22.8	6.2
Procedure 5	19.3	2.0	.01	19.3	0.3	2.0
Procedure 6	38.4	9.7	.38	33.9	36.5	9.1

Table F.3 Miscellaneous extra information concerning the estimation of the mean using 7 perfectly accurate assessments (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.0	0.0	0.74	0.0	0.1	0.0
Procedure 2	0.0	0.0	0.00	0.0	0.0	0.0
Procedure 3	1.1	0.9	0.99	0.0	8.9	0.1
Procedure 4	0.3	0.2	0.97	0.0	2.2	0.1
Procedure 5	0.4	0.3	0.99	0.0	2.9	0.0
Procedure 6	2.6	2.3	0.98	0.1	21.9	0.5

Table F.4 Miscellaneous extra information concerning the estimation of the S.D. using 7 perfectly accurate assessments (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	8.0	1.9	0.21	7.5	4.0	1.9
Procedure 2	2.1	1.2	-0.34	2.5	-3.8	1.1
Procedure 3	34.0	7.5	0.36	30.7	26.4	7.1
Procedure 4	12.2	4.5	0.39	10.1	17.1	4.2
Procedure 5	8.4	2.1	0.39	7.4	7.9	1.9
Procedure 6	30.5	9.8	0.47	24.9	45.1	8.8

Table F.5 Miscellaneous extra information concerning the estimation of the mean using 15 perfectly accurate assessments (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.0	0.0	0.65	0.0	0.0	0.0
Procedure 2	0.0	0.0	-0.05	0.0	0.0	0.0
Procedure 3	0.5	0.4	0.99	0.0	4.2	0.1
Procedure 4	0.1	0.1	0.97	0.0	1.0	0.0
Procedure 5	0.1	0.1	0.98	0.0	0.9	0.0
Procedure 6	1.3	1.2	0.96	-0.1	11.7	0.3

Table F.6 Miscellaneous extra information concerning the estimation of the S.D. using 15 perfectly accurate assessments (Errors are measured as percentage errors)

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	2.2	0.6	0.18	2.0	1.0	0.6
Procedure 2	0.2	0.1	-0.15	0.2	-0.1	0.1
Procedure 3	22.1	6.2	0.38	19.3	23.3	5.8
Procedure 4	6.6	2.8	0.40	5.3	10.9	2.6
Procedure 5	2.3	0.7	0.43	2.0	2.8	0.6
Procedure 6	21.4	8.7	0.52	15.9	44.4	7.4

Table F.7 Miscellaneous extra information concerning the estimation of the mean using 3 assessments; accuracy parameter = 0.2 (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	1.3	1.2	0.47	0.6	5.7	1.1
Procedure 2	1.2	1.2	0.47	0.5	5.5	1.0
Procedure 3	3.5	3.0	0.63	1.2	18.6	2.4
Procedure 4	3.1	2.4	0.36	2.0	8.7	2.3
Procedure 5	1.5	1.0	0.39	1.1	3.7	0.9
Procedure 6	5.3	4.7	0.95	-0.2	44.3	1.5

Table F.8 Miscellaneous extra information concerning the estimation of the S.D. using 3 assessments; accuracy parameter = 0.2 (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	18.3	8.4	0.09	17.4	7.7	8.4
Procedure 2	11.3	7.0	-0.22	13.2	-15.0	6.9
Procedure 3	47.7	6.2	0.33	45.2	20.3	5.9
Procedure 4	24.8	5.6	0.21	23.4	11.4	5.5
Procedure 5	17.1	8.7	-0.36	20.9	-30.6	8.2
Procedure 6	38.4	10.0	0.31	34.6	30.7	9.6

Table F.9 Miscellaneous extra information concerning the estimation of the mean using 7 assessments; accuracy parameter = 0.2 (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	1.1	1.1	0.34	0.7	3.6	1.0
Procedure 2	1.2	1.1	0.33	0.7	3.6	1.1
Procedure 3	2.8	2.4	0.59	1.0	14.1	2.0
Procedure 4	2.6	2.1	0.40	1.6	8.4	2.0
Procedure 5	0.7	0.5	0.01	0.7	0.0	0.5
Procedure 6	4.0	3.1	0.78	1.0	23.8	2.0

Table F.10 Miscellaneous extra information concerning the estimation of the S.D. using 7 assessments; accuracy parameter = 0.2 (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	12.9	9.3	0,08	12.1	7.1	9.3
Procedure 2	20.7	16.6	0.20	16.7	32.3	16.4
Procedure 3	34.1	7.3	0.29	31.5	21.1	7.1
Procedure 4	12.4	6.8	0.13	11.3	9.0	6.8
Procedure 5	5.2	4.0	-0.05	5.5	-2.1	4.0
Procedure 6	31.0	9.5	0.38	26.7	35.5	8.8

Table F.11 Miscellaneous extra information concerning the estimation of the mean using 3 assessments; accuracy parameter = 0.1 (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.8	0.8	0.33	0.5	2.6	0.8
Procedure 2	0.9	0.9	0.41	0.5	3.4	0.8
Procedure 3	3.0	2.4	0.82	0.6	19.3	1.4
Procedure 4	1.7	1.3	0.41	1.0	5.4	1.2
Procedure 5	0.9	1.1	0.73	0.0	7.7	0.7
Procedure 6	4.5	4.1	0.95	0.2	38.2	1.3

Table F.12 Miscellaneous extra information concerning the estimation of the S.D. using 3 assessments; accuracy parameter = 0.1 (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	21.0	6.6	0.26	19.0	16.7	6.4
Procedure 2	10.7	7.2	0.00	10.7	-0.1	7.2
Procedure 3	47.2	8.7	0.30	44.0	25.3	8.4
Procedure 4	24.9	9.9	0.22	22.2	21.5	9.7
Procedure 5	17.4	5.9	-0.20	18.9	-11.5	5.8
Procedure 6	38.6	9.6	0.38	34.2	35.5	9.0

Table F.13 Miscellaneous extra information concerning the estimation of the mean using 7 assessments; accuracy parameter = 0.1 (Errors are measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.8	0.9	0.20	0.6	1.7	0.8
Procedure 2	0.9	0.8	0.19	0.7	1.6	0.8
Procedure 3	1.6	1.5	0.45	0.8	6.5	1.3
Procedure 4	1.3	1.2	0.29	0.9	3.3	1.2
Procedure 5	0.4	0.4	0.45	0.2	1.7	0.4
Procedure 6	3.2	2.9	0.91	0.0	25.5	1.2

Table F.14 Miscellaneous extra information concerning the estimation of the S.D. using 7 assessments; accuracy parameter = 0.1 (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	6.4	4.1	0.05	6.1	2.2	4.1
Procedure 2	9.2	6.9	0.12	8.3	7.9	6.9
Procedure 3	37.1	5.7	0.37	34.5	21.0	5.4
Procedure 4	21.3	4.1	0.15	20.6	5.9	4.1
Procedure 5	5.8	3.9	0.01	5.8	0.4	3.9
Procedure 6	32.3	9.1	0.39	28.0	35.0	8.5

Table F.15 Miscellaneous extra information concerning the estimation of the mean using 3 assessments; accuracy parameter = 0.01 (Errors are measured as a percentage of the range)

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.2	0.1	0.69	0.0	0.9	0.1
Procedure 2	0.5	0.5	0.35	0.3	1.6	0.4
Procedure 3	2.3	2.0	0.99	0.0	19.0	0.2
Procedure 4	0.6	0.5	0.93	0.0	4.8	0.2
Procedure 5	0.9	0.9	0.94	-0.2	8.7	0.3
Procedure 6	4.5	3.8	0.99	-0.1	37.1	0.6

Table F.16 Miscellaneous extra information concerning the estimation of the S.D. using 3 assessments; accuracy parameter = 0.01 (Errors are measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	23.6	4.7	0.19	22.5	8.9	4.7
Procedure 2	12.1	8.3	0.00	12.1	-0.1	8.4
Procedure 3	46.6	7.7	0.32	43.6	24.2	7.4
Procedure 4	20.5	6.7	0.34	17.8	22.1	6.4
Procedure 5	19.3	2.1	0.00	19.3	0.0	2.1
Procedure 6	38.4	9.7	0.38	33.9	36.6	9.1

Table F.17 Miscellaneous extra information concerning the estimation of the mean using 7 assessments; accuracy parameter = 0.01 (Errors measured as a percentage of the range).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	0.1	0.1	0.23	0.1	0.3	0.1
Procedure 2	0.1	0.1	0.21	0.1	0.2	0.1
Procedure 3	1.1	1.0	0.97	0.0	9.1	0.2
Procedure 4	0.3	0.3	0.77	0.0	2.4	0.2
Procedure 5	0.4	0.3	0.97	0.0	2.9	0.1
Procedure 6	2.6	2.3	0.97	0.1	21.9	0.5

Table F.18 Miscellaneous extra information concerning the estimation of the S.D. using 7 assessments; accuracy parameter = 0.01 (Errors measured as percentage errors).

	μ_1	σ_1	ρ	a	b	σ_2
Procedure 1	7.5	2.2	0.22	6.9	4.7	2.2
Procedure 2	1.5	1.2	-0.15	1.7	-1.7	1.2
Procedure 3	34.0	7.6	0.36	30.8	26.5	7.1
Procedure 4	12.3	4.8	0.37	10.2	17.3	4.5
Procedure 5	8.2	2.2	0.38	7.2	8.1	2.0
Procedure 6	30.5	9.8	0.46	24.9	44.9	8.8

Note that when the accuracy parameter is 0 or 0.01, the coefficients of correlation calculated for procedures 1 and 2 are unreliable because of errors in the algorithm which was used to calculate the inverse cumulative of the beta distribution.

ERRORS IN MEANS ASSUMING 3 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	-0.0013	-0.0000	0.0024	0.0000	0.0000
3	4	0.4286	-0.0006	-0.0003	0.0143	0.0041	0.0109	0.0219
3	5	0.3750	-0.0010	0.0026	0.0258	0.0061	0.0151	0.0426
3	6	0.3333	-0.0015	0.0040	0.0352	0.0083	0.0165	0.0614
3	7	0.3000	-0.0020	0.0018	0.0431	0.0103	0.0176	0.0782
3	8	0.2727	-0.0026	-0.0049	0.0497	0.0122	0.0200	0.0934
3	9	0.2500	-0.0030	-0.0105	0.0553	0.0139	0.0238	0.1070
3	10	0.2308	-0.0033	-0.0125	0.0602	0.0155	0.0287	0.1192
3	11	0.2143	-0.0033	-0.0127	0.0644	0.0169	0.0344	0.1302
3	12	0.2000	-0.0031	-0.0122	0.0681	0.0182	0.0404	0.1402
4	4	0.5000	0.0000	-0.0007	-0.0000	0.0020	0.0000	0.0000
4	5	0.4444	-0.0003	-0.0018	0.0117	0.0039	0.0053	0.0197
4	6	0.4000	-0.0005	0.0014	0.0215	0.0059	0.0075	0.0377
4	7	0.3636	-0.0008	0.0049	0.0298	0.0078	0.0089	0.0538
4	8	0.3333	-0.0014	0.0068	0.0368	0.0097	0.0107	0.0684
4	9	0.3077	-0.0022	0.0064	0.0429	0.0113	0.0137	0.0817
4	10	0.2857	-0.0031	0.0030	0.0483	0.0129	0.0177	0.0938
4	11	0.2667	-0.0042	-0.0042	0.0529	0.0143	0.0226	0.1048
4	12	0.2500	-0.0053	-0.0115	0.0571	0.0156	0.0279	0.1148
5	5	0.5000	0.0000	-0.0004	-0.0000	0.0017	0.0000	0.0000
5	6	0.4545	0.0001	-0.0032	0.0099	0.0036	0.0025	0.0177
5	7	0.4167	0.0002	-0.0005	0.0184	0.0055	0.0039	0.0335
5	8	0.3846	0.0001	0.0033	0.0258	0.0073	0.0054	0.0479
5	9	0.3571	-0.0000	0.0068	0.0322	0.0089	0.0077	0.0610
5	10	0.3333	-0.0004	0.0091	0.0378	0.0105	0.0110	0.0730
5	11	0.3125	-0.0010	0.0097	0.0428	0.0119	0.0151	0.0840
5	12	0.2941	-0.0020	0.0081	0.0473	0.0132	0.0198	0.0941
6	6	0.5000	0.0000	-0.0002	-0.0000	0.0014	0.0000	0.0000
6	7	0.4615	0.0002	-0.0044	0.0086	0.0033	0.0013	0.0158
6	8	0.4286	0.0004	-0.0025	0.0161	0.0051	0.0026	0.0301
6	9	0.4000	0.0006	0.0011	0.0227	0.0067	0.0043	0.0431
6	10	0.3750	0.0008	0.0050	0.0285	0.0083	0.0070	0.0551
6	11	0.3529	0.0010	0.0085	0.0338	0.0097	0.0104	0.0661
6	12	0.3333	0.0010	0.0111	0.0384	0.0110	0.0145	0.0762
7	7	0.5000	0.0000	-0.0001	-0.0000	0.0012	0.0000	0.0000
7	8	0.4667	0.0002	-0.0054	0.0076	0.0030	0.0011	0.0143
7	9	0.4375	0.0003	-0.0043	0.0143	0.0047	0.0025	0.0273
7	10	0.4118	0.0006	-0.0012	0.0202	0.0062	0.0046	0.0392
7	11	0.3889	0.0010	0.0026	0.0256	0.0077	0.0075	0.0503
7	12	0.3684	0.0016	0.0065	0.0305	0.0090	0.0111	0.0605
8	8	0.5000	0.0000	-0.0000	-0.0000	0.0011	0.0000	0.0000
8	9	0.4706	-0.0001	-0.0062	0.0067	0.0028	0.0013	0.0130
8	10	0.4444	-0.0001	-0.0060	0.0128	0.0043	0.0031	0.0249
8	11	0.4211	0.0001	-0.0035	0.0183	0.0058	0.0055	0.0359
8	12	0.4000	0.0007	-0.0000	0.0233	0.0071	0.0086	0.0462
9	9	0.5000	0.0000	-0.0000	-0.0000	0.0010	0.0000	0.0001
9	10	0.4737	-0.0003	-0.0069	0.0061	0.0025	0.0017	0.0120
9	11	0.4500	-0.0007	-0.0075	0.0116	0.0040	0.0039	0.0230
9	12	0.4286	-0.0006	-0.0057	0.0167	0.0054	0.0066	0.0332
10	10	0.5000	-0.0000	-0.0000	-0.0000	0.0009	0.0000	-0.0000
10	11	0.4762	-0.0007	-0.0075	0.0056	0.0024	0.0021	0.0111
10	12	0.4545	-0.0013	-0.0088	0.0106	0.0037	0.0047	0.0213
11	11	0.5000	0.0000	0.0000	-0.0000	0.0008	0.0000	-0.0001
11	12	0.4783	-0.0011	-0.0080	0.0051	0.0022	0.0025	0.0102
12	12	0.5000	0.0000	-0.0000	-0.0000	0.0008	0.0000	0.0000

ERRORS IN S.D.S ASSUMING 3 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.1256	0.0354	0.2283	0.0379	0.0940	0.0891
3	4	0.1750	0.1414	0.0150	0.2762	0.0605	0.1345	0.1468
3	5	0.1614	0.1611	0.0304	0.3286	0.0914	0.1629	0.2161
3	6	0.1491	0.1826	0.0705	0.3786	0.1266	0.1760	0.2832
3	7	0.1382	0.2036	0.1298	0.4243	0.1635	0.1782	0.3437
3	8	0.1286	0.2228	0.2016	0.4652	0.2004	0.1767	0.3967
3	9	0.1201	0.2396	0.1950	0.5016	0.2363	0.1779	0.4429
3	10	0.1126	0.2534	0.1975	0.5339	0.2707	0.1860	0.4831
3	11	0.1059	0.2639	0.2006	0.5026	0.3032	0.2018	0.5181
3	12	0.1000	0.2711	0.1983	0.5682	0.3339	0.2240	0.5488
4	4	0.1667	0.1503	0.0675	0.2998	0.0737	0.1562	0.1703
4	5	0.1571	0.1643	0.0400	0.3344	0.0944	0.1759	0.2143
4	6	0.1477	0.1832	0.0429	0.3720	0.1206	0.1866	0.2648
4	7	0.1389	0.2044	0.0634	0.4091	0.1497	0.1889	0.3150
4	8	0.1307	0.2266	0.0989	0.4442	0.1799	0.1866	0.3619
4	9	0.1234	0.2489	0.1476	0.4765	0.2103	0.1843	0.4045
4	10	0.1166	0.2705	0.2067	0.5062	0.2402	0.1859	0.4429
4	11	0.1106	0.2904	0.2729	0.5332	0.2692	0.1934	0.4773
4	12	0.1050	0.3076	0.2738	0.5579	0.2970	0.2071	0.5081
5	5	0.1503	0.1709	0.1020	0.3546	0.1084	0.1900	0.2371
5	6	0.1437	0.1837	0.0703	0.3811	0.1269	0.1986	0.2714
5	7	0.1367	0.1997	0.0654	0.4097	0.1493	0.2007	0.3099
5	8	0.1300	0.2174	0.0732	0.4384	0.1738	0.1982	0.3486
5	9	0.1237	0.2370	0.0925	0.4661	0.1993	0.1946	0.3858
5	10	0.1179	0.2586	0.1236	0.4923	0.2250	0.1929	0.4205
5	11	0.1124	0.2816	0.1657	0.5169	0.2505	0.1955	0.4524
5	12	0.1074	0.3048	0.2163	0.5396	0.2754	0.2033	0.4817
6	6	0.1337	0.1914	0.1371	0.3982	0.1404	0.2064	0.2918
6	7	0.1332	0.2031	0.1029	0.4193	0.1569	0.2087	0.3192
6	8	0.1278	0.2154	0.0928	0.4420	0.1763	0.2067	0.3495
6	9	0.1225	0.2282	0.0915	0.4650	0.1974	0.2028	0.3804
6	10	0.1174	0.2430	0.0987	0.4875	0.2192	0.1996	0.4105
6	11	0.1126	0.2611	0.1161	0.5091	0.2414	0.1995	0.4391
6	12	0.1081	0.2825	0.1443	0.5297	0.2634	0.2035	0.4660
7	7	0.1291	0.2130	0.1717	0.4339	0.1696	0.2117	0.3371
7	8	0.1247	0.2236	0.1362	0.4512	0.1844	0.2106	0.3596
7	9	0.1203	0.2324	0.1224	0.4698	0.2014	0.2071	0.3842
7	10	0.1160	0.2400	0.1148	0.4887	0.2198	0.2035	0.4094
7	11	0.1118	0.2490	0.1130	0.5074	0.2388	0.2020	0.4342
7	12	0.1079	0.2618	0.1191	0.5256	0.2581	0.2037	0.4582
8	8	0.1213	0.2355	0.2049	0.4639	0.1961	0.2104	0.3752
8	9	0.1176	0.2451	0.1689	0.4784	0.2095	0.2076	0.3940
8	10	0.1140	0.2508	0.1528	0.4939	0.2246	0.2042	0.4144
8	11	0.1104	0.2538	0.1409	0.5098	0.2408	0.2020	0.4354
8	12	0.1069	0.2571	0.1323	0.5257	0.2575	0.2025	0.4563
9	9	0.1147	0.2585	0.2362	0.4894	0.2204	0.2055	0.4077
9	10	0.1116	0.2673	0.2004	0.5019	0.2325	0.2024	0.4238
9	11	0.1086	0.2705	0.1829	0.5151	0.2460	0.2000	0.4411
9	12	0.1055	0.2697	0.1681	0.5287	0.2604	0.1997	0.4589
10	10	0.1091	0.2815	0.2654	0.5116	0.2425	0.1995	0.4359
10	11	0.1065	0.2895	0.2302	0.5225	0.2536	0.1970	0.4498
10	12	0.1033	0.2908	0.2119	0.5339	0.2658	0.1962	0.4646
11	11	0.1043	0.3040	0.2925	0.5311	0.2629	0.1943	0.4605
11	12	0.1020	0.3113	0.2582	0.5406	0.2730	0.1930	0.4727
12	12	0.1000	0.3257	0.3175	0.5484	0.2816	0.1911	0.4824

ERRORS IN MEANS ASSUMING 7 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0,5000	0,0000	-0,0003	-0,0000	0,0000	0,0000	0,0000
3	4	0,4286	-0,0000	-0,0001	0,0063	0,0010	0,0023	0,0102
3	5	0,3750	-0,0001	0,0001	0,0116	0,0022	0,0039	0,0210
3	6	0,3333	-0,0001	0,0001	0,0161	0,0033	0,0054	0,0319
3	7	0,3000	-0,0002	-0,0004	0,0198	0,0043	0,0067	0,0422
3	8	0,2727	-0,0002	-0,0001	0,0231	0,0053	0,0078	0,0523
3	9	0,2500	-0,0003	-0,0001	0,0258	0,0062	0,0088	0,0617
3	10	0,2308	-0,0004	0,0002	0,0283	0,0070	0,0096	0,0705
3	11	0,2143	-0,0005	0,0005	0,0304	0,0077	0,0103	0,0788
3	12	0,2000	-0,0006	0,0008	0,0323	0,0084	0,0108	0,0868
4	4	0,5000	0,0000	-0,0004	-0,0000	0,0000	0,0000	-0,0000
4	5	0,4444	-0,0000	0,0000	0,0053	0,0011	0,0016	0,0099
4	6	0,4000	-0,0000	-0,0004	0,0099	0,0021	0,0030	0,0197
4	7	0,3636	-0,0000	-0,0001	0,0138	0,0031	0,0042	0,0290
4	8	0,3333	-0,0000	0,0001	0,0172	0,0041	0,0054	0,0379
4	9	0,3077	-0,0000	0,0002	0,0202	0,0050	0,0064	0,0464
4	10	0,2857	-0,0001	-0,0002	0,0228	0,0057	0,0073	0,0545
4	11	0,2667	-0,0001	-0,0004	0,0451	0,0065	0,0080	0,0621
4	12	0,2500	-0,0002	-0,0006	0,0272	0,0072	0,0087	0,0694
5	5	0,5000	0,0000	-0,0004	-0,0000	0,0000	0,0000	0,0000
5	6	0,4545	0,0000	-0,0000	0,0046	0,0011	0,0014	0,0093
5	7	0,4167	0,0000	-0,0000	0,0086	0,0020	0,0026	0,0182
5	8	0,3846	0,0000	-0,0001	0,0121	0,0030	0,0037	0,0266
5	9	0,3571	0,0000	-0,0003	0,0152	0,0038	0,0048	0,0346
5	10	0,3333	0,0000	-0,0000	0,0179	0,0046	0,0057	0,0422
5	11	0,3125	0,0000	0,0004	0,0203	0,0053	0,0064	0,0495
5	12	0,2941	0,0000	0,0005	0,0225	0,0060	0,0071	0,0564
6	6	0,5000	0,0000	-0,0004	-0,0000	0,0000	0,0000	-0,0000
6	7	0,4615	0,0000	-0,0001	0,0040	0,0010	0,0012	0,0087
6	8	0,4286	0,0000	0,0001	0,0076	0,0019	0,0024	0,0168
6	9	0,4000	0,0000	-0,0002	0,0107	0,0028	0,0034	0,0245
6	10	0,3750	0,0000	-0,0006	0,0136	0,0036	0,0043	0,0319
6	11	0,3529	0,0000	-0,0006	0,0161	0,0043	0,0051	0,0389
6	12	0,3333	0,0000	-0,0002	0,0184	0,0050	0,0058	0,0454
7	7	0,5000	0,0000	-0,0004	-0,0000	0,0000	0,0000	0,0000
7	8	0,4667	-0,0000	-0,0003	0,0036	0,0009	0,0011	0,0081
7	9	0,4375	-0,0000	-0,0000	0,0068	0,0018	0,0021	0,0156
7	10	0,4118	-0,0000	0,0002	0,0097	0,0026	0,0030	0,0228
7	11	0,3889	-0,0000	-0,0005	0,0123	0,0033	0,0039	0,0297
7	12	0,3684	-0,0000	-0,0009	0,0146	0,0040	0,0046	0,0361
8	8	0,5000	0,0000	-0,0004	-0,0000	0,0000	0,0000	0,0000
8	9	0,4706	-0,0000	-0,0004	0,0032	0,0009	0,0010	0,0075
8	10	0,4444	-0,0000	-0,0002	0,0061	0,0017	0,0019	0,0146
8	11	0,4211	-0,0000	0,0003	0,0088	0,0024	0,0028	0,0213
8	12	0,4000	-0,0000	0,0002	0,0112	0,0031	0,0035	0,0277
9	9	0,5000	0,0000	-0,0003	-0,0000	0,0000	0,0000	0,0000
9	10	0,4737	-0,0000	-0,0006	0,0029	0,0008	0,0009	0,0071
9	11	0,4500	-0,0000	-0,0003	0,0056	0,0016	0,0018	0,0137
9	12	0,4286	-0,0000	0,0002	0,0080	0,0023	0,0025	0,0200
10	10	0,5000	0,0000	-0,0003	-0,0000	0,0000	0,0000	-0,0001
10	11	0,4762	0,0000	-0,0008	0,0027	0,0008	0,0008	0,0066
10	12	0,4545	0,0000	-0,0005	0,0051	0,0015	0,0016	0,0129
11	11	0,5000	0,0000	-0,0003	-0,0000	0,0000	0,0000	-0,0000
11	12	0,4783	0,0000	-0,0009	0,0025	0,0007	0,0008	0,0063
12	12	0,5000	-0,0000	-0,0002	-0,0000	0,0000	0,0000	0,0000

ERRORS IN S.D.S ASSUMING 7 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.0351	0.0031	0.1269	0.0170	0.0324	0.0467
3	4	0.1750	0.0405	0.0042	0.1641	0.0288	0.0410	0.0857
3	5	0.1614	0.0473	0.0043	0.2086	0.0464	0.0492	0.1426
3	6	0.1491	0.0552	0.0061	0.2543	0.0680	0.0590	0.2062
3	7	0.1382	0.0638	0.0138	0.2986	0.0921	0.0702	0.2684
3	8	0.1286	0.0730	0.0107	0.3401	0.1176	0.0820	0.3267
3	9	0.1201	0.0829	0.0097	0.3785	0.1439	0.0940	0.3794
3	10	0.1126	0.0931	0.0104	0.4137	0.1703	0.1057	0.4261
3	11	0.1059	0.1037	0.0138	0.4458	0.1964	0.1169	0.4676
3	12	0.1000	0.1146	0.0206	0.4750	0.2218	0.1272	0.5044
4	4	0.1667	0.0439	0.0070	0.1872	0.0352	0.0455	0.0998
4	5	0.1571	0.0490	0.0063	0.2117	0.0473	0.0506	0.1353
4	6	0.1477	0.0550	0.0117	0.2459	0.0634	0.0576	0.1818
4	7	0.1389	0.0617	0.0085	0.2814	0.0822	0.0661	0.2318
4	8	0.1307	0.0690	0.0080	0.3164	0.1027	0.0756	0.2819
4	9	0.1234	0.0769	0.0128	0.3499	0.1243	0.0856	0.3294
4	10	0.1166	0.0854	0.0245	0.3816	0.1464	0.0958	0.3736
4	11	0.1106	0.0943	0.0261	0.4112	0.1687	0.1060	0.4140
4	12	0.1050	0.1037	0.0234	0.4388	0.1910	0.1159	0.4508
5	5	0.1508	0.0527	0.0120	0.2289	0.0549	0.0534	0.1526
5	6	0.1437	0.0576	0.0094	0.2530	0.0666	0.0583	0.1831
5	7	0.1367	0.0630	0.0126	0.2803	0.0812	0.0649	0.2210
5	8	0.1300	0.0690	0.0182	0.3089	0.0978	0.0726	0.2619
5	9	0.1237	0.0754	0.0140	0.3373	0.1157	0.0809	0.3029
5	10	0.1179	0.0822	0.0134	0.3650	0.1344	0.0897	0.3424
5	11	0.1124	0.0893	0.0164	0.3916	0.1536	0.0987	0.3798
5	12	0.1074	0.0967	0.0243	0.4168	0.1731	0.1078	0.4147
6	6	0.1387	0.0615	0.0175	0.2685	0.0745	0.0613	0.2004
6	7	0.1332	0.0661	0.0131	0.2886	0.0856	0.0663	0.2270
6	8	0.1278	0.0713	0.0147	0.3112	0.0989	0.0725	0.2585
6	9	0.1225	0.0768	0.0221	0.3348	0.1137	0.0794	0.2921
6	10	0.1174	0.0827	0.0256	0.3585	0.1296	0.0869	0.3262
6	11	0.1126	0.0888	0.0219	0.3818	0.1461	0.0947	0.3596
6	12	0.1081	0.0952	0.0207	0.4044	0.1631	0.1029	0.3916
7	7	0.1291	0.0700	0.0235	0.3025	0.0935	0.0698	0.2434
7	8	0.1247	0.0745	0.0174	0.3197	0.1039	0.0746	0.2662
7	9	0.1203	0.0794	0.0175	0.3387	0.1160	0.0803	0.2926
7	10	0.1160	0.0847	0.0230	0.3586	0.1294	0.0866	0.3211
7	11	0.1118	0.0902	0.0376	0.3787	0.1437	0.0934	0.3500
7	12	0.1079	0.0960	0.0344	0.3987	0.1585	0.1006	0.3782
8	8	0.1213	0.0783	0.0300	0.3322	0.1117	0.0782	0.2814
8	9	0.1176	0.0826	0.0222	0.3471	0.1214	0.0828	0.3011
8	10	0.1140	0.0873	0.0195	0.3634	0.1325	0.0881	0.3238
8	11	0.1104	0.0923	0.0248	0.3804	0.1447	0.0939	0.3481
8	12	0.1069	0.0976	0.0361	0.3978	0.1576	0.1002	0.3728
9	9	0.1147	0.0863	0.0369	0.3583	0.1289	0.0864	0.3150
9	10	0.1116	0.0905	0.0275	0.3713	0.1380	0.0908	0.3324
9	11	0.1086	0.0950	0.0237	0.3855	0.1483	0.0957	0.3522
9	12	0.1055	0.0998	0.0273	0.4004	0.1594	0.1012	0.3729
10	10	0.1091	0.0942	0.0443	0.3815	0.1452	0.0943	0.3451
10	11	0.1065	0.0932	0.0316	0.3930	0.1538	0.0984	0.3605
10	12	0.1033	0.1025	0.0286	0.4056	0.1633	0.1032	0.3777
11	11	0.1043	0.1013	0.0519	0.4022	0.1607	0.1018	0.3721
11	12	0.1020	0.1057	0.0382	0.4126	0.1687	0.1058	0.3858
12	12	0.1000	0.1093	0.0556	0.4210	0.1753	0.1092	0.3963

ERRORS IN MEANS ASSUMING 15 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000
3	4	0.4286	-0.0000	-0.0000	0.0028	0.0005	0.0006	0.0045
3	5	0.3750	-0.0000	-0.0000	0.0052	0.0010	0.0010	0.0098
3	6	0.3333	-0.0000	-0.0000	0.0073	0.0015	0.0014	0.0155
3	7	0.3000	-0.0000	-0.0001	0.0091	0.0020	0.0018	0.0211
3	8	0.2727	-0.0000	-0.0000	0.0107	0.0025	0.0022	0.0269
3	9	0.2500	-0.0000	-0.0001	0.0121	0.0029	0.0025	0.0324
3	10	0.2308	-0.0000	0.0001	0.0133	0.0033	0.0028	0.0379
3	11	0.2143	-0.0000	0.0000	0.0143	0.0037	0.0031	0.0432
3	12	0.2000	-0.0000	-0.0000	0.0153	0.0040	0.0035	0.0483
4	4	0.5000	0.0000	-0.0000	-0.0000	0.0000	0.0000	-0.0000
4	5	0.4444	-0.0000	0.0000	0.0024	0.0005	0.0004	0.0047
4	6	0.4000	-0.0000	-0.0001	0.0045	0.0010	0.0008	0.0096
4	7	0.3636	-0.0000	-0.0000	0.0064	0.0015	0.0012	0.0145
4	8	0.3333	-0.0000	-0.0000	0.0080	0.0019	0.0015	0.0195
4	9	0.3077	-0.0000	-0.0001	0.0095	0.0023	0.0018	0.0242
4	10	0.2857	-0.0000	-0.0002	0.0107	0.0027	0.0021	0.0289
4	11	0.2667	-0.0000	-0.0000	0.0119	0.0031	0.0024	0.0335
4	12	0.2500	-0.0000	-0.0001	0.0129	0.0034	0.0027	0.0380
5	5	0.5000	0.0000	-0.0001	-0.0000	0.0000	0.0000	-0.0000
5	6	0.4545	0.0000	-0.0000	0.0021	0.0005	0.0004	0.0046
5	7	0.4167	0.0000	-0.0000	0.0040	0.0010	0.0007	0.0092
5	8	0.3846	0.0000	0.0000	0.0057	0.0014	0.0010	0.0137
5	9	0.3571	0.0000	-0.0001	0.0071	0.0018	0.0013	0.0181
5	10	0.3333	0.0000	0.0000	0.0085	0.0022	0.0016	0.0224
5	11	0.3125	0.0000	-0.0001	0.0097	0.0026	0.0019	0.0266
5	12	0.2941	0.0000	-0.0001	0.0107	0.0029	0.0021	0.0306
6	6	0.5000	0.0000	0.0001	-0.0000	0.0000	0.0000	0.0000
6	7	0.4615	0.0000	-0.0000	0.0019	0.0005	0.0003	0.0044
6	8	0.4286	0.0000	-0.0000	0.0036	0.0009	0.0006	0.0087
6	9	0.4000	0.0000	-0.0001	0.0051	0.0013	0.0009	0.0128
6	10	0.3750	0.0000	-0.0000	0.0064	0.0017	0.0012	0.0170
6	11	0.3529	0.0000	-0.0001	0.0077	0.0021	0.0014	0.0209
6	12	0.3333	0.0000	-0.0002	0.0088	0.0024	0.0017	0.0247
7	7	0.5000	0.0000	0.0001	-0.0000	0.0000	0.0000	0.0000
7	8	0.4667	-0.0000	-0.0000	0.0017	0.0004	0.0003	0.0042
7	9	0.4375	-0.0000	0.0000	0.0032	0.0008	0.0006	0.0083
7	10	0.4118	-0.0000	-0.0000	0.0046	0.0012	0.0008	0.0121
7	11	0.3889	-0.0000	-0.0002	0.0058	0.0016	0.0011	0.0160
7	12	0.3684	-0.0000	-0.0000	0.0070	0.0019	0.0013	0.0196
8	8	0.5000	-0.0000	-0.0001	-0.0000	0.0000	0.0000	0.0000
8	9	0.4706	-0.0000	-0.0000	0.0015	0.0004	0.0003	0.0040
8	10	0.4444	-0.0000	-0.0001	0.0029	0.0008	0.0005	0.0078
8	11	0.4211	-0.0000	-0.0000	0.0042	0.0012	0.0008	0.0115
8	12	0.4000	-0.0000	-0.0001	0.0054	0.0015	0.0010	0.0151
9	9	0.5000	0.0000	-0.0001	-0.0000	0.0000	0.0000	-0.0000
9	10	0.4737	0.0000	-0.0000	0.0014	0.0004	0.0003	0.0038
9	11	0.4500	0.0000	-0.0001	0.0027	0.0007	0.0005	0.0074
9	12	0.4286	0.0000	0.0000	0.0039	0.0011	0.0007	0.0109
10	10	0.5000	0.0000	0.0001	-0.0000	0.0000	0.0000	0.0000
10	11	0.4762	0.0000	-0.0000	0.0013	0.0004	0.0002	0.0036
10	12	0.4545	0.0000	-0.0001	0.0025	0.0007	0.0005	0.0071
11	11	0.5000	-0.0000	0.0001	-0.0000	0.0000	0.0000	-0.0000
11	12	0.4783	-0.0000	0.0000	0.0012	0.0003	0.0002	0.0034
12	12	0.5000	-0.0000	0.0001	-0.0000	0.0000	0.0000	0.0000

ERRORS IN S.D.S ASSUMING 15 PERFECT ASSESSMENTS

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.0090	0.0004	0.0619	0.0063	0.0088	0.0206
3	4	0.1750	0.0105	0.0006	0.0854	0.0122	0.0108	0.0423
3	5	0.1614	0.0123	0.0007	0.1160	0.0214	0.0131	0.0801
3	6	0.1491	0.0144	0.0008	0.1499	0.0333	0.0160	0.1278
3	7	0.1332	0.0168	0.0014	0.1851	0.0473	0.0193	0.1803
3	8	0.1236	0.0194	0.0011	0.2201	0.0628	0.0229	0.2342
3	9	0.1201	0.0221	0.0018	0.2542	0.0794	0.0268	0.2861
3	10	0.1126	0.0252	0.0014	0.2869	0.0967	0.0310	0.3351
3	11	0.1059	0.0284	0.0016	0.3180	0.1146	0.0355	0.3806
3	12	0.1000	0.0318	0.0020	0.3473	0.1328	0.0402	0.4220
4	4	0.1667	0.0115	0.0008	0.0970	0.0153	0.0116	0.0501
4	5	0.1571	0.0129	0.0009	0.1175	0.0217	0.0132	0.0731
4	6	0.1477	0.0146	0.0012	0.1428	0.0306	0.0153	0.1069
4	7	0.1389	0.0165	0.0013	0.1705	0.0413	0.0178	0.1469
4	8	0.1307	0.0185	0.0014	0.1992	0.0535	0.0206	0.1906
4	9	0.1234	0.0208	0.0018	0.2280	0.0668	0.0237	0.2348
4	10	0.1166	0.0231	0.0028	0.2564	0.0809	0.0271	0.2784
4	11	0.1106	0.0257	0.0021	0.2840	0.0957	0.0307	0.3203
4	12	0.1050	0.0283	0.0025	0.3106	0.1109	0.0345	0.3598
5	5	0.1508	0.0140	0.0011	0.1296	0.0257	0.0140	0.0841
5	6	0.1437	0.0154	0.0013	0.1476	0.0322	0.0155	0.1066
5	7	0.1367	0.0170	0.0014	0.1690	0.0406	0.0175	0.1364
5	8	0.1300	0.0187	0.0017	0.1922	0.0504	0.0197	0.1710
5	9	0.1237	0.0206	0.0020	0.2164	0.0614	0.0222	0.2079
5	10	0.1179	0.0226	0.0023	0.2408	0.0732	0.0250	0.2455
5	11	0.1124	0.0248	0.0025	0.2650	0.0856	0.0280	0.2830
5	12	0.1074	0.0271	0.0032	0.2887	0.0985	0.0312	0.3192
6	6	0.1387	0.0165	0.0015	0.1593	0.0367	0.0165	0.1188
6	7	0.1332	0.0178	0.0015	0.1752	0.0431	0.0180	0.1395
6	8	0.1278	0.0194	0.0017	0.1936	0.0510	0.0197	0.1662
6	9	0.1225	0.0210	0.0021	0.2136	0.0600	0.0218	0.1960
6	10	0.1174	0.0228	0.0023	0.2343	0.0700	0.0241	0.2281
6	11	0.1136	0.0247	0.0027	0.2554	0.0806	0.0267	0.2608
6	12	0.1031	0.0267	0.0039	0.2764	0.0918	0.0294	0.2933
7	7	0.1291	0.0190	0.0017	0.1863	0.0477	0.0190	0.1525
7	8	0.1247	0.0203	0.0017	0.2004	0.0540	0.0204	0.1716
7	9	0.1203	0.0218	0.0022	0.2166	0.0614	0.0221	0.1952
7	10	0.1160	0.0233	0.0023	0.2340	0.0698	0.0240	0.2213
7	11	0.1118	0.0251	0.0033	0.2521	0.0789	0.0262	0.2495
7	12	0.1079	0.0269	0.0029	0.2705	0.0886	0.0285	0.2780
8	8	0.1213	0.0214	0.0022	0.2108	0.0587	0.0214	0.1843
8	9	0.1176	0.0227	0.0021	0.2236	0.0647	0.0228	0.2017
8	10	0.1140	0.0241	0.0028	0.2379	0.0717	0.0244	0.2227
8	11	0.1104	0.0257	0.0025	0.2533	0.0795	0.0263	0.2460
8	12	0.1069	0.0273	0.0030	0.2693	0.0880	0.0283	0.2708
9	9	0.1147	0.0239	0.0028	0.2332	0.0694	0.0239	0.2137
9	10	0.1116	0.0251	0.0025	0.2448	0.0751	0.0252	0.2299
9	11	0.1086	0.0265	0.0031	0.2577	0.0818	0.0268	0.2487
9	12	0.1055	0.0280	0.0030	0.2714	0.0891	0.0285	0.2695
10	10	0.1091	0.0263	0.0033	0.2538	0.0798	0.0263	0.2415
10	11	0.1065	0.0275	0.0028	0.2644	0.0853	0.0276	0.2561
10	12	0.1038	0.0289	0.0034	0.2760	0.0916	0.0291	0.2733
11	11	0.1043	0.0287	0.0039	0.2728	0.0898	0.0287	0.2671
11	12	0.1020	0.0299	0.0033	0.2825	0.0952	0.0300	0.2806
12	12	0.1000	0.0310	0.0044	0.2903	0.0996	0.0310	0.2909

ERRORS IN MEANS ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.20)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	-0.0000	-0.0023	0.0000	0.0028	-0.0129	0.0000
3	4	0.4286	-0.0036	0.0020	-0.0036	-0.0202	0.0129	0.0159
3	5	0.3750	0.0000	0.0000	0.0000	-0.0278	0.0129	0.0694
3	6	0.3333	0.0000	0.0000	0.0189	-0.0029	0.0099	0.1111
3	7	0.3000	-0.0250	-0.0139	0.0523	0.0304	0.0432	0.1111
3	8	0.2727	-0.0227	-0.0227	0.0717	0.0398	-0.0114	0.1037
3	9	0.2500	-0.0250	-0.0361	0.0944	0.0625	0.0113	0.1185
3	10	0.2308	-0.0058	-0.0160	0.1130	0.0768	0.0207	0.1378
3	11	0.2143	0.0107	-0.0004	0.0862	0.0439	0.0372	0.1345
3	12	0.2000	-0.0250	-0.0375	0.1004	0.0582	0.0515	0.1369
4	4	0.5000	0.0000	0.0000	0.0000	0.0028	-0.0156	0.0000
4	5	0.4444	-0.0278	-0.0278	0.0556	0.0583	0.0185	0.0000
4	6	0.4000	-0.0250	-0.0250	0.0450	0.0083	0.0054	0.0444
4	7	0.3636	0.0114	0.0114	-0.0114	-0.0332	-0.0060	0.0808
4	8	0.3333	0.0417	0.0417	0.0111	-0.0208	0.0238	0.0778
4	9	0.3077	-0.0327	-0.0216	0.0367	0.0048	-0.0225	0.1034
4	10	0.2857	-0.0107	0.0004	0.0580	0.0219	-0.0133	0.1254
4	11	0.2667	-0.0167	-0.0167	0.0771	0.0409	0.0057	0.1444
4	12	0.2500	-0.0250	-0.0361	0.0878	0.0543	0.0224	0.1611
5	5	0.5000	0.0000	0.0000	0.0000	0.0028	0.0114	0.0000
5	6	0.4545	0.0205	0.0093	0.0455	0.0482	-0.0022	-0.0101
5	7	0.4167	-0.0417	-0.0417	0.0833	0.0819	0.0098	0.0278
5	8	0.3846	-0.0096	-0.0096	-0.0402	-0.0721	-0.0078	0.0598
5	9	0.3571	0.0179	0.0179	-0.0128	-0.0446	0.0005	0.0873
5	10	0.3333	-0.0208	-0.0069	0.0104	-0.0257	0.0243	0.0778
5	11	0.3125	-0.0375	-0.0264	0.0312	-0.0049	-0.0273	0.0986
5	12	0.2941	-0.0191	-0.0080	0.0496	0.0135	-0.0089	0.1170
6	6	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0114	0.0000
6	7	0.4615	0.0135	0.0024	0.0385	0.0370	-0.0092	-0.0171
6	8	0.4286	0.0089	-0.0050	0.0689	0.0713	0.0238	0.0159
6	9	0.4000	-0.0250	-0.0250	0.0353	0.0201	-0.0232	0.0444
6	10	0.3750	0.0000	0.0000	-0.0313	-0.0674	0.0018	0.0694
6	11	0.3529	0.0221	0.0221	-0.0092	-0.0454	0.0238	0.0582
6	12	0.3333	-0.0208	-0.0069	0.0104	-0.0257	0.0243	0.0778
7	7	0.5000	0.0000	0.0000	0.0000	-0.0015	-0.0198	0.0000
7	8	0.4667	0.0083	-0.0028	0.0333	0.0323	-0.0143	0.0333
7	9	0.4375	0.0000	-0.0139	0.0625	0.0614	0.0149	0.0069
7	10	0.4118	0.0257	0.0118	0.0436	0.0083	-0.0106	0.0327
7	11	0.3889	-0.0139	-0.0139	0.0465	0.0312	0.0123	0.0556
7	12	0.3684	0.0066	0.0066	-0.0247	-0.0608	0.0083	0.0427
8	8	0.5000	0.0000	0.0000	0.0000	-0.0011	0.0151	0.0000
8	9	0.4706	0.0044	-0.0067	0.0294	0.0283	0.0144	0.0294
8	10	0.4444	0.0306	0.0194	0.0556	0.0545	-0.0111	0.0000
8	11	0.4211	0.0039	-0.0086	0.0143	-0.0010	-0.0199	0.0234
8	12	0.4000	0.0250	0.0125	0.0353	0.0201	0.0011	0.0444
9	9	0.5000	0.0000	0.0000	0.0000	-0.0011	0.0151	0.0000
9	10	0.4737	0.0013	-0.0098	0.0263	0.0252	0.0113	0.0263
9	11	0.4500	0.0250	0.0139	0.0500	0.0489	-0.0167	0.0500
9	12	0.4286	-0.0036	-0.0161	0.0714	0.0704	0.0048	0.0159
10	10	0.5000	0.0000	0.0000	0.0000	-0.0011	0.0151	0.0000
10	11	0.4762	-0.0012	-0.0123	0.0238	0.0227	0.0088	0.0238
10	12	0.4545	0.0205	0.0093	0.0455	0.0444	-0.0212	0.0455
11	11	0.5000	0.0000	0.0000	0.0000	-0.0011	0.0151	0.0000
11	12	0.4783	-0.0033	-0.0144	0.0217	0.0207	0.0067	0.0217
12	12	0.5000	0.0000	0.0000	0.0000	-0.0007	0.0151	0.0000

ERRORS IN S.D.S ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.20)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.2441	0.1846	0.2577	0.1370	0.0901	0.0742
3	4	0.1750	0.1275	0.0871	0.3511	0.2853	0.1398	0.1279
3	5	0.1614	0.0715	-0.1538	0.3798	0.2577	0.1417	0.1956
3	6	0.1491	0.2000	0.0658	0.4104	0.2527	0.2115	0.2569
3	7	0.1382	0.0280	-0.0750	0.4535	0.3074	0.2692	0.3191
3	8	0.1286	0.1093	0.1093	0.4603	0.2015	-0.0172	0.3900
3	9	0.1201	0.1552	0.0656	0.4958	0.2541	0.0498	0.4356
3	10	0.1126	0.2079	0.1239	0.5218	0.2730	0.1257	0.4708
3	11	0.1059	0.2547	0.1757	0.5505	0.2588	0.1774	0.5152
3	12	0.1000	0.1891	0.0214	0.5757	0.3004	0.2236	0.5507
4	4	0.1667	-0.1547	-0.1547	0.3453	0.2389	0.1338	0.1835
4	5	0.1571	0.1567	0.0153	0.3828	0.2824	0.2024	0.2167
4	6	0.1477	0.1501	-0.0561	0.4522	0.3966	0.2039	0.2637
4	7	0.1389	0.2010	0.0071	0.4507	0.3039	0.0468	0.3078
4	8	0.1307	0.2478	0.0652	0.4511	0.1879	0.2921	0.3557
4	9	0.1234	0.1323	0.0403	0.4822	0.2339	-0.0550	0.3921
4	10	0.1166	0.1795	0.0925	0.5046	0.2470	0.0490	0.4252
4	11	0.1106	0.2341	0.2341	0.5305	0.2863	0.0986	0.4552
4	12	0.1050	0.2612	0.1829	0.5511	0.2946	0.1438	0.4824
5	5	0.1508	-0.0445	-0.0445	0.4078	0.3116	0.1889	0.2263
5	6	0.1437	-0.0111	-0.1183	0.4354	0.3436	0.1548	0.2835
5	7	0.1367	0.2133	0.0223	0.4079	0.1590	0.2416	0.3184
5	8	0.1300	0.2519	0.0703	0.4541	0.1924	0.0715	0.3519
5	9	0.1237	0.2882	0.1154	0.4806	0.2316	0.1508	0.3833
5	10	0.1179	0.0942	-0.0842	0.4995	0.2391	0.1911	0.4192
5	11	0.1124	0.2092	0.1254	0.5226	0.2742	0.0385	0.4460
5	12	0.1074	0.2445	0.1644	0.5439	0.3066	0.0814	0.4707
6	6	0.1387	0.0392	0.0392	0.3995	0.1470	0.2539	0.2883
6	7	0.1332	0.0628	-0.0366	0.4231	0.1805	0.2166	0.3359
6	8	0.1278	0.0179	-0.1755	0.4404	0.1828	0.2487	0.3631
6	9	0.1225	0.2953	0.1243	0.4510	0.1256	0.1254	0.3895
6	10	0.1174	0.2724	0.0796	0.5014	0.2420	0.1616	0.4147
6	11	0.1126	0.3020	0.1171	0.5216	0.2728	0.1957	0.4449
6	12	0.1081	0.1688	0.0051	0.5407	0.3018	0.2577	0.4670
7	7	0.1291	0.1056	0.1056	0.4410	0.2059	0.2226	0.3374
7	8	0.1247	0.1226	0.0296	0.4473	0.1662	0.2666	0.3599
7	9	0.1203	0.0752	-0.1069	0.4668	0.1956	0.2925	0.4003
7	10	0.1160	0.1084	-0.0672	0.4800	0.1719	0.0967	0.4218
7	11	0.1118	0.3070	0.1234	0.4987	0.2016	0.1291	0.4425
7	12	0.1079	0.3316	0.1545	0.5419	0.3036	0.2298	0.4684
8	8	0.1213	0.1598	0.1598	0.4626	0.1893	0.1918	0.3776
8	9	0.1176	0.1724	0.0847	0.4787	0.2135	0.2160	0.3962
8	10	0.1140	0.1981	0.1131	0.4948	0.2379	-0.1096	0.4318
8	11	0.1104	0.1048	-0.0804	0.5051	0.2118	0.1403	0.4497
8	12	0.1069	0.1331	-0.0461	0.5208	0.2368	0.1676	0.4671
9	9	0.1147	0.2053	0.2053	0.4917	0.2331	0.2355	0.4113
9	10	0.1116	0.2146	0.1313	0.5053	0.2536	0.2559	0.4270
9	11	0.1086	0.2363	0.1554	0.5189	0.2742	-0.0567	0.4428
9	12	0.1055	0.1445	-0.0325	0.5325	0.2947	-0.0269	0.4741
10	10	0.1091	0.2441	0.2441	0.5165	0.2706	0.2729	0.4400
10	11	0.1065	0.2510	0.1716	0.5282	0.2882	0.2904	0.4535
10	12	0.1038	0.2696	0.1922	0.5399	0.3059	-0.0106	0.4671
11	11	0.1043	0.2777	0.2777	0.5380	0.3030	0.3052	0.4649
11	12	0.1020	0.2827	0.2067	0.5482	0.3183	0.3205	0.4767
12	12	0.1000	0.3072	0.3072	0.5460	0.2668	0.3336	0.4868

ERRORS IN MEANS ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.20)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	0.0000	0.0000	-0.0000	0.0078	-0.0000
3	4	0.4286	0.0089	0.0089	-0.0161	-0.0246	-0.0026	0.0636
3	5	0.3750	0.0000	0.0000	0.0493	0.0214	0.0083	0.0115
3	6	0.3333	0.0292	0.0333	0.0677	0.0597	-0.0054	0.0471
3	7	0.3000	-0.0125	-0.0167	0.0548	0.0205	0.0148	0.0684
3	8	0.2727	-0.0227	-0.0239	0.0534	0.0395	0.0003	0.0509
3	9	0.2500	-0.0125	-0.0083	0.0790	0.0639	0.0017	0.0705
3	10	0.2308	-0.0433	-0.0433	0.0982	0.0831	-0.0072	0.0647
3	11	0.2143	-0.0268	-0.0268	0.0513	0.0302	0.0030	0.0767
3	12	0.2000	-0.0125	-0.0125	0.0513	0.0289	0.0173	0.0878
4	4	0.5000	0.0000	0.0000	0.0000	-0.0001	-0.0160	-0.0000
4	5	0.4444	-0.0069	-0.0069	-0.0319	-0.0405	0.0075	0.0496
4	6	0.4000	-0.0250	-0.0250	0.0010	-0.0070	-0.0080	0.0357
4	7	0.3636	-0.0011	0.0030	0.0304	0.0220	-0.0100	0.0721
4	8	0.3333	-0.0208	-0.0208	0.0015	-0.0128	0.0138	0.0956
4	9	0.3077	0.0048	0.0048	0.0213	0.0062	-0.0103	0.0701
4	10	0.2857	0.0018	-0.0024	0.0433	0.0282	0.0050	0.0446
4	11	0.2667	0.0208	0.0167	0.0623	0.0472	0.0022	0.0578
4	12	0.2500	-0.0000	-0.0012	0.0790	0.0639	0.0109	0.0745
5	5	0.5000	0.0000	0.0000	0.0000	0.0000	0.0124	0.0000
5	6	0.4545	0.0330	0.0371	-0.0543	-0.0627	-0.0008	0.0412
5	7	0.4167	-0.0292	-0.0333	-0.0181	-0.0272	0.0068	0.0196
5	8	0.3846	-0.0096	-0.0108	0.0140	0.0048	-0.0059	0.0517
5	9	0.3571	0.0054	0.0095	0.0434	0.0338	0.0112	0.0666
5	10	0.3333	-0.0208	-0.0208	-0.0043	-0.0195	0.0027	0.0925
5	11	0.3125	0.0000	0.0000	0.0165	0.0014	-0.0098	0.1100
5	12	0.2941	-0.0066	-0.0108	0.0349	0.0198	0.0033	0.0863
6	6	0.5000	0.0000	-0.0012	0.0000	0.0000	0.0124	0.0000
6	7	0.4615	0.0260	0.0301	0.0351	0.0352	-0.0078	-0.0229
6	8	0.4286	0.0089	0.0089	-0.0280	-0.0376	0.0163	0.0100
6	9	0.4000	-0.0250	-0.0262	0.0006	-0.0091	-0.0161	0.0309
6	10	0.3750	-0.0000	-0.0012	0.0456	0.0159	0.0037	0.0508
6	11	0.3529	-0.0154	-0.0127	-0.0239	-0.0391	0.0084	0.0695
6	12	0.3333	-0.0208	-0.0208	-0.0043	-0.0195	0.0177	0.0892
7	7	0.5000	0.0000	-0.0012	0.0000	-0.0000	-0.0029	0.0000
7	8	0.4667	-0.0292	-0.0292	0.0333	0.0333	-0.0037	-0.0281
7	9	0.4375	0.0000	0.0000	-0.0369	-0.0466	0.0074	-0.0039
7	10	0.4118	0.0007	-0.0034	-0.0112	-0.0208	-0.0004	0.0168
7	11	0.3889	-0.0139	-0.0150	0.0021	-0.0065	0.0079	0.0336
7	12	0.3684	0.0066	0.0054	-0.0394	-0.0545	-0.0027	0.0541
8	8	0.5000	0.0000	-0.0012	0.0000	-0.0000	0.0028	0.0000
8	9	0.4706	-0.0081	-0.0053	0.0294	0.0294	0.0090	-0.0395
8	10	0.4444	-0.0069	-0.0069	0.0556	0.0555	-0.0096	-0.0159
8	11	0.4211	0.0164	0.0164	-0.0205	-0.0301	-0.0021	0.0014
8	12	0.4000	0.0125	0.0083	-0.0090	-0.0176	-0.0032	0.0225
9	9	0.5000	0.0000	-0.0012	0.0000	-0.0000	0.0028	0.0000
9	10	0.4737	-0.0112	-0.0084	0.0263	0.0263	0.0059	-0.0447
9	11	0.4500	-0.0125	-0.0125	0.0500	0.0500	-0.0046	-0.0275
9	12	0.4286	0.0089	0.0089	0.0655	0.0663	0.0063	-0.0061
10	10	0.5000	0.0000	-0.0012	0.0000	-0.0000	0.0028	0.0000
10	11	0.4762	-0.0137	-0.0109	0.0238	0.0238	0.0156	-0.0484
10	12	0.4545	-0.0170	-0.0170	0.0396	0.0403	-0.0091	-0.0321
11	11	0.5000	0.0000	-0.0012	0.0000	-0.0000	0.0028	0.0000
11	12	0.4783	-0.0158	-0.0130	0.0417	0.0217	0.0135	0.0189
12	12	0.5000	0.0000	-0.0012	0.0000	-0.0000	0.0082	0.0000

ERRORS IN S.D.S ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.20)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	-0.3093	-0.3093	0.1875	0.1351	0.0283	0.0030
3	4	0.1750	0.0303	0.0303	0.1474	-0.0117	-0.0050	0.0777
3	5	0.1614	-0.1180	-0.1180	0.2106	0.0505	0.0069	0.1510
3	6	0.1491	-0.1252	-0.1757	0.2584	0.0987	0.0269	0.2180
3	7	0.1382	-0.3761	-0.4742	0.2644	0.0070	0.1234	0.2787
3	8	0.1286	-0.0285	-0.1094	0.3144	0.0587	-0.0164	0.3223
3	9	0.1201	0.0935	0.0528	0.3424	0.0631	0.0064	0.3663
3	10	0.1126	-0.1795	-0.3956	0.3834	0.1216	-0.0266	0.4306
3	11	0.1059	-0.1097	-0.3130	0.4510	0.2329	0.0585	0.4687
3	12	0.1000	-0.0474	-0.2394	0.4686	0.2876	0.1114	0.5041
4	4	0.1667	-0.1547	-0.1547	0.2577	0.1870	0.0317	0.1207
4	5	0.1571	0.1291	0.1291	0.2343	0.0914	0.0047	0.1648
4	6	0.1477	-0.0234	-0.0234	0.2652	0.1070	0.0824	0.1739
4	7	0.1389	-0.0482	-0.0952	0.2874	0.1126	-0.0284	0.2234
4	8	0.1307	-0.3695	-0.6204	0.3039	0.0604	0.0872	0.2614
4	9	0.1234	-0.2920	-0.5288	0.3246	0.0377	-0.0730	0.3285
4	10	0.1166	-0.1634	-0.2445	0.3613	0.0901	0.0162	0.3769
4	11	0.1106	-0.1027	-0.1795	0.3946	0.1376	0.0351	0.4149
4	12	0.1050	0.1598	0.0938	0.4249	0.1807	0.1231	0.4442
5	5	0.1508	-0.0445	-0.0445	0.2087	0.0037	-0.0464	0.1954
5	6	0.1437	-0.0850	-0.1337	0.2717	0.1043	-0.0359	0.2311
5	7	0.1367	-0.0321	-0.0784	0.2905	0.1005	0.0728	0.2305
5	8	0.1300	-0.0402	-0.1220	0.3253	0.1446	-0.0899	0.2683
5	9	0.1237	0.0662	0.0243	0.3464	0.1596	-0.0019	0.2912
5	10	0.1179	-0.2344	-0.4606	0.3547	0.0806	0.0615	0.3192
5	11	0.1124	-0.1775	-0.3933	0.3844	0.1230	0.0088	0.3453
5	12	0.1074	-0.0712	-0.1458	0.4119	0.1622	0.0658	0.3933
6	6	0.1387	-0.1094	-0.1967	0.2373	-0.0164	0.0375	0.2562
6	7	0.1332	-0.0057	-0.0508	0.2569	-0.0045	0.0398	0.2460
6	8	0.1278	-0.3384	-0.5836	0.3249	0.1321	0.0908	0.2769
6	9	0.1225	0.0202	-0.0569	0.3529	0.1681	-0.0128	0.2930
6	10	0.1174	0.0607	-0.0132	0.3797	0.2024	0.0158	0.3217
6	11	0.1126	-0.1235	-0.1755	0.3832	0.1213	-0.0277	0.3440
6	12	0.1081	-0.1328	-0.3403	0.4078	0.1563	0.0479	0.3702
7	7	0.1291	-0.0328	-0.1140	0.2692	-0.0036	0.0136	0.3076
7	8	0.1247	-0.3064	-0.5457	0.2940	0.0305	0.0905	0.2942
7	9	0.1203	-0.2603	-0.4912	0.3643	0.1827	0.1439	0.3080
7	10	0.1160	-0.1570	-0.2377	0.3871	0.2120	0.0244	0.3246
7	11	0.1118	0.1053	0.0349	0.4058	0.2175	0.0189	0.3486
7	12	0.1079	0.1371	0.0692	0.4094	0.1586	0.0341	0.3718
8	8	0.1213	0.0299	-0.0464	0.3135	0.0573	-0.0569	0.3496
8	9	0.1176	-0.1734	-0.2278	0.3340	0.0855	0.0466	0.3170
8	10	0.1140	-0.1941	-0.4128	0.3547	0.1138	-0.0749	0.3362
8	11	0.1104	-0.1564	-0.3683	0.4167	0.2501	0.0697	0.3570
8	12	0.1069	-0.0663	-0.1406	0.4320	0.2520	0.0622	0.3752
9	9	0.1147	0.0823	0.0102	0.3507	0.1083	0.0003	0.3092
9	10	0.1116	-0.1136	-0.1651	0.3680	0.1321	0.0952	0.3461
9	11	0.1086	-0.1371	-0.3455	0.3855	0.1561	-0.0614	0.3655
9	12	0.1055	-0.1051	-0.3076	0.3975	0.1559	0.0052	0.3833
10	10	0.1091	0.1271	0.0585	0.3824	0.1518	0.0491	0.3437
10	11	0.1065	-0.0620	-0.1112	0.3972	0.1723	0.0939	0.3787
10	12	0.1038	-0.0875	-0.2868	0.4071	0.1694	-0.0151	0.3932
11	11	0.1043	0.1659	0.1003	0.3989	0.1387	0.0914	0.3729
11	12	0.1020	-0.0170	-0.0641	0.4121	0.1576	0.1323	0.3832
12	12	0.1000	0.2000	0.1371	0.4234	0.1738	0.1491	0.3914

ERRORS IN MEANS ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.10)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	-0.0013	0.0000	-0.0015	0.0000	0.0000
3	4	0.4286	-0.0286	-0.0286	0.0298	0.0187	0.0129	0.0381
3	5	0.3750	-0.0000	-0.0000	0.0500	0.0317	0.0231	0.0302
3	6	0.3333	-0.0139	-0.0102	0.0167	-0.0155	0.0004	0.0556
3	7	0.3000	-0.0000	-0.0000	0.0500	0.0178	0.0270	0.0778
3	8	0.2727	0.0114	0.0053	0.0773	0.0451	0.0273	0.0939
3	9	0.2500	-0.0139	-0.0213	0.0795	0.0402	0.0113	0.1167
3	10	0.2308	-0.0058	-0.0169	0.0442	-0.0058	0.0255	0.1275
3	11	0.2143	-0.0143	-0.0286	0.0607	0.0107	0.0372	0.1429
3	12	0.2000	-0.0000	-0.0143	0.0750	0.0250	0.0515	0.1571
4	4	0.5000	0.0000	-0.0013	0.0000	-0.0015	0.0000	0.0000
4	5	0.4444	-0.0139	-0.0176	0.0139	0.0028	0.0016	0.0000
4	6	0.4000	-0.0250	-0.0250	0.0250	0.0067	0.0154	0.0444
4	7	0.3636	0.0114	0.0114	0.0014	0.0430	0.0210	0.0622
4	8	0.3333	-0.0083	-0.0028	0.0167	-0.0155	0.0157	0.0854
4	9	0.3077	-0.0009	0.0029	0.0423	0.0101	0.0016	0.0590
4	10	0.2857	-0.0218	-0.0144	0.0643	0.0321	0.0169	0.0810
4	11	0.2667	-0.0167	-0.0167	0.0629	0.0235	0.0057	0.1000
4	12	0.2500	-0.0139	-0.0213	0.0795	0.0402	0.0167	0.1147
5	5	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
5	6	0.4545	-0.0045	-0.0045	0.0455	0.0440	0.0038	0.0207
5	7	0.4167	0.0083	0.0028	0.0083	-0.0100	-0.0130	0.0278
5	8	0.3846	-0.0096	-0.0096	0.0404	0.0221	0.0084	0.0412
5	9	0.3571	-0.0071	-0.0024	0.0406	0.0163	0.0100	0.0616
5	10	0.3333	-0.0038	0.0027	0.0167	-0.0155	-0.0086	0.0854
5	11	0.3125	0.0125	0.0181	0.0375	0.0053	0.0122	0.0921
5	12	0.2941	-0.0191	-0.0080	0.0559	0.0237	0.0225	0.0706
6	6	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
6	7	0.4615	-0.0184	-0.0221	0.0385	0.0370	0.0027	0.0137
6	8	0.4286	-0.0036	-0.0091	-0.0036	-0.0219	0.0006	0.0467
6	9	0.4000	0.0000	-0.0048	0.0250	0.0067	0.0157	0.0187
6	10	0.3750	0.0250	0.0202	0.0500	0.0317	0.0018	0.0437
6	11	0.3529	-0.0029	0.0018	0.0448	0.0205	-0.0095	0.0658
6	12	0.3333	0.0167	0.0214	0.0644	0.0402	0.0003	0.0855
7	7	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
7	8	0.4667	0.0083	-0.0028	0.0533	0.0319	-0.0024	0.0086
7	9	0.4375	-0.0125	-0.0181	0.0284	0.0195	-0.0083	0.0333
7	10	0.4118	-0.0118	-0.0165	0.0132	-0.0051	0.0174	0.0591
7	11	0.3889	0.0111	0.0063	0.0361	0.0178	-0.0007	0.0299
7	12	0.3684	0.0066	0.0066	0.0493	0.0051	0.0198	0.0504
8	8	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
8	9	0.4706	0.0044	-0.0067	0.0294	0.0279	0.0102	0.0002
8	10	0.4444	0.0139	-0.0000	0.0415	0.0126	-0.0000	0.0285
8	11	0.4211	-0.0211	-0.0258	0.0039	-0.0144	-0.0050	0.0519
8	12	0.4000	0.0000	-0.0048	0.0250	0.0067	0.0011	0.0524
9	9	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
9	10	0.4737	0.0013	-0.0098	0.0263	0.0248	0.0036	-0.0007
9	11	0.4500	0.0083	-0.0056	0.0159	0.0070	-0.0056	0.0229
9	12	0.4286	0.0214	0.0071	0.0373	0.0284	-0.0125	0.0444
10	10	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
10	11	0.4762	-0.0012	-0.0123	-0.0103	-0.0192	0.0011	-0.0033
10	12	0.4545	0.0038	-0.0101	0.0114	0.0025	0.0038	0.0184
11	11	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000
11	12	0.4783	-0.0033	-0.0144	-0.0124	-0.0213	-0.0010	0.0217
12	12	0.5000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0000

ERRORS IN S.D.S ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.10)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.1173	0.0261	0.1817	-0.1624	0.1042	0.1121
3	4	0.1750	0.0757	-0.0886	0.2564	-0.0171	0.1398	0.1485
3	5	0.1614	0.1115	-0.0842	0.3080	0.0151	0.1810	0.2251
3	6	0.1491	0.1379	-0.0244	0.3783	0.1043	0.1191	0.2874
3	7	0.1382	0.2131	0.1112	0.4238	0.1698	0.1875	0.3452
3	8	0.1286	0.2702	0.2189	0.4638	0.2275	0.2425	0.3976
3	9	0.1201	0.1641	0.1110	0.4986	0.2555	0.0498	0.4373
3	10	0.1126	0.2079	0.1239	0.5503	0.3383	0.1184	0.4782
3	11	0.1059	0.2175	0.0798	0.5769	0.3774	0.1774	0.5112
3	12	0.1000	0.2615	0.1315	0.6007	0.4124	0.2236	0.5386
4	4	0.1667	0.2215	0.1411	0.2783	-0.0251	0.1472	0.1569
4	5	0.1571	0.0913	-0.0798	0.3322	0.0865	0.1743	0.2167
4	6	0.1477	0.1501	-0.0561	0.3666	0.0985	0.1945	0.2637
4	7	0.1389	0.2010	0.0071	0.4045	0.1525	0.2407	0.3106
4	8	0.1307	0.2146	0.0465	0.4548	0.2144	0.2139	0.3518
4	9	0.1234	0.2753	0.1551	0.4856	0.2588	0.0856	0.4221
4	10	0.1166	0.1881	0.1366	0.5136	0.2991	0.1451	0.4535
4	11	0.1106	0.2341	0.2341	0.5385	0.3146	0.0986	0.4820
4	12	0.1050	0.2690	0.2226	0.5616	0.3489	0.1580	0.5088
5	5	0.1508	-0.0445	-0.0445	0.3472	0.0727	0.1807	0.2373
5	6	0.1437	0.1814	0.0754	0.3776	0.1159	0.2263	0.2688
5	7	0.1367	0.1786	0.0028	0.4136	0.1655	0.1710	0.3184
5	8	0.1300	0.2519	0.0703	0.4424	0.2065	0.2257	0.3545
5	9	0.1237	0.1901	-0.0217	0.4673	0.2157	0.2313	0.3867
5	10	0.1179	0.2673	0.0983	0.5085	0.2919	0.0935	0.4158
5	11	0.1124	0.3246	0.1801	0.5312	0.3245	0.1353	0.4493
5	12	0.1074	0.2445	0.1644	0.5521	0.3547	0.1914	0.4977
6	6	0.1387	0.0392	0.0392	0.3995	0.1470	0.1753	0.2985
6	7	0.1332	0.2172	0.0874	0.4231	0.1805	0.1891	0.3222
6	8	0.1278	0.2324	0.0681	0.4520	0.2202	0.1742	0.3500
6	9	0.1225	0.1982	-0.0114	0.4748	0.2525	0.2380	0.3928
6	10	0.1174	0.2313	0.0303	0.4965	0.2834	0.1616	0.4179
6	11	0.1126	0.2626	0.0698	0.5150	0.2859	0.0706	0.4416
6	12	0.1081	0.2920	0.1069	0.5343	0.3144	0.1434	0.4615
7	7	0.1291	0.1056	0.1056	0.4410	0.2059	0.2322	0.3469
7	8	0.1247	0.1226	0.0296	0.4599	0.2329	0.2409	0.3656
7	9	0.1203	0.2772	0.1225	0.4735	0.2274	0.2224	0.3837
7	10	0.1160	0.2406	0.0420	0.5025	0.2920	0.2503	0.4058
7	11	0.1118	0.2678	0.0764	0.5204	0.3174	0.1733	0.4431
7	12	0.1079	0.2195	0.0099	0.5356	0.3162	0.2027	0.4629
8	8	0.1213	0.1598	0.1598	0.4749	0.2541	0.1598	0.3865
8	9	0.1176	0.1724	0.0847	0.4906	0.2764	0.2479	0.3973
8	10	0.1140	0.1751	0.0459	0.5011	0.2680	0.2200	0.4125
8	11	0.1104	0.2773	0.0883	0.5266	0.3262	0.0670	0.4310
8	12	0.1069	0.3001	0.1171	0.5415	0.3476	0.1676	0.4548
9	9	0.1147	0.2053	0.2053	0.5033	0.2945	0.2053	0.4067
9	10	0.1116	0.2146	0.1313	0.5165	0.3133	0.1056	0.4246
9	11	0.1086	0.2144	0.0914	0.5249	0.3029	0.2572	0.4405
9	12	0.1055	0.2208	0.0836	0.5383	0.3225	0.1084	0.4562
10	10	0.1091	0.2441	0.2441	0.5275	0.3289	0.2441	0.4357
10	11	0.1065	0.2510	0.1716	0.5340	0.3162	0.1470	0.4512
10	12	0.1038	0.2487	0.1310	0.5456	0.3333	0.2487	0.4649
11	11	0.1043	0.2777	0.2777	0.5486	0.3587	0.2777	0.4608
11	12	0.1020	0.2827	0.2067	0.5538	0.3452	0.1831	0.4726
12	12	0.1000	0.3072	0.3072	0.5670	0.3849	0.0202	0.4828

ERRORS IN MEANS ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.10)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	-0.0007	0.0000	0.0000	0.0000	0.0000
3	4	0.4286	0.0089	0.0089	-0.0015	-0.0078	0.0078	0.0356
3	5	0.3750	0.0250	0.0264	0.0521	0.0458	0.0030	0.0544
3	6	0.3333	-0.0333	-0.0330	0.0592	0.0490	-0.0060	0.0526
3	7	0.3000	-0.0000	0.0003	0.0221	0.0067	0.0077	0.0530
3	8	0.2727	-0.0102	-0.0102	-0.0023	-0.0195	0.0127	0.0770
3	9	0.2500	-0.0069	-0.0081	0.0130	-0.0056	0.0034	0.0998
3	10	0.2308	-0.0058	-0.0058	0.0322	0.0137	0.0043	0.0773
3	11	0.2143	0.0073	0.0074	0.0230	0.0021	0.0061	0.0885
3	12	0.2000	-0.0011	0.0005	0.0300	0.0088	0.0173	0.0991
4	4	0.5000	-0.0000	-0.0003	0.0000	0.0000	0.0000	0.0000
4	5	0.4444	-0.0069	-0.0069	0.0219	0.0181	-0.0097	0.0222
4	6	0.4000	0.0250	0.0253	0.0212	0.0139	0.0065	0.0255
4	7	0.3636	-0.0095	-0.0107	0.0141	0.0035	0.0079	0.0364
4	8	0.3333	0.0069	0.0082	0.0394	0.0286	0.0053	0.0441
4	9	0.3077	-0.0066	-0.0055	0.0465	0.0361	0.0016	0.0453
4	10	0.2857	-0.0107	-0.0107	0.0424	0.0291	0.0033	0.0673
4	11	0.2667	-0.0167	-0.0173	-0.0042	-0.0227	0.0003	0.0831
4	12	0.2500	-0.0069	-0.0081	0.0051	-0.0137	0.0010	0.0832
5	5	0.5000	0.0000	-0.0011	0.0000	0.0000	0.0000	-0.0000
5	6	0.4545	-0.0170	-0.0170	0.0090	0.0054	-0.0018	0.0121
5	7	0.4167	0.0083	0.0087	0.0371	0.0339	-0.0050	0.0500
5	8	0.3846	0.0112	0.0125	-0.0091	-0.0197	0.0040	0.0552
5	9	0.3571	-0.0071	-0.0071	0.0184	0.0065	0.0091	0.0203
5	10	0.3333	0.0167	0.0167	0.0208	0.0105	0.0013	0.0197
5	11	0.3125	0.0125	0.0126	0.0158	0.0025	0.0047	0.0405
5	12	0.2941	0.0045	0.0033	0.0276	0.0144	0.0093	0.0589
6	6	0.5000	0.0000	-0.0011	0.0000	-0.0000	0.0000	0.0000
6	7	0.4615	-0.0115	-0.0119	-0.0087	-0.0119	0.0027	0.0051
6	8	0.4286	-0.0024	-0.0014	0.0309	0.0270	0.0053	0.0381
6	9	0.4000	-0.0000	-0.0000	0.0236	0.0163	0.0118	0.0348
6	10	0.3750	-0.0069	-0.0081	0.0006	-0.0113	-0.0022	0.0483
6	11	0.3529	-0.0029	-0.0029	0.0016	-0.0087	-0.0021	0.0449
6	12	0.3333	0.0167	0.0167	0.0192	0.0088	0.0004	0.0197
7	7	0.5000	0.0000	-0.0011	0.0000	-0.0001	0.0000	0.0000
7	8	0.4667	0.0083	0.0083	-0.0072	-0.0111	0.0009	0.0000
7	9	0.4375	-0.0114	-0.0104	0.0199	0.0158	0.0004	0.0023
7	10	0.4118	-0.0118	-0.0118	0.0128	0.0055	0.0106	0.0230
7	11	0.3889	-0.0139	-0.0145	0.0182	0.0126	0.0012	0.0332
7	12	0.3684	-0.0004	-0.0015	-0.0138	-0.0242	0.0103	0.0511
8	8	0.5000	-0.0000	-0.0009	0.0000	-0.0000	0.0000	0.0000
8	9	0.4706	0.0044	0.0044	-0.0119	-0.0159	0.0054	0.0294
8	10	0.4444	0.0306	0.0306	0.0143	0.0103	0.0058	-0.0097
8	11	0.4211	-0.0211	-0.0211	0.0377	0.0337	-0.0013	0.0022
8	12	0.4000	-0.0000	-0.0000	0.0057	0.0001	0.0003	0.0224
9	9	0.5000	0.0000	-0.0006	0.0000	-0.0000	0.0000	0.0000
9	10	0.4737	0.0013	0.0013	-0.0150	-0.0190	-0.0021	0.0263
9	11	0.4500	0.0250	0.0250	0.0087	0.0047	0.0054	0.0121
9	12	0.4286	-0.0050	-0.0061	0.0115	0.0093	-0.0042	-0.0061
10	10	0.5000	0.0000	-0.0006	0.0000	-0.0000	0.0000	0.0000
10	11	0.4762	-0.0012	-0.0012	0.0238	0.0238	-0.0046	0.0238
10	12	0.4545	0.0205	0.0205	-0.0145	-0.0167	-0.0035	0.0067
11	11	0.5000	0.0000	-0.0006	0.0000	-0.0000	0.0000	0.0006
11	12	0.4783	-0.0033	-0.0033	0.0217	0.0217	-0.0021	0.0131
12	12	0.5000	0.0000	-0.0006	0.0000	-0.0000	0.0000	0.0000

ERRORS IN S.D.S ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.10)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.0103	-0.0208	0.1719	0.0872	0.0409	0.1062
3	4	0.1750	-0.1760	-0.2428	0.2368	0.1513	0.0598	0.1306
3	5	0.1614	0.0079	-0.0324	0.2961	0.2172	0.0180	0.1568
3	6	0.1491	-0.0176	-0.0776	0.3070	0.2001	0.0191	0.2017
3	7	0.1382	0.0568	0.0012	0.3500	0.2327	0.0748	0.2631
3	8	0.1286	-0.0704	-0.1505	0.3758	0.2027	0.1218	0.3162
3	9	0.1201	-0.0916	-0.2205	0.3963	0.1983	0.0085	0.3613
3	10	0.1126	0.0469	-0.0568	0.4340	0.2483	0.0472	0.4274
3	11	0.1059	0.0681	-0.0462	0.4535	0.2294	0.0472	0.4651
3	12	0.1000	0.1535	0.0770	0.4619	0.2571	0.1114	0.4974
4	4	0.1667	0.0572	0.0218	0.2697	0.1950	0.0572	0.1340
4	5	0.1571	-0.0562	-0.1161	0.2850	0.1871	0.0175	0.1333
4	6	0.1477	-0.0083	-0.0677	0.3351	0.2561	0.0512	0.2185
4	7	0.1389	-0.1267	-0.2049	0.3169	0.1829	0.0909	0.2552
4	8	0.1307	-0.0888	-0.1947	0.3450	0.2066	0.0880	0.2982
4	9	0.1234	0.1198	0.0607	0.3543	0.1654	0.0049	0.3421
4	10	0.1166	0.0127	-0.0947	0.3607	0.1681	0.0491	0.3779
4	11	0.1106	-0.0328	-0.1395	0.4403	0.2519	0.0575	0.4120
4	12	0.1050	0.0454	-0.0673	0.4637	0.2721	0.0777	0.4481
5	5	0.1508	-0.1329	-0.1926	0.3394	0.2718	-0.0133	0.1896
5	6	0.1437	0.0339	-0.0210	0.3016	0.1873	0.0384	0.2072
5	7	0.1367	0.0666	0.0116	0.3174	0.1913	0.0107	0.2458
5	8	0.1300	-0.0549	-0.1281	0.3515	0.2175	0.1072	0.2739
5	9	0.1237	-0.0472	-0.1611	0.3576	0.2038	0.0872	0.3359
5	10	0.1179	0.0025	-0.1060	0.3831	0.2026	0.0163	0.3715
5	11	0.1124	0.0884	0.0246	0.3988	0.1874	0.0745	0.4005
5	12	0.1074	-0.0854	-0.2258	0.4217	0.2060	0.0927	0.4272
6	6	0.1387	-0.0421	-0.0970	0.3496	0.2663	0.0030	0.2155
6	7	0.1332	0.0905	0.0369	0.3302	0.2030	0.0088	0.2651
6	8	0.1278	0.0882	0.0270	0.3296	0.1812	0.0284	0.2952
6	9	0.1225	-0.0366	-0.1494	0.3393	0.1596	0.0893	0.3108
6	10	0.1174	-0.0672	-0.1933	0.3903	0.2444	-0.0118	0.3373
6	11	0.1126	0.0466	-0.0371	0.4057	0.2272	0.0132	0.3675
6	12	0.1081	0.0846	-0.0149	0.4252	0.2471	0.0565	0.4232
7	7	0.1291	0.0299	-0.0215	0.3434	0.2273	0.0718	0.2697
7	8	0.1247	-0.0557	-0.1705	0.3457	0.2008	0.0789	0.3121
7	9	0.1203	0.1414	0.0838	0.3647	0.2207	0.0828	0.3281
7	10	0.1160	0.0181	-0.0886	0.3699	0.1950	0.1147	0.3472
7	11	0.1118	-0.0448	-0.1527	0.3844	0.1786	0.0506	0.3683
7	12	0.1079	0.0196	-0.0962	0.4309	0.2600	0.0470	0.3909
8	8	0.1213	-0.0627	-0.1681	0.3364	0.1556	0.0299	0.3140
8	9	0.1176	0.0042	-0.1041	0.3743	0.2294	0.0990	0.3345
8	10	0.1140	0.0351	-0.0698	0.3937	0.2533	0.0871	0.3585
8	11	0.1104	0.0656	-0.0361	0.4129	0.2768	0.0081	0.3769
8	12	0.1069	0.0951	-0.0033	0.4072	0.2031	0.0643	0.3873
9	9	0.1147	-0.0716	-0.1823	0.3723	0.2013	0.0823	0.3511
9	10	0.1116	0.0550	-0.0478	0.4062	0.2687	-0.0006	0.3530
9	11	0.1086	0.0811	-0.0188	0.4227	0.2889	0.1216	0.3596
9	12	0.1055	-0.0663	-0.2042	0.4195	0.2431	0.0458	0.3953
10	10	0.1091	-0.0193	-0.1246	0.4029	0.2403	0.1271	0.3677
10	11	0.1065	0.0987	0.0007	0.4173	0.2586	0.0457	0.3830
10	12	0.1038	0.1212	0.0256	0.4288	0.2551	0.0935	0.3869
11	11	0.1043	0.0260	-0.0746	0.3936	0.1321	0.1596	0.3945
11	12	0.1020	0.1369	0.0431	0.4070	0.1511	0.0863	0.3975
12	12	0.1000	0.0658	-0.0307	0.4184	0.1675	0.0202	0.3975

ERRORS IN MEANS ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.01)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	0.0000	-0.0013	0.0000	-0.0024	0.0000	0.0000
3	4	0.4286	0.0009	0.0011	0.0139	0.0037	0.0107	0.0235
3	5	0.3750	-0.0025	0.0014	0.0253	0.0054	0.0148	0.0412
3	6	0.3333	-0.0038	-0.0018	0.0342	0.0067	0.0154	0.0626
3	7	0.3000	-0.0025	-0.0015	0.0450	0.0123	0.0184	0.0770
3	8	0.2727	-0.0038	-0.0072	0.0523	0.0160	0.0197	0.0932
3	9	0.2500	-0.0038	-0.0122	0.0575	0.0163	0.0258	0.1063
3	10	0.2308	-0.0033	-0.0132	0.0617	0.0170	0.0279	0.1177
3	11	0.2143	-0.0043	-0.0140	0.0632	0.0152	0.0345	0.1318
3	12	0.2000	-0.0025	-0.0117	0.0639	0.0127	0.0410	0.1392
4	4	0.5000	0.0000	-0.0007	0.0000	0.0020	0.0000	0.0000
4	5	0.4444	0.0031	0.0012	0.0131	0.0060	0.0056	0.0211
4	6	0.4000	-0.0025	-0.0004	0.0200	0.0043	0.0073	0.0389
4	7	0.3636	-0.0013	0.0002	0.0290	0.0070	0.0094	0.0534
4	8	0.3333	-0.0033	0.0017	0.0392	0.0126	0.0106	0.0672
4	9	0.3077	-0.0027	0.0037	0.0448	0.0136	0.0128	0.0825
4	10	0.2857	-0.0023	0.0017	0.0497	0.0147	0.0175	0.0919
4	11	0.2667	-0.0042	-0.0057	0.0503	0.0110	0.0231	0.1027
4	12	0.2500	-0.0050	-0.0122	0.0575	0.0161	0.0266	0.1167
5	5	0.5000	-0.0000	-0.0004	0.0000	-0.0017	-0.0000	0.0000
5	6	0.4545	0.0005	-0.0026	0.0114	0.0022	0.0018	0.0175
5	7	0.4167	0.0033	0.0022	0.0183	0.0058	0.0048	0.0338
5	8	0.3846	-0.0021	-0.0033	0.0254	0.0039	0.0060	0.0479
5	9	0.3571	-0.0021	0.0011	0.0329	0.0101	0.0065	0.0606
5	10	0.3333	-0.0008	0.0057	0.0392	0.0125	0.0113	0.0715
5	11	0.3125	-0.0000	0.0086	0.0423	0.0115	0.0143	0.0843
5	12	0.2941	-0.0016	0.0068	0.0487	0.0152	0.0188	0.0945
6	6	0.5000	0.0000	-0.0001	0.0000	-0.0015	0.0000	0.0000
6	7	0.4615	-0.0007	-0.0054	0.0075	0.0026	0.0024	0.0170
6	8	0.4286	0.0014	-0.0016	0.0189	0.0059	0.0033	0.0312
6	9	0.4000	-0.0000	-0.0044	0.0200	0.0039	0.0044	0.0446
6	10	0.3750	-0.0000	-0.0000	0.0275	0.0046	0.0082	0.0549
6	11	0.3529	-0.0004	0.0040	0.0346	0.0083	0.0094	0.0644
6	12	0.3333	0.0013	0.0080	0.0366	0.0092	0.0139	0.0767
7	7	0.5000	0.0000	-0.0001	0.0000	0.0012	0.0000	0.0000
7	8	0.4667	-0.0017	-0.0072	0.0073	0.0003	-0.0004	0.0151
7	9	0.4375	-0.0025	-0.0112	0.0125	0.0031	0.0023	0.0281
7	10	0.4118	0.0007	-0.0061	0.0407	0.0046	0.0054	0.0388
7	11	0.3889	0.0010	-0.0021	0.0257	0.0056	0.0062	0.0490
7	12	0.3684	0.0016	0.0027	0.0291	0.0078	0.0115	0.0609
8	8	0.5000	0.0000	-0.0001	0.0000	0.0012	0.0000	0.0000
8	9	0.4706	-0.0006	-0.0067	0.0069	0.0009	0.0011	0.0138
8	10	0.4444	-0.0019	-0.0123	0.0106	-0.0005	0.0037	0.0246
8	11	0.4211	-0.0011	-0.0096	0.0189	0.0045	0.0049	0.0354
8	12	0.4000	-0.0000	-0.0054	0.0200	0.0038	0.0088	0.0463
9	9	0.5000	0.0000	-0.0000	0.0000	-0.0010	0.0000	0.0000
9	10	0.4737	0.0013	-0.0054	0.0078	0.0026	0.0009	0.0109
9	11	0.4500	-0.0025	-0.0137	0.0075	-0.0004	0.0025	0.0217
9	12	0.4286	-0.0011	-0.0114	0.0146	0.0011	0.0052	0.0331
10	10	0.5000	0.0000	-0.0000	0.0000	-0.0010	0.0000	0.0000
10	11	0.4762	-0.0002	-0.0090	0.0049	-0.0002	-0.0005	0.0110
10	12	0.4545	-0.0020	-0.0140	0.0114	0.0029	0.0067	0.0202
11	11	0.5000	0.0000	-0.0000	0.0000	-0.0008	0.0000	0.0000
11	12	0.4783	-0.0023	-0.0111	0.0067	0.0026	0.0051	0.0093
12	12	0.5000	0.0000	0.0000	0.0000	-0.0008	0.0000	0.0000

ERRORS IN S.D.S ASSUMING 3 IMPERFECT ASSESSMENTS (ACC. PAR. =0.01)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.1173	0.0261	0.2276	0.0355	0.0932	0.0886
3	4	0.1750	0.1447	0.0193	0.2730	0.0471	0.1334	0.1457
3	5	0.1614	0.1511	0.0171	0.3283	0.0899	0.1625	0.2170
3	6	0.1491	0.1690	0.0129	0.3778	0.1212	0.1688	0.2828
3	7	0.1382	0.2000	0.0947	0.4218	0.1526	0.1813	0.3446
3	8	0.1286	0.2145	0.1734	0.4647	0.2007	0.1781	0.3971
3	9	0.1201	0.2344	0.1766	0.5017	0.2405	0.1901	0.4433
3	10	0.1126	0.2547	0.1875	0.5518	0.2608	0.1832	0.4840
3	11	0.1059	0.2597	0.1885	0.5640	0.3104	0.1960	0.5169
3	12	0.1000	0.2807	0.2077	0.5883	0.3279	0.2343	0.5497
4	4	0.1667	0.1487	0.0660	0.2986	0.0687	0.1555	0.1718
4	5	0.1571	0.1669	0.0444	0.3360	0.1024	0.1765	0.2141
4	6	0.1477	0.1847	0.0444	0.3719	0.1190	0.1868	0.2642
4	7	0.1389	0.1938	-0.0025	0.4074	0.1399	0.1929	0.3147
4	8	0.1307	0.2212	0.0501	0.4451	0.1873	0.1877	0.3624
4	9	0.1234	0.2431	0.1090	0.4752	0.2049	0.1843	0.4038
4	10	0.1166	0.2753	0.1915	0.5076	0.2503	0.1846	0.4440
4	11	0.1106	0.2926	0.2643	0.5335	0.2674	0.1969	0.4785
4	12	0.1050	0.3121	0.2680	0.5591	0.3050	0.1998	0.5069
5	5	0.1508	0.1743	0.1048	0.3564	0.1168	0.1919	0.2373
5	6	0.1437	0.1876	0.0752	0.3797	0.1220	0.1982	0.2711
5	7	0.1367	0.2030	0.0705	0.4083	0.1420	0.2029	0.3107
5	8	0.1300	0.2091	0.0035	0.4372	0.1694	0.1993	0.3486
5	9	0.1237	0.2336	0.0358	0.4648	0.1926	0.1881	0.3860
5	10	0.1179	0.2587	0.0829	0.4933	0.2319	0.1953	0.4213
5	11	0.1124	0.2807	0.1339	0.5156	0.2422	0.1868	0.4521
5	12	0.1074	0.3071	0.2000	0.5392	0.2746	0.1898	0.4815
6	6	0.1387	0.1761	0.1257	0.3995	0.1470	0.2084	0.2918
6	7	0.1332	0.1873	0.0875	0.4193	0.1570	0.2081	0.3186
6	8	0.1278	0.2250	0.1033	0.4417	0.1777	0.2076	0.3496
6	9	0.1225	0.2189	0.0165	0.4648	0.1948	0.2082	0.3798
6	10	0.1174	0.2414	0.0361	0.4862	0.2136	0.2043	0.4107
6	11	0.1126	0.2545	0.0577	0.5099	0.2490	0.1932	0.4399
6	12	0.1081	0.2793	0.1020	0.5306	0.2667	0.2011	0.4657
7	7	0.1291	0.1968	0.1607	0.4331	0.1655	0.2138	0.3382
7	8	0.1247	0.2244	0.1350	0.4523	0.1912	0.2084	0.3594
7	9	0.1203	0.2251	0.0621	0.4705	0.2050	0.2035	0.3844
7	10	0.1160	0.2380	0.0484	0.4879	0.2172	0.2095	0.4094
7	11	0.1118	0.2513	0.0499	0.5082	0.2455	0.1963	0.4348
7	12	0.1079	0.2632	0.0628	0.5260	0.2592	0.2140	0.4579
8	8	0.1213	0.2455	0.2116	0.4675	0.2161	0.2058	0.3746
8	9	0.1176	0.2549	0.1762	0.4768	0.2009	0.2101	0.3941
8	10	0.1140	0.2405	0.0905	0.4944	0.2277	0.2006	0.4146
8	11	0.1104	0.2545	0.0758	0.5118	0.2539	0.1940	0.4354
8	12	0.1069	0.2593	0.0653	0.5267	0.2614	0.2093	0.4564
9	9	0.1147	0.2491	0.2304	0.4894	0.2201	0.2053	0.4073
9	10	0.1116	0.2787	0.2103	0.5028	0.2384	0.2053	0.4239
9	11	0.1086	0.2652	0.1282	0.5152	0.2450	0.2009	0.4413
9	12	0.1055	0.2645	0.0972	0.5273	0.2522	0.1970	0.4588
10	10	0.1091	0.2857	0.2680	0.5143	0.2581	0.2092	0.4362
10	11	0.1065	0.2742	0.2010	0.5204	0.2415	0.1938	0.4497
10	12	0.1038	0.2848	0.1603	0.5331	0.2615	0.1984	0.4648
11	11	0.1043	0.3175	0.3005	0.5297	0.2540	0.2030	0.4606
11	12	0.1020	0.3049	0.2349	0.5402	0.2708	0.1943	0.4729
12	12	0.1000	0.3072	0.3072	0.5489	0.2845	0.1848	0.4823

ERRORS IN MEANS ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.01)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.5000	-0.0000	-0.0003	0.0000	-0.0001	0.0000	-0.0000
3	4	0.4286	-0.0024	-0.0026	0.0052	-0.0004	0.0024	0.0081
3	5	0.3750	-0.0038	-0.0036	0.0135	0.0043	0.0030	0.0216
3	6	0.3333	-0.0021	-0.0021	0.0170	0.0042	0.0054	0.0323
3	7	0.3000	-0.0000	-0.0005	0.0175	0.0020	0.0066	0.0392
3	8	0.2727	0.0023	0.0023	0.0246	0.0066	0.0071	0.0526
3	9	0.2500	-0.0025	-0.0026	0.0288	0.0100	0.0097	0.0623
3	10	0.2308	-0.0008	-0.0004	0.0352	0.0147	0.0089	0.0744
3	11	0.2143	-0.0018	-0.0009	0.0320	0.0093	0.0100	0.0802
3	12	0.2000	0.0000	0.0013	0.0279	0.0040	0.0123	0.0872
4	4	0.5000	-0.0000	-0.0004	0.0000	-0.0001	0.0000	0.0000
4	5	0.4444	0.0031	0.0028	0.0043	-0.0000	0.0021	0.0114
4	6	0.4000	-0.0025	-0.0022	0.0087	0.0014	0.0030	0.0205
4	7	0.3636	-0.0057	-0.0060	0.0151	0.0041	0.0039	0.0283
4	8	0.3333	-0.0008	-0.0009	0.0111	-0.0024	0.0050	0.0415
4	9	0.3077	-0.0002	-0.0000	0.0223	0.0074	0.0059	0.0474
4	10	0.2857	-0.0032	-0.0036	0.0255	0.0082	0.0067	0.0565
4	11	0.2667	-0.0004	-0.0011	0.0258	0.0072	0.0084	0.0612
4	12	0.2500	-0.0000	-0.0007	0.0246	0.0047	0.0084	0.0658
5	5	0.5000	0.0000	-0.0004	0.0000	0.0000	0.0000	0.0000
5	6	0.4545	0.0030	0.0031	0.0057	0.0018	0.0003	0.0090
5	7	0.4167	-0.0017	-0.0017	0.0109	0.0046	0.0027	0.0222
5	8	0.3846	-0.0009	-0.0011	0.0116	0.0023	0.0035	0.0265
5	9	0.3571	-0.0021	-0.0027	0.0109	-0.0003	0.0043	0.0346
5	10	0.3333	0.0004	0.0002	0.0188	0.0054	0.0059	0.0410
5	11	0.3125	0.0012	0.0015	0.0225	0.0074	0.0052	0.0501
5	12	0.2941	-0.0004	-0.0002	0.0196	0.0032	0.0056	0.0573
6	6	0.5000	-0.0000	-0.0004	0.0000	-0.0000	0.0000	0.0000
6	7	0.4615	-0.0013	-0.0011	-0.0003	-0.0031	0.0020	0.0110
6	8	0.4286	0.0014	0.0020	0.0102	0.0042	0.0024	0.0144
6	9	0.4000	0.0009	0.0011	0.0110	0.0033	0.0033	0.0246
6	10	0.3750	-0.0013	-0.0021	0.0150	0.0053	0.0046	0.0332
6	11	0.3529	-0.0013	-0.0021	0.0171	0.0053	0.0051	0.0389
6	12	0.3333	0.0039	0.0034	0.0200	0.0068	0.0063	0.0462
7	7	0.5000	-0.0000	-0.0004	0.0000	-0.0000	0.0000	0.0000
7	8	0.4667	0.0008	0.0009	0.0030	0.0001	-0.0001	0.0107
7	9	0.4375	-0.0025	-0.0024	0.0087	0.0036	0.0017	0.0168
7	10	0.4118	0.0007	0.0012	0.0107	0.0040	0.0036	0.0224
7	11	0.3889	0.0024	0.0024	0.0127	0.0040	0.0028	0.0314
7	12	0.3684	0.0022	0.0024	0.0159	0.0054	0.0057	0.0323
8	8	0.5000	-0.0000	-0.0004	0.0000	0.0000	0.0000	0.0000
8	9	0.4706	-0.0006	-0.0008	0.0032	0.0006	0.0012	0.0097
8	10	0.4444	-0.0006	-0.0003	0.0043	0.0001	0.0025	0.0150
8	11	0.4211	-0.0025	-0.0018	0.0065	0.0003	0.0029	0.0215
8	12	0.4000	-0.0000	0.0009	0.0163	0.0084	0.0044	0.0287
9	9	0.5000	0.0000	-0.0003	0.0000	-0.0000	0.0000	0.0000
9	10	0.4737	0.0018	0.0014	0.0042	0.0021	0.0007	0.0069
9	11	0.4500	-0.0025	-0.0029	0.0062	0.0020	0.0017	0.0134
9	12	0.4286	-0.0011	-0.0007	0.0102	0.0043	0.0021	0.0219
10	10	0.5000	0.0000	-0.0003	0.0000	-0.0000	0.0000	-0.0000
10	11	0.4762	0.0005	-0.0000	0.0022	0.0000	0.0001	0.0064
10	12	0.4545	-0.0008	-0.0015	0.0057	0.0018	0.0032	0.0137
11	11	0.5000	0.0000	-0.0002	0.0000	-0.0000	0.0000	0.0007
11	12	0.4783	0.0005	-0.0010	0.0042	0.0025	0.0013	0.0044
12	12	0.5000	-0.0000	-0.0006	0.0000	-0.0000	0.0000	-0.0000

ERRORS IN S.D.S ASSUMING 7 IMPERFECT ASSESSMENTS (ACC. PAR. =0.01)

N1	N2	TRUE VALUE	ERROR METHOD 1	ERROR METHOD 2	ERROR METHOD 3	ERROR METHOD 4	ERROR METHOD 5	ERROR METHOD 6
3	3	0.1890	0.0575	0.0271	0.1266	0.0160	0.0310	0.0427
3	4	0.1750	0.0260	-0.0144	0.1600	0.0226	0.0397	0.0845
3	5	0.1614	0.0411	-0.0046	0.2115	0.0549	0.0424	0.1425
3	6	0.1491	0.0361	-0.0204	0.2533	0.0658	0.0596	0.2072
3	7	0.1382	0.0636	0.0110	0.3025	0.1006	0.0658	0.2706
3	8	0.1286	0.0805	0.0267	0.3333	0.0998	0.0793	0.3248
3	9	0.1201	0.0637	-0.0203	0.3784	0.1417	0.1016	0.3795
3	10	0.1126	0.0843	-0.0062	0.4185	0.1916	0.1035	0.4237
3	11	0.1059	0.0982	0.0030	0.4426	0.1871	0.1104	0.4667
3	12	0.1000	0.1236	0.0291	0.4736	0.2116	0.1424	0.5024
4	4	0.1667	0.0508	0.0133	0.1812	0.0333	0.0417	0.1000
4	5	0.1571	0.0462	-0.0002	0.2052	0.0337	0.0489	0.1391
4	6	0.1477	0.0542	0.0078	0.2452	0.0609	0.0569	0.1788
4	7	0.1389	0.0421	-0.0202	0.2826	0.0845	0.0597	0.2317
4	8	0.1307	0.0674	0.0027	0.3095	0.0801	0.0731	0.2815
4	9	0.1234	0.0683	-0.0000	0.3489	0.1240	0.0838	0.3262
4	10	0.1166	0.0601	-0.0137	0.3848	0.1574	0.0945	0.3713
4	11	0.1106	0.0956	0.0196	0.4132	0.1739	0.1093	0.4145
4	12	0.1050	0.1111	0.0257	0.4390	0.1899	0.1152	0.4549
5	5	0.1508	0.0505	0.0006	0.2294	0.0542	0.0597	0.1524
5	6	0.1437	0.0410	-0.0191	0.2531	0.0695	0.0529	0.1824
5	7	0.1367	0.0725	0.0195	0.2831	0.0874	0.0566	0.2209
5	8	0.1300	0.0615	0.0044	0.3086	0.1006	0.0711	0.2592
5	9	0.1237	0.0761	0.0093	0.3590	0.1199	0.0758	0.3033
5	10	0.1179	0.0775	0.0059	0.3658	0.1353	0.0890	0.3436
5	11	0.1124	0.0976	0.0237	0.3944	0.1653	0.0909	0.3812
5	12	0.1074	0.0897	0.0069	0.4177	0.1735	0.0946	0.4129
6	6	0.1387	0.0392	-0.0180	0.2709	0.0828	0.0701	0.2034
6	7	0.1332	0.0437	-0.0228	0.2917	0.0942	0.0723	0.2273
6	8	0.1278	0.0744	0.0073	0.3091	0.0935	0.0693	0.2606
6	9	0.1225	0.0652	-0.0068	0.3521	0.1033	0.0817	0.2930
6	10	0.1174	0.0847	0.0216	0.3615	0.1376	0.0866	0.3260
6	11	0.1126	0.0827	0.0110	0.3849	0.1575	0.0903	0.3568
6	12	0.1081	0.1054	0.0313	0.4052	0.1697	0.1017	0.3905
7	7	0.1291	0.0534	-0.0048	0.2986	0.0819	0.0778	0.2450
7	8	0.1247	0.0759	0.0089	0.3212	0.1078	0.0711	0.2658
7	9	0.1203	0.0664	-0.0175	0.3402	0.1225	0.0767	0.2922
7	10	0.1160	0.0826	0.0020	0.3571	0.1252	0.0908	0.3203
7	11	0.1118	0.0754	0.0022	0.3817	0.1539	0.0863	0.3482
7	12	0.1079	0.0789	-0.0102	0.4067	0.1863	0.1102	0.3800
8	8	0.1213	0.0887	0.0317	0.3270	0.1003	0.0665	0.2803
8	9	0.1176	0.0926	0.0233	0.3477	0.1275	0.0782	0.2998
8	10	0.1140	0.0787	-0.0100	0.3652	0.1380	0.0797	0.3236
8	11	0.1104	0.0787	-0.0121	0.3859	0.1619	0.0840	0.3492
8	12	0.1069	0.0951	0.0125	0.3968	0.1557	0.1008	0.3734
9	9	0.1147	0.0699	0.0117	0.3540	0.1116	0.0823	0.3173
9	10	0.1116	0.1072	0.0380	0.3733	0.1408	0.0901	0.3337
9	11	0.1086	0.0830	-0.0100	0.3903	0.1604	0.0946	0.3544
9	12	0.1055	0.0882	-0.0061	0.3949	0.1431	0.0970	0.3715
10	10	0.1091	0.0965	0.0399	0.3856	0.1549	0.0990	0.3447
10	11	0.1065	0.0727	-0.0151	0.3993	0.1707	0.0958	0.3608
10	12	0.1038	0.0918	-0.0033	0.4040	0.1600	0.1039	0.3776
11	11	0.1043	0.1176	0.0626	0.3988	0.1506	0.1060	0.3723
11	12	0.1020	0.1016	0.0162	0.4123	0.1695	0.1046	0.3855
12	12	0.1000	0.0710	-0.0250	0.4234	0.1853	0.1043	0.3962

APPENDIX G

OUTPUT FROM SENSITIVITY ANALYSES

Tables G.1 - G.5 show the results of carrying out sensitivity analyses for case studies A - E. The data used for the variables is shown in appendix C. For all case studies the performance measure has been assumed to be NPV with a discount rate of 10%.

The values of the performance measure which would be calculated using most likely values for all variables are as follows:

Case A:	5880	(in \$'000s)
Case B:	5670	(in \$'000s)
Case C:	59.3	(in Lire M)
Case D:	8923	(in £'000s)
Case E:	2262	(in \$'000s)

Table G.1 Sensitivity Analysis: Case A

Variable	NPV at lowest value (\$'000s)	NPV at highest value (\$'000s)	Range of NPV (\$'000s)	Range Coefficient
Selling Price	-19934	19303	39237	1.00
Operating Cost	-16836	19303	36139	0.92
Mkt. Share	-5736	12334	18070	0.46
Init. Mkt. Size	-3413	11456	14870	0.38
Life of Invest.	9450	1163	8287	0.21
Mkt. Growth	4217	7804	3587	0.09
Init. Invest.	4880	8380	3500	0.09
Fixed Costs	6187	5419	768	0.02
Residual Value	6072	5494	578	0.01

Table G.2 Sensitivity Analysis: Case B

Variable	NPV at lowest value ($'000s)	NPV at highest value ($'000s)	Range of NPV ($'000s)	Range Coefficient
Cost Growth Rate	9899	-10550	20449	1.00
Production	-1004	13680	14684	0.72
Price Growth	-597	11815	12412	0.61
Sales Price	2853	8271	5418	0.26
Life of Project	3164	6410	3247	0.16
Extra Var. Costs 2	6207	2988	3218	0.16
Fixed Costs	6148	4236	1913	0.09
Var. Costs Now	6314	4812	1502	0.07
Capital Costs	6656	5177	1479	0.07
Extra Var. Costs 1	6207	4866	1341	0.07
Extra Var. Costs 3	5752	5215	536	0.03
Working Capital	5889	5533	356	0.02

Table G.3 Sensitivity Analysis: Case C

Variable	NPV at lowest value (Lire M)	NPV at highest value (Lire M)	Range of NPV (Lire M)	Range Coefficient
Market Share	-42.7	277.1	319.8	1.00
Init. Mkt. Size	-18.2	168.2	186.4	0.58
Mkt. Growth	3.1	122.2	119.0	0.37
Var. Sales Exp.	99.6	-2.7	102.4	0.32
Cost of Goods	99.6	-2.7	102.4	0.32
Fixed Costs	85.8	0.0	85.8	0.27
Price Adj. Factor	34.2	76.0	41.8	0.13

Table G.4 Sensitivity Analysis: Case D

Variable	NPV at lowest value £'000s	NPV at highest value £'000s	Range of NPV £'000s	Range Coefficient
Life of Plant	8069	9935	1866	1.00
Prodn. Capacity	8180	9244	1064	0.57
Start Year	8499	9406	907	0.49
Price (Home Mkt)	8532	9314	782	0.42
Fixed Costs Prodn.	9314	8727	587	0.31
Sales Vol (Home Mkt)	8415	8923	508	0.27
Var. Costs (Home Mkt)	9080	8689	391	0.21
Capital Costs	9003	8883	120	0.06
Var. Costs Prodn.	8962	8884	78	0.04
Fixed costs (Home Mkt).	8962	8884	78	0.04
Working Capital	8955	8891	65	0.03
Fixed Costs (Exp. Mktd).	8943	8904	39	0.02
Start Costs	8934	8913	21	0.01
Salvage Value	8919	8927	8	0.00
Sales Vol (Exp. Mkt)	8923	8923	0	0.00
Var. Costs (Exp Mkt)	8923	8923	0	0.00
Price (Exp Mkt)	8923	8923	0	0.00

Table G.5 Sensitivity Analysis: Case E

Variable	NPV at lowest value ($'000s)	NPV at highest value ($'000s)	Range of NPV ($'000s)	Range Coefficient
Year: 4th Gen. Equip	837	7165	6328	1.00
Year: Hostile Act	2082	5895	3813	0.60
Mkt: Int. Growth	1994	3379	1386	0.22
Mkt: Ext Growth	2100	2966	867	0.14

APPENDIX H

AN EXAMINATION OF THE INTERPRETATION WHICH CAN BE PUT ON THE RESULTS FROM A SENSITIVITY ANALYSIS

This appendix investigates whether, in case A, the results from a sensitivity analysis provide a good indication as to:

(i) the relative importance of errors in the means of different variables.

(ii) the relative importance of errors in the standard deviations of different variables.

and (iii) the relative importance of dependencies between different pairs of variables.

Table H.1 summarises the data in tables 6.3 and 7.5. Table H.2 provides the following ratios:

Ratio I : $\left|\dfrac{\text{Effect of error in mean of variable on mean of NPV}}{\text{Sensitivity Coefficient}}\right|$

Ratio II : $\left|\dfrac{\text{Effect of error in mean of variable on S.D. of NPV}}{\text{Sensitivity Coefficient}}\right|$

Ratio III : $\left|\dfrac{\text{Effect of error in mean of variable on S.D. of NPV}}{\text{Square of Sensitivity Coefficient}}\right|$

Ratio IV : $\left|\dfrac{\text{Effect of error in S.D. of variable on S.D. of NPV}}{\text{Sensitivity Coefficient}}\right|$

Ratio V : $\left|\dfrac{\text{Effect of error in S.D. of variable on S.D. of NPV}}{\text{Square of Sensitivity Coefficient}}\right|$

Table H.1 Sensitivity Coefficients compared with effect of errors in means and standard deviations ($'000s)

Variable	Sensitivity Coefficient $\lvert S_i \rvert$	Effect of error in mean equal to 5% of range on		Effect of 30% error in S.D. on S.D. of NPV
		Mean of NPV	S.D. of NPV	
Init. Mkt. Size	14870	349	464	201
Mkt. Growth	3587	83	121	27
Selling Price	39237	1626	177	1591
Mkt. Share	18070	457	579	295
Init. Invest.	3500	-175	0	12
Life of Invest.	8287	107	316	10
Residual Value	578	27	0	2
Op. Costs	36139	-1498	131	1280
Fixed Costs	768	-39	0	-1

Table H.2 Ratios calculated from table H.1

Variable	Ratio I	Ratio II	Ratio III $\times 10^6$	Ratio IV	Ratio V $\times 10^6$
Init. Mkt. Size	0.023	0.031	2.1	0.014	0.91
Mkt. Growth	0.023	0.034	9.4	0.008	2.10
Selling Price	0.041	0.005	0.1	0.041	1.03
Mkt. Share	0.025	0.032	1.8	0.016	0.90
Init. Invest.	0.050	0.000	0.0	0.003	0.98
Life of Invest.	0.013	0.038	4.6	0.001	0.15
Residual Value	0.047	0.000	0.0	0.003	5.98
Operating Costs	0.041	0.004	0.1	0.035	0.98
Fixed Costs	0.051	0.000	0.0	0.001	1.70

Ratio I shows that the magnitude of a sensitivity coefficient provides a moderately good indication of the effect on the mean of NPV of errors in the mean of a variable. Ratios II and III show that neither the sensitivity coefficient nor the square of the sensitivity coefficient provides a particularly good indication as to the effect on the S.D. of NPV of errors in the mean of a variable. Ratios IV and V are similarly discouraging as far as the effect on the S.D. of NPV of errors in the S.D. of a variable.

Table H.3 compares the effect of total dependence between different pairs of variables with the product of their sensitivity coefficients. It provides the following ratios:

Ratio VI : $$\frac{|\text{Effect on Mean of NPV of total dependence}|}{\sqrt{\text{Product of Sensitivity Coefficients}}}$$

Ratio VII : $$\left|\frac{\text{Effect on S.D. of NPV of total dependence}}{\text{Product of Sensitivity Coefficients}}\right|$$

(The square root of the product of sensitivity coefficients was taken in ratio VI because of the results produced for the second of the two models considered in section 7.2).

Table H.3 Effect of total dependence between different pairs of variables compared with the product of their sensitivity coefficients

First Variable	Second Variable	Product of Sensitivity Coefficients x 10^6	Effect on Mean of NPV of Total Dependence	Effect on S.D. of NPV of Total Dependence	Ratio VI	Ratio VII
Init. Mkt. Size	Mkt. Growth	53.3	+42	+178	5.7	3.3
Init. Mkt. Size	Selling Price	583.5	+1571	+1466	65.0	2.5
Init. Mkt. Size	Mkt. Share	268.7	+671	+1430	40.9	5.3
Init. Mkt. Size	Init. Invest.	52.0	-3	-116	0.4	2.2
Init. Mkt. Size	Life of Invest.	123.2	+235	+424	21.2	3.4
Init. Mkt. Size	Residual Value	8.6	+6	+15	2.0	1.7
Init. Mkt. Size	Op. Costs	537.4	-1429	-443	61.6	0.8
Init. Mkt. Size	Fixed Costs	11.4	-4	-25	1.2	2.2
Mkt. Growth	Selling Price	140.7	+1045	+1026	88.1	7.3
Mkt. Growth	Mkt. Share	64.8	+604	+914	73.0	14.1
Mkt. Growth	Init. Invest.	12.6	-47	-42	13.2	3.3
Mkt. Growth	Life of Invest.	29.7	+197	+190	36.1	6.4
Mkt. Growth	Residual Value	2.1	+11	+6	7.6	2.9
Mkt. Growth	Op. Costs	129.6	-1022	+446	89.8	3.4
Mkt. Growth	Fixed Costs	2.8	-16	-7	9.6	2.5
Selling Price	Mkt. Share	709.0	+1925	+1383	72.3	1.9
Selling Price	Init. Invest.	137.3	-1	-464	0.1	3.4
Selling Price	Life of Invest.	325.2	+879	+194	48.7	0.6
Selling Price	Residual Value	22.7	+7	+66	1.5	2.9
Selling Price	Op. Costs	1418.0	-26	-6831	0.7	4.8
Selling Price	Fixed Costs	30.1	-3	-99	0.6	3.3
Mkt. Share	Init. Invest.	63.2	+48	-96	6.0	1.5
Mkt. Share	Life of Invest.	149.7	+179	+412	14.6	2.7
Mkt. Share	Residual Value	10.4	-2	+14	0.6	1.3
Mkt. Share	Op. Costs	653.0	-1248	+18	48.8	0.0
Mkt. Share	Fixed Costs	13.9	+8	-18	2.1	1.3
Init. Invest.	Life of Invest.	29.0	+31	-69	5.8	2.4
Init. Invest.	Residual Value	2.0	+6	-9	4.2	4.5
Init. Invest.	Op. Costs	126.4	-229	+890	20.4	7.0
Init. Invest.	Fixed Costs	2.7	-2	+12	1.2	4.4
Life of Invest.	Residual Value	4.8	-6	-1	2.7	0.2
Life of Invest.	Op. Costs	299.5	-1574	+685	90.9	2.3
Life of Invest.	Fixed Costs	6.4	-37	0	14.6	0
Residual Value	Op. Costs	20.9	-136	+191	29.7	9.1
Residual Value	Fixed Costs	0.4	0	+1	0	2.5
Op. Costs	Fixed Costs	27.8	-3	+95	0.6	3.4

Ratio VI shows that the magnitude of the product of the sensitivity coefficients of two different variables provides a very poor indication as to the effect on the mean of NPV of total dependence between the variables. Ratio VII shows that it provides a slightly better - but not by no means perfect - indication as to the effect on the S.D. of NPV of total dependence between the variables.

Section 7.2 shows that in the situation where all variables are either inflows or outflows of money a sensitivity analysis does provide a good indication as to the relative importance of errors in the means and standard deviations of, and the coefficients of correlation between, the variables. This appendix has shown that it would be dangerous to extend the results in section 7.2 to deal with situations where the cash flow model is non-linear.

APPENDIX I

THE RELATIONSHIP BETWEEN THE MEAN AND STANDARD DEVIATION OF NPV AND THE MEAN AND STANDARD DEVIATION OF INDIVIDUAL VARIABLES IN CASE A

Tables I.1, I.2 and I.3 show the effect on the mean and standard deviation of NPV of different errors in the means and standard deviations of the variables in case A. (The results are based on 1000 simulation runs and all amounts are in $'000s).

Table I.1 Effect on mean of NPV of different errors in means of variables in case A.

Variable	Effect on mean of NPV of error in mean of variable equal to		
	5% of range	10% of range	15% of range
Init. Mkt. Size	349	698	1047
Mkt. Growth	83	168	254
Selling Price	1626	3252	4878
Mkt. Share	457	914	1371
Init. Invest.	-175	-350	-525
Life of Invest.	107	219	324
Residual Value	27	55	82
Op. Costs	-1498	-2995	-4493
Fixed Costs	-39	-77	-116

Table I.2 Effect on S.D. of NPV of different errors in means of variables in case A.

Variable	Effect on S.D. of NPV of error in mean of variable equal to		
	5% of range	10% of range	15% of range
Init. Mkt. Size	464	930	1397
Mkt. Growth	121	244	369
Selling Price	177	388	631
Mkt. Share	579	1162	1748
Init. Invest.	0	0	0
Life of Invest.	316	629	928
Residual Value	0	0	0
Op. Costs	-131	-229	-295
Fixed Costs	0	0	0

Table I.3 Effect on S.D. of NPV of different errors in S.D.s of variables in case A.

Variable	Effect on S.D. of NPV of error in S.D. of variable to		
	10%	30%	50%
Init. Mkt. Size	61	201	363
Mkt. Growth	8	27	49
Selling Price	505	1591	2755
Mkt. Share	89	295	535
Init. Invest.	3	12	23
Life of Invest.	0	10	23
Residual Value	1	2	3
Op. Costs	402	1280	2241
Fixed Costs	0	-1	-1

The procedure suggested in sections 7.7, 7.8 and 7.9 makes, inter alia, the following assumptions:

μ_P is linearly dependent on μ_i

σ_P is linearly dependent on μ_i

σ_P is linearly dependent on σ_i

(For the notation here, see section 7.2)

Tables I.1, I.2 and I.3 provide some empirical evidence to support these assumptions. For example:

(i) The ratio:

$$\frac{\text{Increase in } \mu_P \text{ when } \mu_i \text{ increases by 10\% of range}}{\text{Increase in } \mu_P \text{ when } \mu_i \text{ increases by 5\% of range}}$$

lies for all 9 variables between 1.97 and 2.05

(ii) The ratio:

$$\frac{\text{Increase in } \mu_P \text{ when } \mu_i \text{ increases by 15\% of range}}{\text{Increase in } \mu_P \text{ when } \mu_i \text{ increases by 5\% of range}}$$

lies for all 9 variables between 2.97 and 3.06

(iii) The ratio:

$$\frac{\text{Increase in } \sigma_P \text{ when } \mu_i \text{ increases by 10\% of range}}{\text{Increase in } \sigma_P \text{ when } \mu_i \text{ increases by 5\% of range}}$$

lies between 1.75 and 2.19 for all variables.

(iv) The ratio:

$$\frac{\text{Increase in } \sigma_P \text{ when } \mu_i \text{ increases by 15\% of range}}{\text{Increase in } \sigma_P \text{ when } \mu_i \text{ increases by 5\% of range}}$$

lies between 2.25 and 3.56 for all variables.

(v) The ratio:

$$\frac{\text{Increase in } \sigma_P \text{ when } \sigma_i \text{ increases by 30\%}}{\text{Increase in } \sigma_P \text{ when } \sigma_i \text{ increases by 10\%}}$$

lies between 3.15 and 3.31 for all variables where numerator is greater than 20.

(vi) The ratio:

$$\frac{\text{Increase in } \sigma_P \text{ when } \sigma_i \text{ increases by 30\%}}{\text{Increase in } \sigma_P \text{ when } \sigma_i \text{ increases by 50\%}}$$

lies between 0.55 and 0.58 for all variables where denominator is greater than 20.

APPENDIX J

CALCULATION OF THE POTENTIAL EFFECTS OF DIFFERENT FACTORS

This appendix outlines the way in which tables 7.9 and 7.10 were produced.

In order to produce table 7.9 it was necessary to assume that, once highest, lowest and most likely values for a distribution had been specified, the range of possibilities for the 'true' mean and the 'true' standard deviation was limited in some way. Suppose that μ_T and σ_T are the mean and standard deviation of the triangular distribution illustrated in figure J.1. Then it was assumed:

(i) that the 'true' mean of the distribution lay between the mid point of the range and the mode.

and (ii) that the 'true' standard deviation of the distribution lay between 0.5 σ_T and 1.5 σ_T

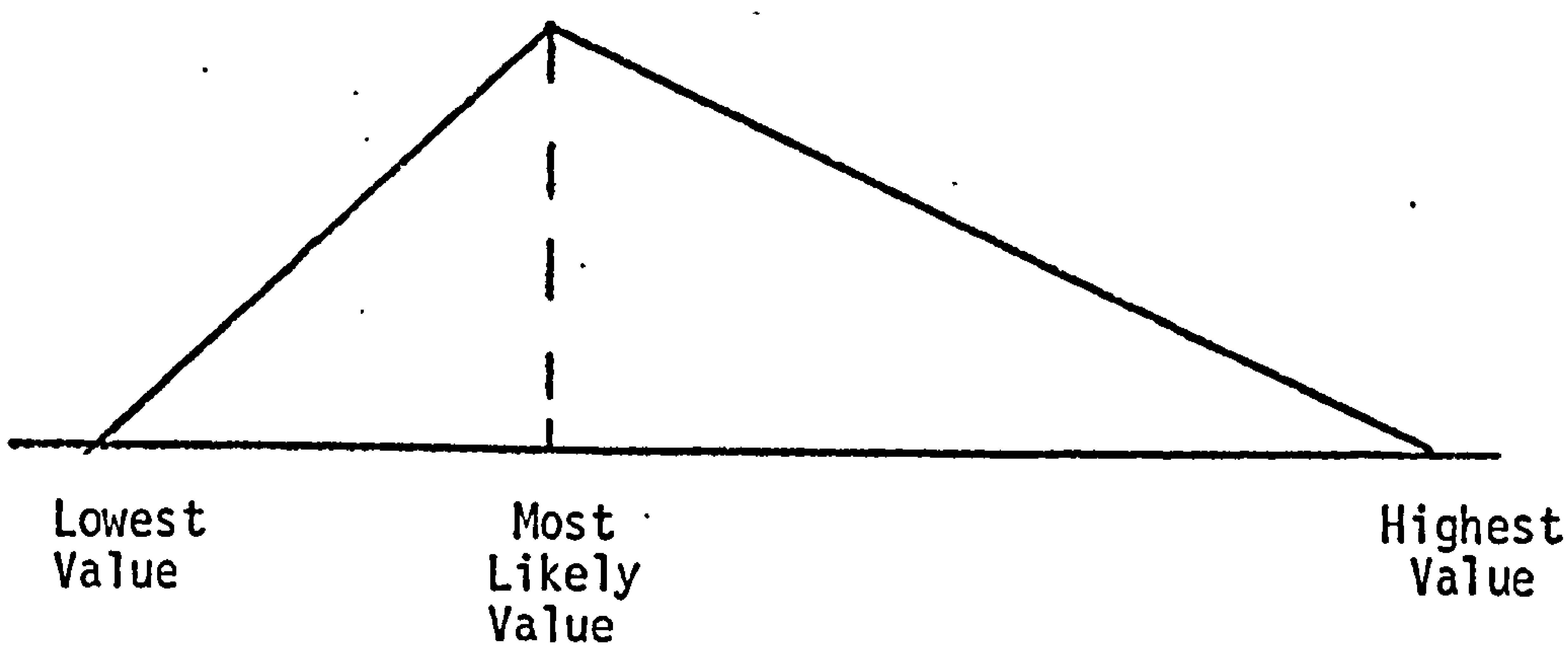

Figure J.1 Triangular distribution fitted to lowest, most likely and highest values.

The calculations necessary to produce table 7.9 will be illustrated with reference to the variable 'selling price'. The worst assumptions for this variable are:

(i) Standard deviation equals 1.5 σ_T

(ii) Mean equals mid point of range.

The mid point of the range is, for selling price less than the mean of the triangular distribution by an amount equal to 5.26% of the range. The effect of the assumptions can therefore be calculated from tables 7.7 and 7.8 as the following:

μ_P decreases by $\frac{5.26}{5.00} \times 1620 = 1704$

σ_P increases by $\frac{5.26}{5.00} \times 170 + \frac{50}{30} \times 1620 = 2521$

giving a decrease in $\mu_P -1.28\ \sigma_P$ of approximately 4900. Similarly the effect of assuming the best possible assumptions for the variable (i.e. Standard deviation equals 0.5 σ_T and mean equals most likely value) is an increase of 6400 in $\mu_P -1.28\ \sigma_P$. (These calculations assume (a) that σ_P depends linearly on σ_i and μ_i and (b) that μ_P depends linearly on μ_i but not at all on σ_i. See section 7.9 for a discussion of the assumptions.)

As far as table 6.10 is concerned extra assumptions were necessary to produce the figures corresponding to the following factors:

errors in selling price distribution

errors in operating costs distribution

selling price dependent on operating costs

For the first two factors analyses along the lines indicated in section 5.7 were carried out. As figures are not available in appendix F for situations where 4 assessments are made some interpolation was necessary. The general procedure will be illustrated with reference to 'selling price'. The skewness of this variable is 0.16. Tables F.15 and F.16 do therefore suggest that with 3 assessments and accuracy parameter equal to 0.01

Max. error in mean = 0.3% of range

Max. error in S.D. = 31.3% (Sig. Level = 0.05)

Tables F.17 and F.18 suggest that with 7 assessments and accuracy parameters equal to 0.01.

Max. error in mean = 0.3% of range

Max. error in S.D. = 11.1% (Sig. Level = 0.05)

With 4 assessments and an accuracy parameter of 0.01 the following assumptions were therefore made for the variable 'selling price'.

Max. error in mean = C.3% of range

Max. error in S.D. = 26.2%

These assumptions led, using tables 7.7 and 7.8 in the same way as above, to the figures shown in table 7.10.

For the last factor (i.e. the dependence between selling price and operating costs) it was assumed that the coefficient of correlation had been determined to ± 0.15 and that μ_P and σ_P were approximately linearly dependent on ρ. In view of the results in section 6.8, the first assumption is not unreasonable. The second assumption is discussed in section 7.9.